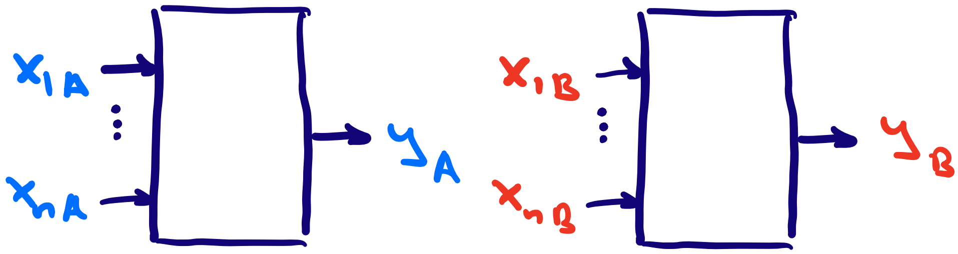
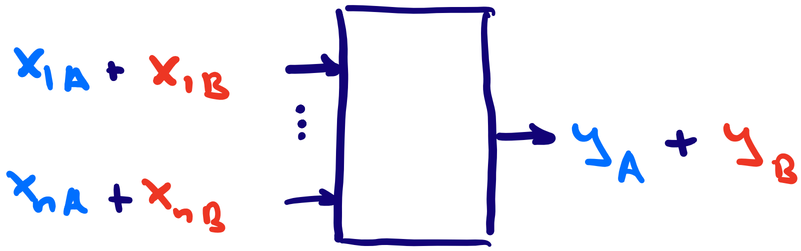
Recall: Linear systems have important properties including

Superposition: If and



then

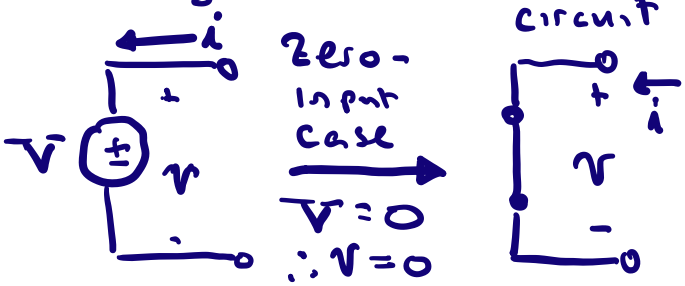


A sum of inputs yields a response that is the sum of the responses to the individual inputs

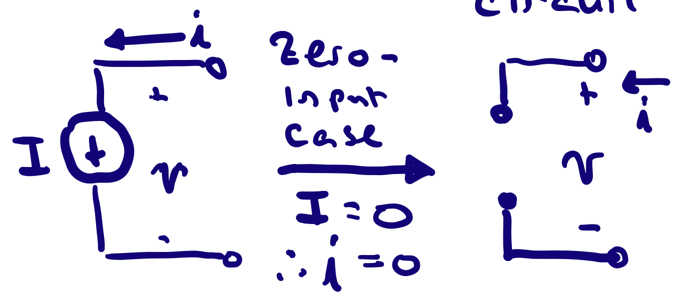
For circuits, our inputs are independent voltage + current sources, and our outputs are the component voltages and currents.

For linear circuits, the total response with all inputs can be found as the sum of the responses to individual inputs with other inputs set to zero (by superposition)

