Foundations of Analog and Digital Electronic Circuits

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ANALYSIS OF NONLINEAR CIRCUITS

4

Thus far we have discussed a variety of circuits containing linear devices such as resistors and voltage sources. We have also discussed methods of analyzing linear circuits built out of these elements. In this chapter, we extend our repertoire of network elements and corresponding analysis techniques by introducing a nonlinear two-terminal device called a *nonlinear resistor*. Recall, from Section 1.5.2, a nonlinear resistor is an element that has a nonlinear, algebraic relation between its instantaneous terminal current and its instantaneous terminal voltage. A diode is an example of a device that behaves like a nonlinear resistor. In this chapter, we will introduce methods of analyzing general circuits containing nonlinear elements, trying whenever possible to use analysis methods already introduced in the preceding chapters. Chapter 7 will develop further the basic ideas on nonlinear analysis and Chapter 8 will expand on the concept of incremental analysis introduced in this chapter. Chapter 16 will elaborate on diodes.

4.1 INTRODUCTION TO NONLINEAR FLEMENTS

Before we begin our analysis of nonlinear resistors, we will describe as examples several nonlinear resistive devices, by their v-i characteristics, just as we did for the resistor, the battery, etc. The first of the nonlinear devices that we discuss is the diode. Figure 4.1 shows the symbol for a diode. The diode is a two-terminal, nonlinear resistor whose current is exponentially related to the voltage across its terminals.

An analytical expression for the nonlinear relation between the voltage v_D and the current i_D for the diode is the following:

$$i_{\rm D} = I_{\rm s}(e^{\nu_{\rm D}/V_{TH}} - 1).$$
 (4.1)

For silicon diodes the constant I_s is typically 10^{-12} A and the constant V_{TH} is typically 0.025 V. This function is plotted in Figure 4.2.

An analytical expression for the relationship between voltage v_H and current i_H for another hypothetical nonlinear device is shown in Equation 4.2. In the equation, I_K is a constant. The relationship is plotted in Figure 4.3.

$$i_H = I_K \nu_H^3. \tag{4.2}$$

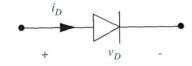


FIGURE 4.1 The symbol for a diode.

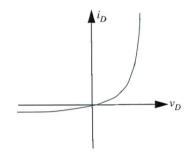


FIGURE 4.2 *v*–*i* characteristics of a silicon diode.

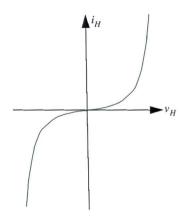


FIGURE 4.3 Another nonlinear *v*–*i* characteristics.

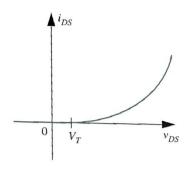


FIGURE 4.4 The *v*-*i* characteristics for a square law device.

The v-i relationship for yet another two-terminal nonlinear device is shown in Equation 4.3. Figure 8.11 in Chapter 8 introduces such a nonlinear device. For this device the current is related to the square of the terminal voltage. In this equation, K and V_T are constants. The variables i_{DS} and v_{DS} are the terminal variables for the device. The relationship is plotted in Figure 4.4.

$$i_{DS} = \begin{cases} \frac{K(v_{DS} - V_T)^2}{2} & \text{for } v_{DS} \ge V_T \\ 0 & \text{for } v_{DS} < V_T \end{cases}$$
 (4.3)

EXAMPLE 4.1 SQUARE LAW DEVICE For the nonlinear resistor device following the square law in Figure 4.4, determine the value of i_{DS} for $v_{DS} = 2$ V. We are given that $V_T = 1$ V and K = 4 mA/V².

For the parameters that we have been given ($v_{DS} = 2 \text{ V}$ and $V_T = 1 \text{ V}$), it is easy to see that

$$\nu_{DS} \geq V_T$$
.

From Equation 4.3, the expression for i_{DS} when $v_{DS} \ge V_T$ is

$$i_{DS} = \frac{K(\nu_{DS} - V_T)^2}{2}.$$

Substituting the known numerical values,

$$i_{DS} = \frac{4 \times 10^{-3} (2-1)^2}{2} = 2 \text{ mA}.$$

How does i_{DS} change if v_{DS} is doubled?

