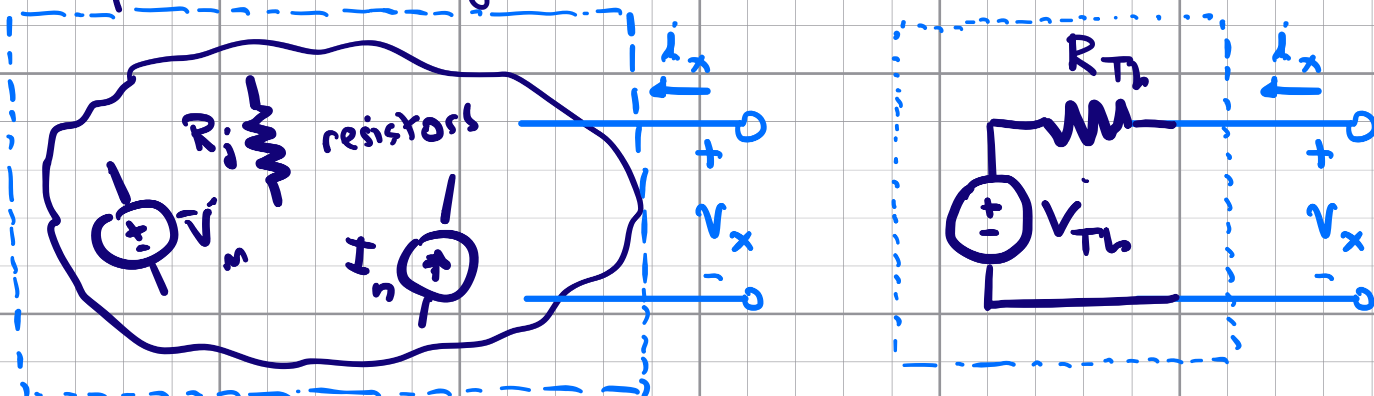
We have seen that Superposition provides a powerful result for linear circuits: We can model the i-v characteristic at a port of an arbitrarily complex linear network (linear elements and independent sources) with a very simple model - a Thévenin equivalent circuit comprising an independent voltage source and a resistance:



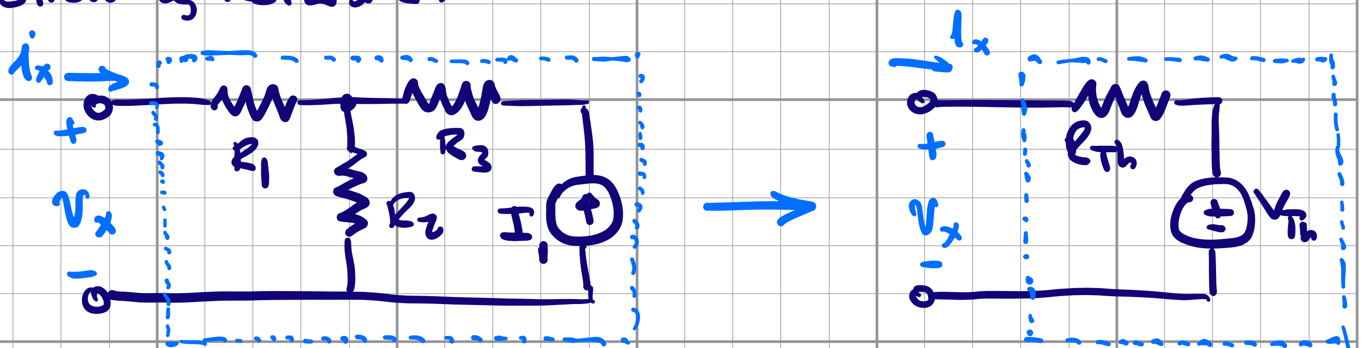
Contains V sources, I sources and linear elements (like fixed resistors)

- The value V_{Th} is the "open-circuit" voltage v_{oc} of the original network: $V_{Th} = v_{oc} = v_x \big|_{i_x = 0}$
- The value R_{Th} is the resistance looking into the port with all independent sources set to zero ("killed"). That is, with all V_m's replaced with short circuits, and all I_n's replaced with open circuits.
- We can also find R_{Th} as the ratio of the "open circuit" voltage v_{oc} to the "short circuit" current i_{sc} of the original network:

$$R_{Th} = \frac{v_{oc}}{i_{sc}} = \frac{v_x \big|_{i_x=0}}{-i_x \big|_{v_x=0}}$$

so long as v_{oc}, i_{sc} are not both 0.