Voltage source → Short circuit

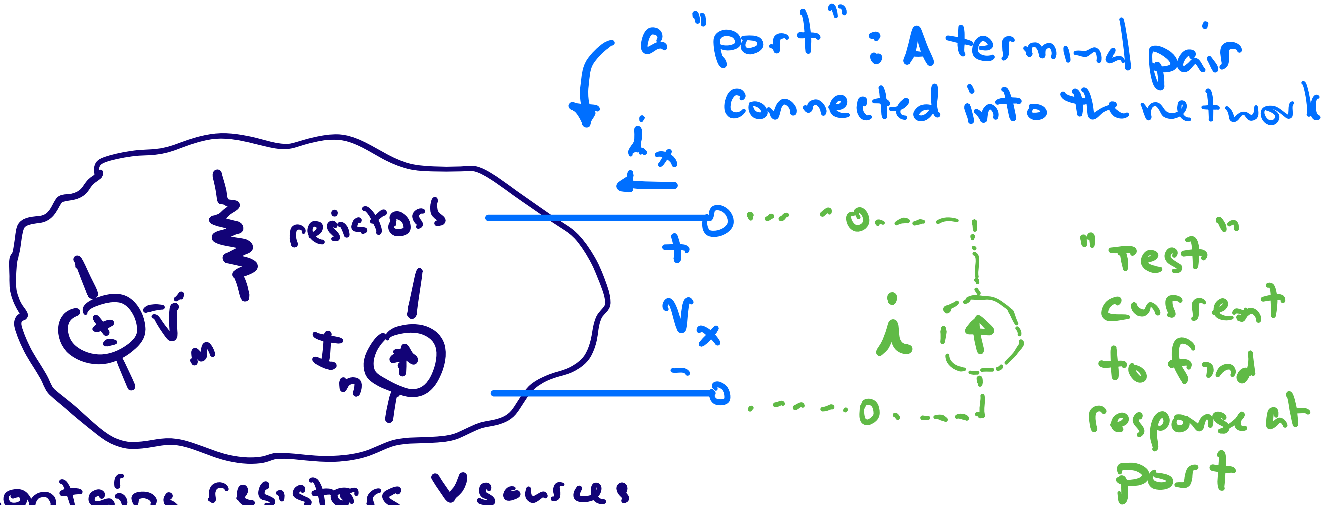


Current source → open circuit



Superposition enables a powerful modeling technique for linear circuits: The Thévenin equivalent circuit

Consider an arbitrary "linear resistor plus source" network



Contains resistors, V sources and I sources

By superposition

$$v_x = \left\{ \sum_m \alpha_m \cdot V_m + \sum_n \beta_n I_n \right\} + R_{Th} \cdot i$$

α_m: unitless β_n: units of resistance R_{Th}: units of resistance

Response at port with $i = 0$ ($\therefore i_x = 0$).

This is the "open circuit" voltage V_{Th} found when $i_x = 0$.

(The voltage difference between any terminal pair is a difference between node voltages $= k \cdot e = k \cdot G^{-1} S$.)
(v_x response from internal sources only)

R_{Th}: equivalent resistance looking into terminal pair with all independent sources inside network "killed"

$\therefore$ all V_m's $= 0$ "shorted"

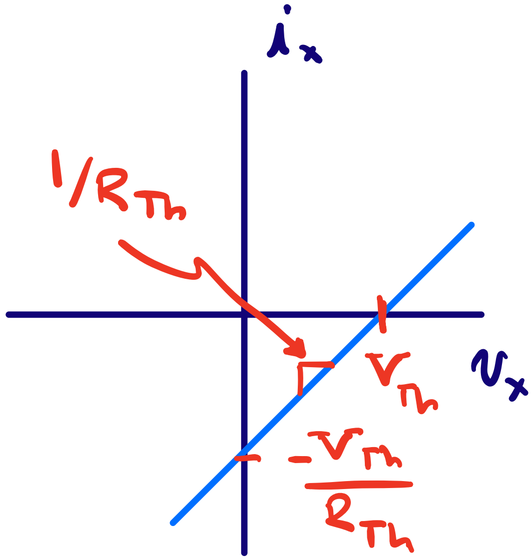
all I_n's $= 0$ "opened"

(v_x from external i only)

By superposition, the response at the terminal pair must have the form:

$$v_x = V_{Th} + R_{Th} \cdot i_x$$

So what is the i-v characteristic of our <u>arbitrary</u> linear resistor + sources network?



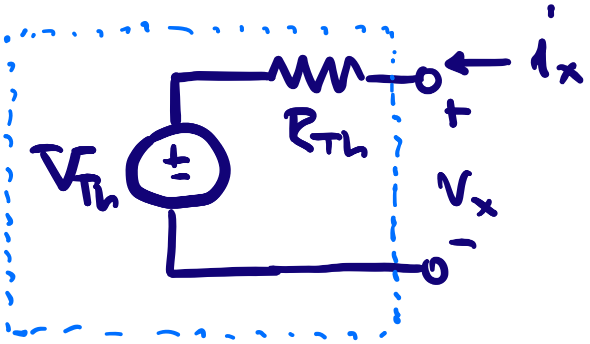
Open-circuit voltage ($i_x = 0$)

$$v_x = \sum_m \alpha_m V_m + \sum_n \beta_n I_n = V_{Th}$$

Short-circuit current ($v_x = 0$)