If v_{DS} is doubled to 4 V,

$$i_{DS} = \frac{K(\nu_{DS} - V_T)^2}{2} = \frac{4 \times 10^{-3} (4 - 1)^2}{2} = 18 \text{ mA}$$

In other words, i_{DS} increases to 18 mA when v_{DS} is doubled.

What is the value of i_{DS} if v_{DS} is changed to 0.5 V?

For $v_{DS} = 0.5 \text{ V}$ and $V_T = 1 \text{ V}$,

$$v_{DS} < V_T$$
.

From Equation 4.3, we get

$$i_{DS}=0.$$

When operating within some circuit, the current through our square law device is measured to be 4 mA. What must be the voltage across the device?

We are given that $i_{DS} = 4$ mA. Since there is a current through the device, the equation that applies is

$$i_{DS} = \frac{K(v_{DS} - V_T)^2}{2}.$$

Substituting known values,

$$8 \times 10^{-3} = \frac{4 \times 10^{-3} (\nu_{DS} - 1)^2}{2}.$$

Solving for v_{DS} , we get

$$v_{DS} = 3 \text{ V}.$$

EXAMPLE 4.2 DIODE EXAMPLE For the diode shown in Figure 4.1, determine the value of i_D for $\nu_D=0.5$ V, 0.6 V, and 0.7 V. We are given that $V_{TH}=0.025$ V and $I_5=1$ pA.

From the device law for a diode given in Equation 4.1, the expression for i_D is

$$i_D = I_s(e^{\nu_D/V_{TH}} - 1).$$

Substituting the known numerical values for $v_D = 0.5 \text{ V}$, we get

$$i_D = 1 \times 10^{-12} (e^{0.5/0.025} - 1) = 0.49 \text{ mA}.$$

Similarly, for $v_D=0.6$ V, $i_D=26$ mA, and for $v_D=0.7$ V, $i_D=1450$ mA. Notice the dramatic increase in current as v_D increases beyond 0.6 V.

What is the value of i_D if v_D is -0.2 V?

$$i_D = I_s(e^{\nu_D/V_{TH}} - 1) = 1 \times 10^{-12}(e^{-0.2/0.025} - 1) = -0.9997 \times 10^{-12} \text{A}.$$

The negative sign for i_D simply reflects the fact that when v_D is negative, so is the current.

When operating within some circuit, the current through the diode is measured to be 8 mA. What must be the voltage across the diode?

We are given that $i_D = 8$ mA. Using the diode equation, we get

$$8 \times 10^{-3} = I_s(e^{\nu_D/V_{TH}} - 1) = 1 \times 10^{-12}(e^{\nu_D/0.025} - 1).$$

Simplifying, we get

$$e^{\nu_D/0.025} = 8 \times 10^9 + 1.$$

Taking logs on both sides, and solving for ν_D , we get

$$v_D = 0.025 \ln(8 \times 10^9 + 1) = 0.57 \text{ V}.$$

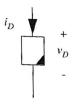


FIGURE 4.5 A nonlinear device.

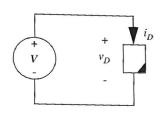


FIGURE 4.6 A circuit containing the nonlinear device.

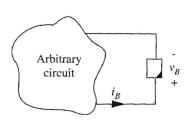


FIGURE 4.7 The nonlinear device connected to an arbitrary circuit.

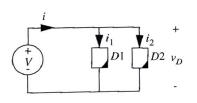


FIGURE 4.8 Nonlinear devices connected in parallel.

EXAMPLE 4.3 ANOTHER SQUARE LAW DEVICE PROBLEM
The nonlinear device shown in Figure 4.5 is characterized by this device equation:

$$i_D = 0.1 v_D^2 \quad \text{for } v_D \ge 0,$$
 (4.4)

 i_D is given to be 0 for $v_D < 0$.

Given that V = 2 V, determine i_D for the circuit in Figure 4.6.