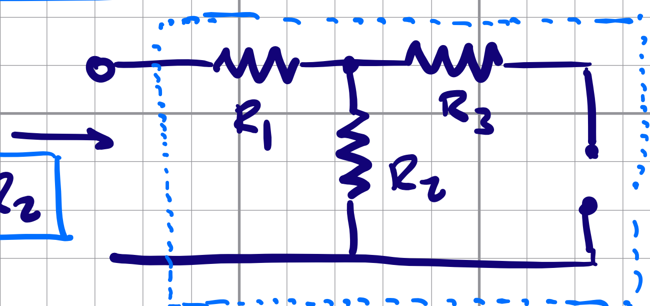
For example, consider the $i_x - v_x$ relationship of the following network:



$$\boxed{V_{Th} = V_{oc} = I_1 \cdot R_2}$$

$$v_x = V_{Th} + i_x R_{Th}$$

R_{Th}:

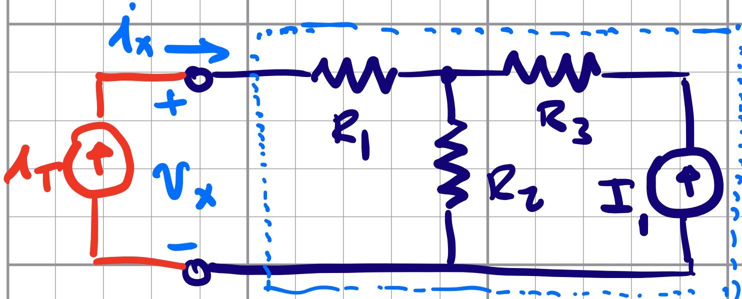


$$\boxed{R_{Th} = R_1 + R_2}$$

$$v_x = (I_1 R_2) + i_x (R_1 + R_2)$$

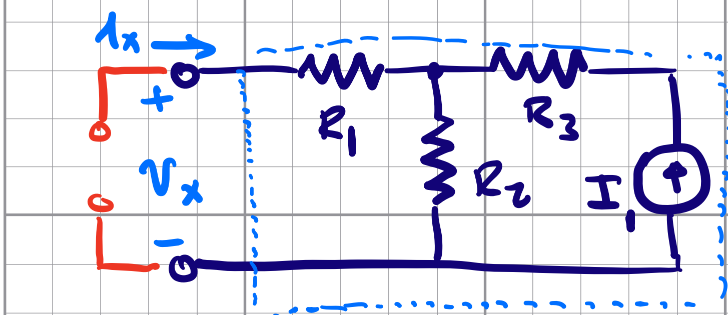
<u>Why</u> does this model work? SUPERPOSITION!

Consider our network of interest driven from an external current (which we model with a test source i_T ($= i_x$))



We can find voltage v_x by superposition of I_1, i_T

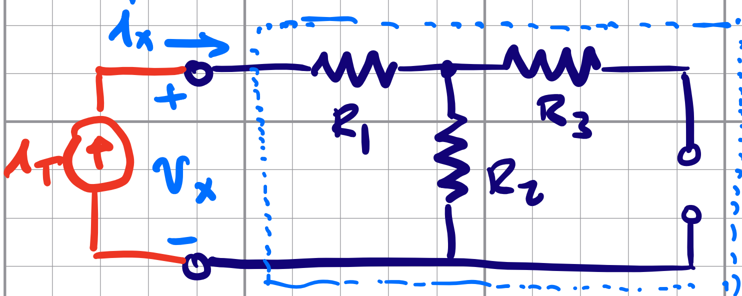
$$v_x = v_x\Big|_{i_T=0} + v_x\Big|_{I_1=0}$$