$$i_x = -\frac{V_{Th}}{R_{Th}}$$

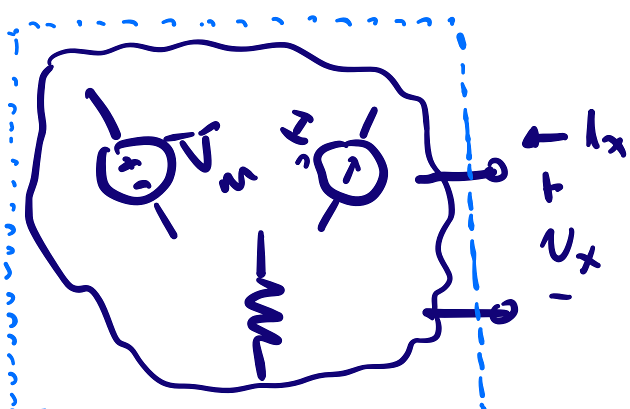
The terminal characteristics i_x-v_x of our arbitrary network are the same as:



where

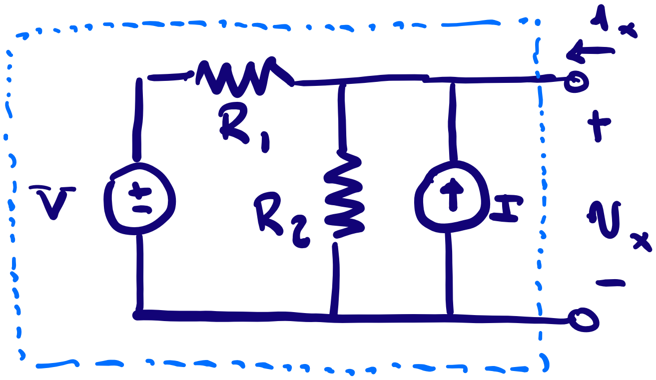
1. V_{Th} is the voltage at v_x with nothing connected externally ($i_x = 0$)
2. R_{Th} is the resistance looking into the i_x-v_x "port" with all internal independent sources "killed" (set to zero), i.e.
   I_n's $= 0$ "opened"
   V_m's $= 0$ "shorted"

same behavior as

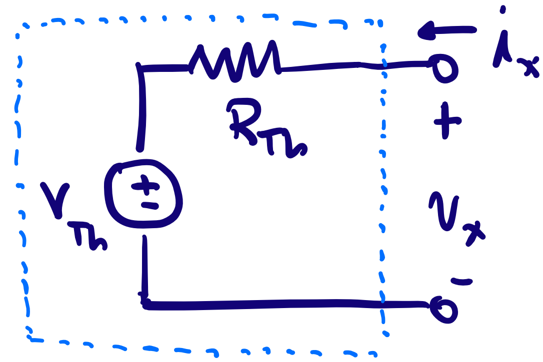


This V_{Th}, R_{Th} model for a circuit is called a "Thévenin equivalent circuit"

Example: Suppose we will make connections to the following circuit and would like a simpler model for its behavior:

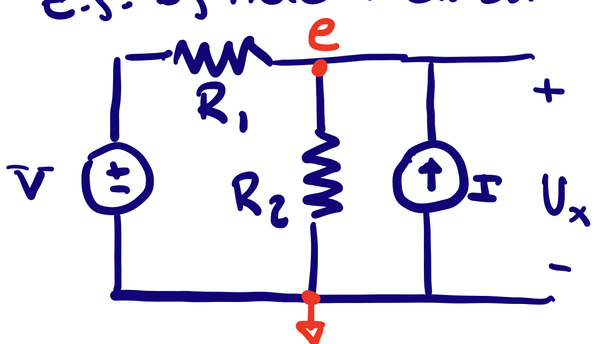


we can model as



• What is the Thévenin voltage V_{Th}? It is the open-circuit voltage v_x with $i_x = 0$.