Using the device equation for $v_D \ge 0$,

$$i_D = 0.1 \nu_D^2 = 0.1 \times 2^2 = 0.4 \text{ A}$$
 (4.5)

The nonlinear device is connected to some arbitrary circuit as shown in Figure 4.7. Following the associated variables discipline, the branch variables v_B and i_B for the device are defined as shown in the same figure. Suppose that a measurement reveals that $i_B = -1$ mA. What must be the value of v_B ?

Notice that the polarity of the branch variables has been reversed in Figure 4.7 from those in Figure 4.5. With this definition of the branch variables, the device equation becomes

$$-i_B = 0.1\nu_B^2 \quad \text{for } \nu_B \le 0.$$
 (4.6)

Furthermore, i_B is 0 for $v_B > 0$.

Given that $i_B = -1$ mA, Equation 4.6 yields

$$-(-1 \times 10^{-3}) = 0.1\nu_B^2$$
 where $\nu_B \le 0$.

In other words, $v_B = -0.1 \text{ V}$.

Given that V = 2 V, determine *i* for the circuit in Figure 4.8.

Since the voltage across each of the nonlinear devices connected in parallel is $v_D = 2 \text{ V}$, the current through each nonlinear device is the same as that calculated in Equation 4.5. In other words,

$$i_1 = i_2 = 0.4 \text{ A}.$$

Therefore, $i = i_1 + i_2 = 0.8$ A.

Given the analytical expression for the characteristic of a nonlinear device, such as that for the diode in Equation 4.1, how can we calculate the voltages and currents in a simple circuit such as Figure 4.9? In the following sections we will discuss four methods for solving such nonlinear circuits:

- Analytical solutions
- 2. Graphical analysis
- 3. Piecewise linear analysis
- 4. Incremental or small signal analysis

42 ANALYTICAL SOLUTIONS

We first try to solve the simple nonlinear resistor circuit in Figure 4.9 by analytical methods. Assume that the hypothetical nonlinear resistor in the figure is characterized by the following v-i relationship:

$$i_D = \begin{cases} K \nu_D^2 & \text{for } \nu_D > 0\\ 0 & \text{for } \nu_D \le 0. \end{cases}$$

$$(4.7)$$

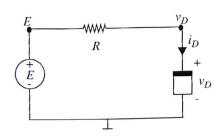
The constant *K* is positive.

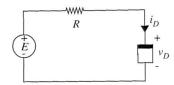
This circuit is amenable to a straightforward application of the node method. Recall that the node method and its foundational Kirchhoff's voltage and current laws are derived from Maxwell's Equations with no assumptions about linearity. (Note, however, that the superposition method, the Thévenin method, and the Norton method do require a linearity assumption.)

To apply the node method, we first choose a ground node and label the node voltages as illustrated in Figure 4.10. v_D is our only unknown node voltage.

Next, following the node method, we write KCL for the node that has an unknown node voltage. As prescribed by the node method, we will use KVL and the device relation ($i_D = Kv_D^2$) to obtain the currents directly in terms of the node voltage differences and element parameters. For the node with voltage v_D ,

$$\frac{\nu_D - E}{R} + i_D = 0 {(4.8)}$$





Nonlinear resistor $\downarrow^{i_{D}} + i_{D} = Kv_{D}^{2} \text{ for } v_{D} > 0$ $\downarrow^{i_{D}} + i_{D} = Kv_{D}^{2} \text{ for } v_{D} > 0$ otherwise

FIGURE 4.9 A simple circuit with a nonlinear resistor.

FIGURE 4.10 The nonlinear circuit with the ground node chosen and node voltages labeled.

Note that this is not quite our node equation, because of the presence of the i_D term. To get the node equation we need to substitute for i_D in terms of node voltages. Recall that the nonlinear device v-i relation is

$$i_{\rm D} = K v_{\rm D}^2. \tag{4.9}$$

Note that this device equation applies for positive v_D . We are given that $i_D = 0$ when $v_D \le 0$.