$v_x =$

$v_x\big|_{i_T=0} = I_1 R_2$

+

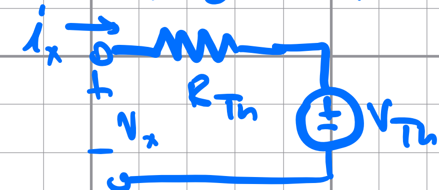


$v_x\big|_{I_1=0} = i_T \cdot (R_1 + R_2)$

$$v_x = v_x\big|_{i_T=0} + v_x\big|_{I_1=0}$$

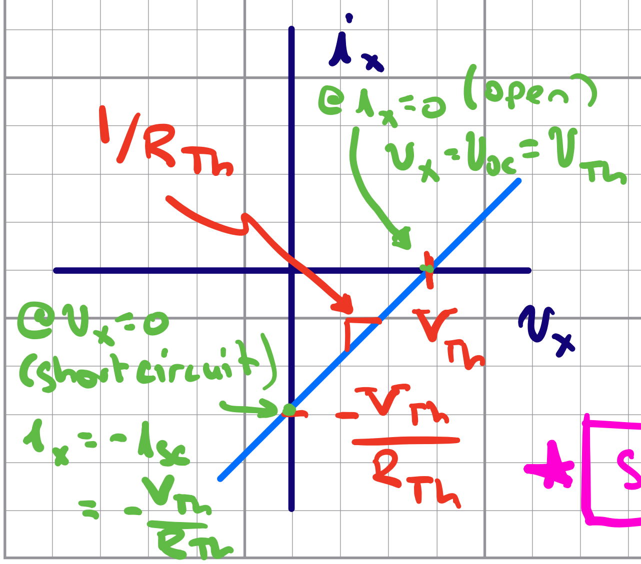
$$= I_1 R_2 + i_T (R_1 + R_2) = I_1 R_2 + i_x (R_1 + R_2)$$

$$v_x = \underbrace{V_{Th}} + \underbrace{i_x \cdot R_{Th}}$$



V_{Th} is v_x response due to sources inside the circuit being modeled with no external current

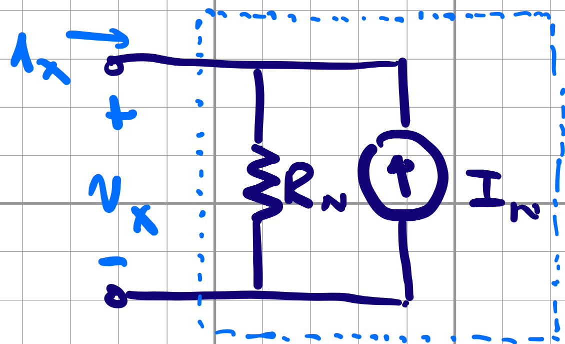
$i_x \cdot R_{Th}$ is the v_x response owing to the external current entering the network with no internal sources

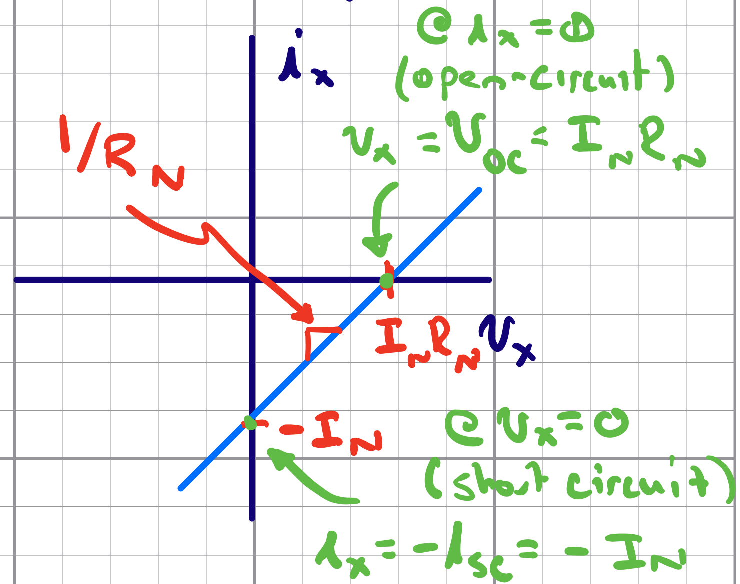


All linear circuits have the same form of i_x - v_x relationship at a port x: a line in the i_x-v_x plane that can be defined by an intercept and a slope (or a Thévenin voltage + Thévenin resistance or an open circuit voltage + short circuit current)