e.g. by node method



$$(V - e)\frac{1}{R_1} - e\frac{1}{R_2} + I = 0$$

$$e\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{1}{R_1}V + I$$

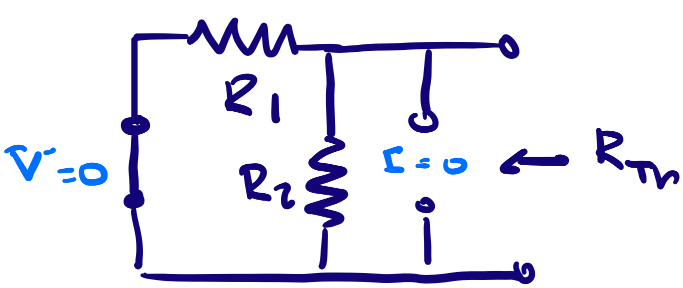
$$e[R_1 + R_2] = R_2 V + R_1 R_2 I$$

$$e = \frac{R_2}{R_1+R_2}V + \frac{R_1 R_2}{R_1+R_2} I$$

$$\therefore \boxed{V_{Th} = \frac{R_1 R_2}{R_1+R_2} I + \frac{R_2}{R_1+R_2} \cdot V}$$

(Thévenin equiv. voltage)

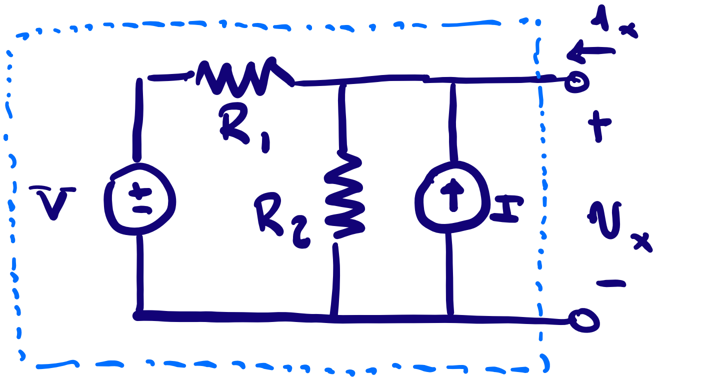
• The Thévenin equivalent resistance R_{Th} of our model is the input resistance with sources "killed"

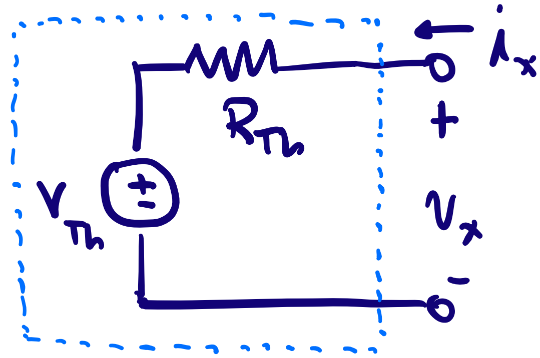


$$\boxed{R_{Th} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}}$$

(Thévenin equiv. current)

So we can model as

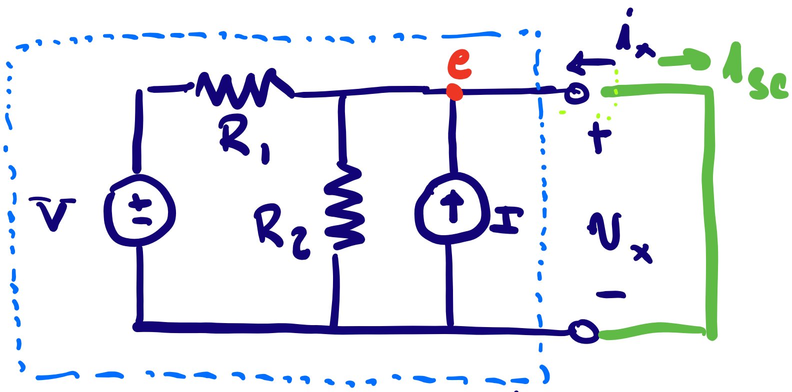




where $R_{Th} = \frac{R_1 R_2}{R_1+R_2}$ $\quad V_{Th} = \frac{R_2}{R_1+R_2}V + \frac{R_1R_2}{R_1+R_2} I$

Both networks have the same v_x - i_x terminal relationship: electrically they do the exact same thing at the terminals.