Substituting the nonlinear device v-i relationship for i_D in Equation 4.8, we get the required node equation in terms of the node voltages:

$$\frac{\nu_D - E}{R} + K\nu_D^2 = 0. {(4.10)}$$

For our device, note that Equation 4.9 holds only for $v_D > 0$. For $v_D \le 0$, i_D is 0.

Simplifying Equation 4.10, we obtain the following quadratic equation.

$$RK\nu_D^2 + \nu_D - E = 0.$$

Solving for v_D and choosing the positive solution

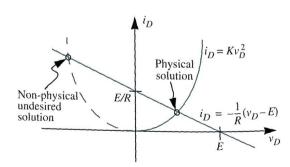
$$v_D = \frac{-1 + \sqrt{1 + 4RKE}}{2RK}. (4.11)$$

The corresponding expression for i_D can be obtained by substituting the previous expression for v_D into Equation 4.9 as follows:

$$i_D = K \left[\frac{-1 + \sqrt{1 + 4RKE}}{2RK} \right]^2$$
 (4.12)

It is worth discussing why we ignored the negative solution. As shown in Figure 4.11, two *mathematical* solutions are possible when we solve Equations 4.10 and 4.9. However, the dotted curve in Figure 4.11 is part of

FIGURE 4.11 Solutions to equations Equations 4.10 and 4.9.



Equation 4.9 but not the *physical* device. Because, recall, Equation 4.9 applies only for positive v_D . When E is negative, i_D will be equal to 0 and v_D will be equal to E.

EXAMPLE 4.4 ONE NONLINEAR DEVICE, SEVERAL SOURCES, AND RESISTORS Shown in Figure 4.12 is a circuit of no obvious value, which we use to illustrate how to solve nonlinear circuits with more than one source present, using the nonlinear analysis method just discussed. Let us assume that we wish to calculate the nonlinear device current *i*_D.

Assume that the nonlinear device is characterized by the following v-i relationship:

$$i_D = \begin{cases} K \nu_D^2 & \text{for } \nu_D > 0\\ 0 & \text{for } \nu_D \le 0. \end{cases}$$
 (4.13)

The terminal variables for the nonlinear device are defined as shown in Figure 4.9, and the constant *K* is positive.

Linear analysis techniques such as superposition cannot be applied to the whole circuit because of the nonlinear element. But because there is only one nonlinear device, it is permissible to find the Thévenin (or Norton) equivalent circuit *faced by the nonlinear device* (see Figures 4.13a and b), because this part of the circuit is linear. Then we can compute easily the terminal voltage and current for the nonlinear device using the circuit in Figure 4.13b from Equations 4.11 and 4.12.

First, to find the open-circuit voltage, we draw the linear circuit as seen from the nonlinear device terminals in Figure 4.13c. Superposition or any other linear analysis method can now be used to calculate the open-circuit voltage:

$$V_{TH} = V \frac{R_2}{R_1 + R_2} - I_0 R_3. (4.14)$$

The Thévenin equivalent resistance, R_{TH} , the resistance seen at the terminals in Figure 4.13d, with the sources set to zero is

$$R_{TH} = (R_1||R_2) + R_3. (4.15)$$

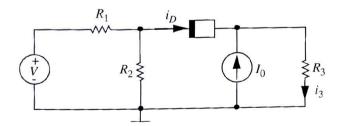


FIGURE 4.12 Circuit with several sources and resistors.

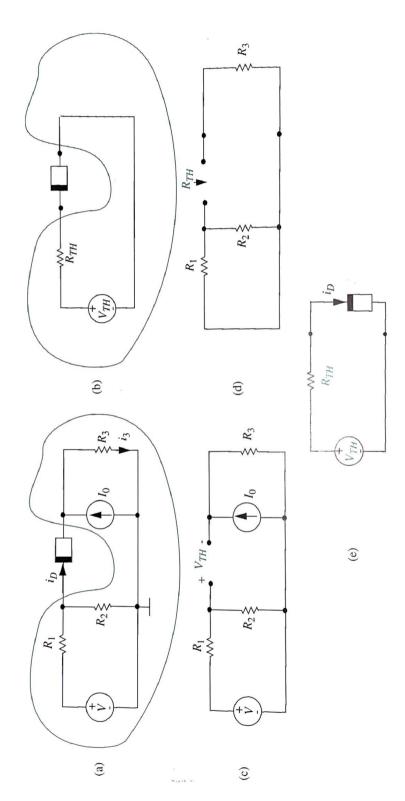


FIGURE 4.13 Analysis using Thévenin's Theorem.