★ $\boxed{\text{See Thévenin v-i match demo!}}$ ★

- The Thévenin equivalent model is not the only simple equivalent circuit for an arbitrary "linear plus sources" circuit. Consider the i_x-v_x characteristic of the following circuit, called the <u>Norton equivalent circuit</u>





$$v_x = (I_N + i_x)R_N$$
$$= I_N R_N + R_N \cdot i_x$$

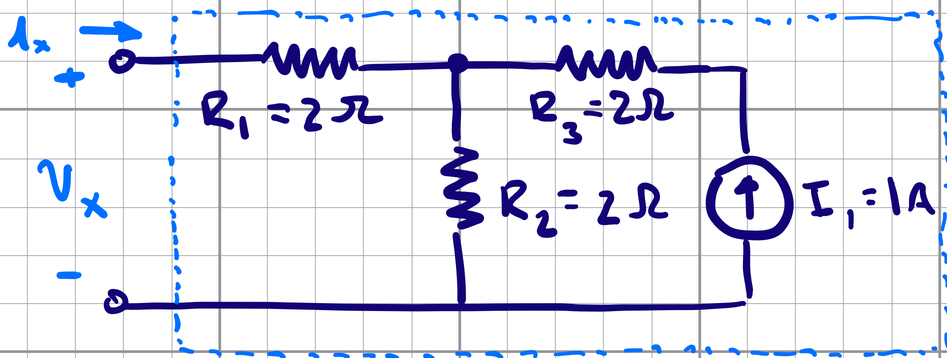
(Fun fact: the Norton equivalent circuit is named for Edward Norton of Bell Labs, who received his S.B. in EE at MIT in 1922)

- Like the Thévenin equivalent circuit, the Norton equivalent circuit maps out a line in the i_x-v_x plane and can be used to match the i_x-v_x characteristics of any "linear plus sources" circuit
- I_N is the "short circuit" current that comes out of the network with the terminals shorted $\left(I_N = -i_x\big|_{v_x=0}\right)$
- R_N is the resistance looking into the network with all independent sources $= 0$ ("killed"). exactly the same as R_{Th}
- The parameters of the Thévenin + Norton models may be matched as follows:

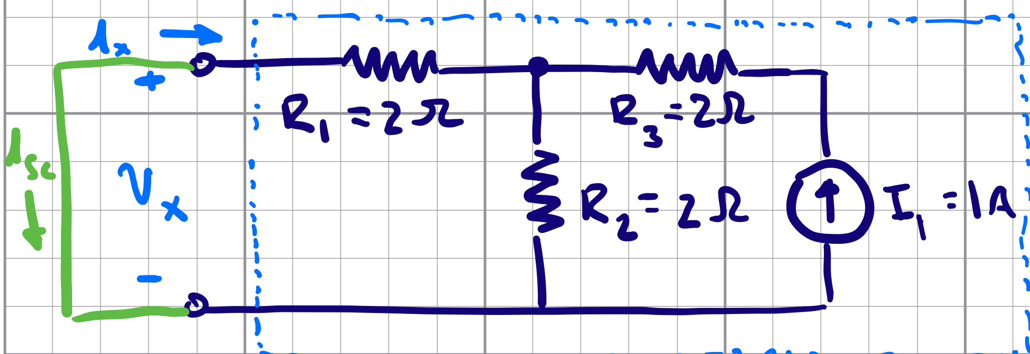
$$R_N = R_{Th}\ ;\ I_N = V_{Th}/R_{Th} \quad \text{or} \quad V_{Th} = I_N \cdot R_N$$