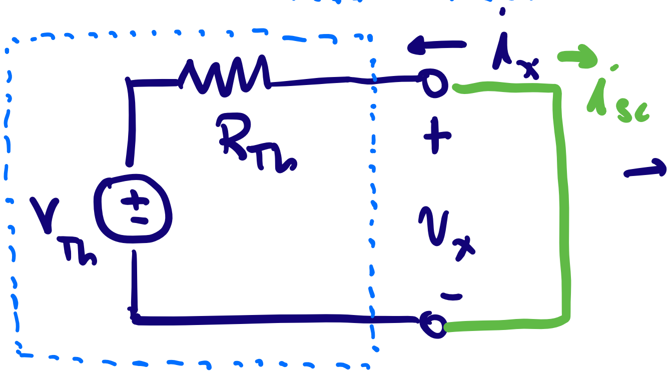
Let's check this: Suppose we impose $v_x = 0$ (e.g. by putting a short circuit at the terminals) and look at the resulting current: ($i_{sc} = -i_x$)



KCL @ node e, where $e = v_x = 0$

$$\frac{V}{R_1} - \frac{0}{R_2} + I - i_{sc} = 0$$

$$\boxed{i_{sc} = \frac{V}{R_1} + I}$$



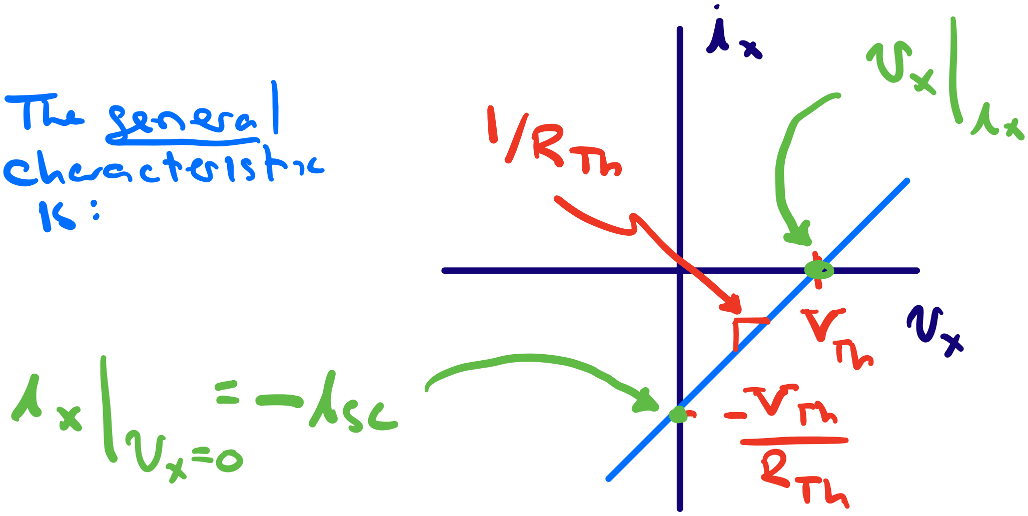
$$\rightarrow i_{sc} = \frac{V_{Th}}{R_{Th}}$$

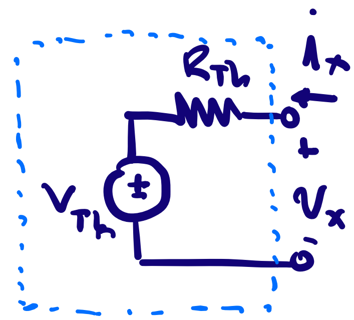
$$= \frac{\frac{R_2}{R_1+R_2}V + \frac{R_1R_2}{R_1+R_2}I}{\left(\frac{R_1R_2}{R_1+R_2}\right)} = \frac{V}{R_1} + I \quad ✓$$

we get the same val. of i_{sc}!

Another check: Lets set $i_x = 0$ (open circuit):
@ $i_x = 0$, open circuit $V_x = V_{oc} = V_{Th}$ in both circuits

The general characteristic is:





$$R_{Th} = \frac{\Delta V}{\Delta i} = \frac{V_x\big|_{i_x=0}}{-i_x\big|_{V_x=0}} = \frac{V_{oc}}{i_{sc}}$$
$\leftarrow$ open ckt voltage
$\leftarrow$ short ckt current

So in principle we could put an open circuit at the terminals and measure the voltage ($V_{oc} = V_{Th}$), put a short circuit + measure the current (i_{sc}), and then get R_{Th} as their ratio, finding the full Thévenin model.

$\Rightarrow$ Be careful, however! many circuits may be damaged if one does this in practice!

Why is Thévenin so powerful? When working with linear circuits having many (e.g. 100's) of components, we are able to focus on a small portion of the circuit and replace everything else with a simple equivalent circuit! This helps us break down a large circuit into smaller blocks that are each tractable, and focus only on one part of a circuit at a time.