When we reconnect the nonlinear device to this Thévenin circuit, as in Figure 4.13e, we are back to a familiar example: one nonlinear device, one source, and one resistor. The desired device current i_D can be found by a nonlinear analysis method, such as that used to solve the circuit in Figure 4.9.

One further comment: If in the problem statement we had been asked to find one of the resistor currents, say i_3 , rather than i_D , then the Thévenin circuit, Figure 4.13e, would not give this current directly, because the identity of currents internal to the Thévenin network are in general lost, as noted in Chapter 3. Nonetheless, the Thévenin approach is probably the best, as it is a simple matter to work back through the linear part of the network to relate i_3 to i_D . In this case, once we have computed i_D , we can easily determine i_3 from Figure 4.13a using KCL,

$$i_3 = i_D + I_0. (4.16)$$

www EXAMPLE 4.5 NODE METHOD

EXAMPLE 4.6 ANOTHER SIMPLE NONLINEAR CIRCUIT Let us try to solve the nonlinear circuit containing a diode in Figure 4.16 by analytical methods. Following the node method, we first choose the ground node and label the node voltages as illustrated in Figure 4.17.

Next, we write KCL for the node with the unknown node voltage, and substitute for the diode current using the diode equation

$$\frac{\nu_D - E}{R} + i_D = 0 {(4.18)}$$

$$i_D = I_s(e^{\nu_D/V_{TH}} - 1).$$
 (4.19)

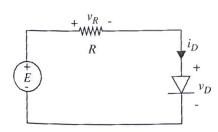


FIGURE 4.16 A simple non-linear circuit containing a diode.

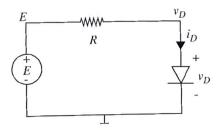


FIGURE 4.17 The circuit with the ground node and the node voltages marked.

If i_D is eliminated by substituting Equation 4.19 into Equation 4.18, the following transcendental equation results:

$$\frac{\nu_D - E}{R} + I_s(e^{\nu_D/V_{TH}} - 1) = 0.$$

This equation must be solved by trial and error. Easy via computer, but n_{ot} very insightful.

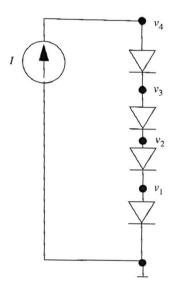


FIGURE 4.18 Series connected diodes.

EXAMPLE 4.7 SERIES-CONNECTED DIODES Referring to the series-connected diodes in Figure 4.18, determine v_1 , v_2 , v_3 , and v_4 , given that I = 2 A. The parameters in the diode relation are given to be $I_s = 10^{-12}$ A, $V_{TH} = 0.025$ V.

We will first use the node method to solve this problem. Figure 4.18 shows the ground node and the node voltages. There are four unknown node voltages. Next, we write KCL for each of the nodes. As prescribed by the node method, we will use KVL and the diode relation (Equation 4.1) to obtain the currents directly in terms of the node voltage differences and element parameters. For the node with voltage v_1 ,

$$10^{-12}(e^{\nu_1/0.025} - 1) = 10^{-12}(e^{(\nu_2 - \nu_1)/0.025} - 1).$$
(4.20)

The term on the left-hand side is the current through the lowermost device expressed in terms of node voltages. Similarly, the term on the right-hand side is the current through the device that is second from the bottom.

Similarly, we can write the node equations for the nodes with voltages v_2 , v_3 , and v_4 as follows:

$$10^{-12} (e^{(\nu_2 - \nu_1)/0.025} - 1) = 10^{-12} (e^{(\nu_3 - \nu_2)/0.025} - 1)$$
(4.21)

$$10^{-12} (e^{(\nu_3 - \nu_2)/0.025} - 1) = 10^{-12} (e^{(\nu_4 - \nu_3)/0.025} - 1)$$
(4.22)

$$10^{-12}(e^{(\nu_4-\nu_3)/0.025}-1)=I. (4.23)$$

Simplifying, and taking the log on both sides of Equations 4.20 through 4.23, we get

$$\nu_1 = \nu_2 - \nu_1 \tag{4.24}$$

$$\nu_2 - \nu_1 = \nu_3 - \nu_2 \tag{4.25}$$

$$\nu_3 - \nu_2 = \nu_4 - \nu_3 \tag{4.26}$$

$$v_4 - v_3 = 0.025 \ln(10^{12}I + 1).$$
 (4.27)

Given that I = 2 A, we can solve for v_1 , v_2 , v_3 , and v_4 , to get

$$v_1 = 0.025 \ln(10^{12}I + 1) = 0.025 \ln(10^{12} \times 2 + 1) = 0.71 \text{ V}$$

$$v_2 = 2v_1 = 1.42 \text{ V}$$

$$v_3 = 3v_1 = 2.13 \text{ V}$$

$$v_4 = 4v_1 = 2.84 \text{ V}.$$

Notice that we could have also solved the circuit intuitively by observing that the same 2-A current flows through each of the four identical diodes. Thus, the same voltage must drop across each of the diodes. In other words,

$$I = 10^{-12} (e^{\nu_1/0.025} - 1)$$

or,

$$v_1 = 0.025 \ln(10^{12}I + 1).$$

For I = 2 A,

$$v_1 = 0.025 \ln(10^{12} \times 2A + 1) = 0.71 \text{ V}.$$

Once the value of v_1 is known, we can easily compute the rest of the node voltages from

$$v_1 = v_2 - v_1 = v_3 - v_2 = v_4 - v_3$$
.

www example 4.8 Maring simplifying assumptions

www EXAMPLE 4.9 VOLTAGE-CONTROLLED NONLINEAR RESISTOR

4.3 GRAPHICAL ANALYSIS

Unfortunately, the preceding examples are a rather special case. There are many nonlinear circuits that cannot be solved analytically. The simple circuit in Figure 4.16 is one such example. Usually we must resort to trial-and-error solutions on a computer. Such solutions provide answers, but usually give little insight about circuit performance and design. Graphical solutions, on the other hand, provide insight at the expense of accuracy. So let us re-examine the circuit in Figure 4.16 with a graphical solution in mind. For concreteness, we will assume that E=3 V and R=500 Ω , and that we are required to determine v_D , i_D , and v_R .

We have already found the two simultaneous equations, Equations 4.18 and 4.19, that describe the circuit. For convenience, let us rewrite these equations here after moving a few terms around:

$$i_{\rm D} = -\frac{\nu_{\rm D} - E}{R} \tag{4.31}$$

$$i_D = I_s(e^{\nu_D/V_{TH}} - 1).$$
 (4.32)

To solve these expressions graphically, we plot both on the same coordinates and find the point of intersection. Because we are assuming that we have a graph of the nonlinear function, in this case Figure 4.2, the simplest course of action is to plot the linear expression, Equation 4.31, on this graph, as shown in Figure 4.20. The linear constraint of Equation 4.31 is usually called a "load line" for historical reasons arising from amplifier design (as we will see in Chapter 7).

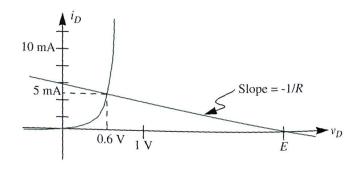
Equation 4.31 plots as a straight line of slope -1/R intersecting the v_D axis, $(i_D = 0)$ at $v_D = E$. (The negative sign may be a bit distressing, but does not represent a negative resistance, just the fact that i_D and v_D are not associated variables for the resistors.) For the particular values in this circuit, the graph indicates that i_D must be about 5 mA, and v_D , about 0.6 V. Once we know that i_D is 5 mA, it immediately follows that

$$v_R = i_D R = 5 \times 10^{-3} \times 500 = 2.5 \text{ V}.$$

It is easy to see from the construction that if E were made three times as large, the voltage across the diode would increase by only a small amount, perhaps to about 0.65 V. This illustrates the kind of insight available from graphical analysis.

The graphical method described here is really more general than it might at first appear. For circuits containing many resistors and sources, but only one nonlinear element, the rest of the circuit, exclusive of the one nonlinear element, is by definition linear. Hence, as described previously in Example 4.4, regardless of circuit complexity we can reduce the circuit to the form in Figure 4.16 by

FIGURE 4.20 Graphical solution for diode circuit. The graph assumes that E=3 V and R=500 Ω .



the application of Thévenin's Theorem to the linear circuit facing the nonlinear element.

For circuits with two nonlinear elements, the method is less useful, as it involves sketching one nonlinear characteristic on another. Nonetheless, crude sketches can still provide much insight.

EXAMPLE 4.10 HALF-WAVE RECTIFIER Let us carry the dioderesistor example of Figure 4.16 and Figure 4.20 a step further, and allow the driving voltage to be a sinusoid rather than DC. That is, let $v_I = E_0 \cos(\omega t)$. Also, for reasons that will become evident, let us calculate the voltage across the resistor rather than the diode voltage. The graphical solution is no different than before, except that now we must solve for the voltage assuming a succession of values of v_I , and visualize how the resultant time waveform should appear.

The circuit now looks like Figure 4.21a. The diode characteristic with a number of different plots of Equation 4.31 (or load lines), corresponding to a representative set of values of v_I , is shown in Figure 4.21b. In Figures 4.21c and 4.21d we show the input sinusoid $v_I(t)$, and the corresponding succession of values of $v_O(t)$ derived from the graphical analysis in Figure 4.21b. Note from Figure 4.21a (or Equation 4.31) that

$$v_{O} = v_{I} - v_{D}$$

$$v_{I} = E_{o} \cos(\omega t) + v_{D}$$

$$v_{I} = V_{O} + v_{D$$

FIGURE 4.21 Half-wave rectifier.

and thus in the graph v_O is the horizontal distance from the load line intersection on the v_D axis to v_I .

A number of interesting conclusions can be drawn from this simple example. First, we really do not have to repeat the load line construction fifty times to visualize the output wave. It is clear from the graph that whenever the input voltage is negative, the diode current is so small that v_O is almost zero. Also, for large positive values of v_I , the diode voltage stays relatively constant at about 0.6 volts (due to the nature of the exponential), so the voltage across the resistor will be approximately $v_I - 0.6 \text{ V}$. This kind of insight is the principal value of the graphical method.

Second, in contrast to all previous examples, the output waveform in this circuit is a gross distortion of the input waveform. Note in particular that the input voltage waveform has no average value, (no DC value), whereas the output has a significant DC component, roughly $0.3\ E_o$. The DC motors in most toys, for example, will run nicely if connected across the resistor in the circuit of Figure 4.21a, whereas they will not run if driven directly by the sinusoid $v_l(t)$. This circuit is called a half-wave rectifier, because it reproduces only half of the input wave. Rectifiers are present in power supplies of most electronic equipment to generate DC from the 60-Hz "sinusoidal" wave from 110-V AC power line.

4.4 PIECEWISE LINEAR ANALYSIS

In the third of the four major methods of analysis for networks containing nonlinear elements, we represent the nonlinear v-i characteristics of each nonlinear element by a succession of straight-line segments, then make calculations within each straight-line segment using the linear analysis tools already developed. This is called *piecewise linear analysis*. We will first illustrate piecewise linear analysis by using as an example a very simple piecewise linear model for the diode called *the ideal diode model*.

First, let us develop a simple piecewise linear model for the diode, and then use the piecewise linear method to analyze the circuit in Figure 4.16.

As can be seen from Figure 4.22a, the essential property of a diode is that for an applied positive voltage v_D in excess of 0.6 volts, large amounts of current

FIGURE 4.22 *v-i* characteristics of a silicon diode plotted using different scales.