Thus: $R_{Th} = R_N = V_{Th}/I_N = V_{oc}/i_{sc}$

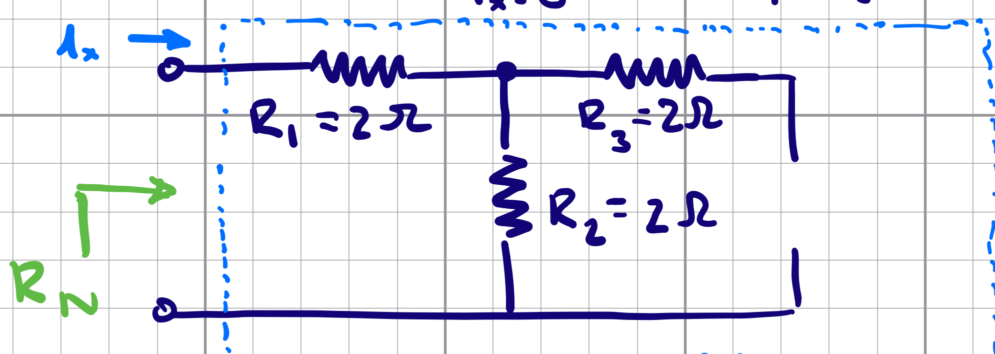
Example: Consider a Norton model for this circuit



To find I_N we could find the short-circuit current:

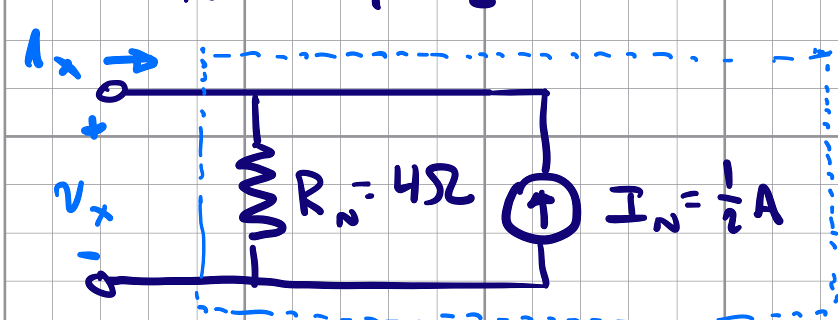


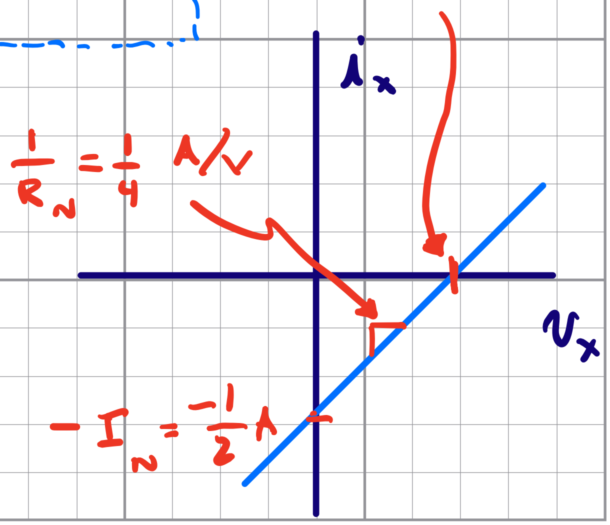
$$i_N = i_{sc} = -i_x\Big|_{v_x=0} = I_1 \cdot \frac{R_2}{R_1+R_2} = \frac{1}{2}\,A$$



$V_{Th} = I_N R_N = 2V$

$$R_N = R_1 + R_2 = 4\Omega$$





Note that the "open circuit voltage for this system is

$$v_{oc} = I_1 \cdot R_2 = 2V \quad \{= V_{Th}\}$$

we could have also found the Norton (or Thévenin) resistance as:

$$R_N = \frac{v_{oc}}{i_{sc}} = \frac{V_{Th}}{I_N} = \frac{2V}{\frac{1}{2}A} = 4\Omega \;(= R_1 + R_2)$$

So we can calculate the two needed parameters in various ways.

Also note the correspondence to the Thévenin model parameters

$$R_{Th} = R_N = R_1 + R_2 = 4\Omega$$

$$V_{Th} = I_N R_N = I_1 \cdot R_2 = 2V$$

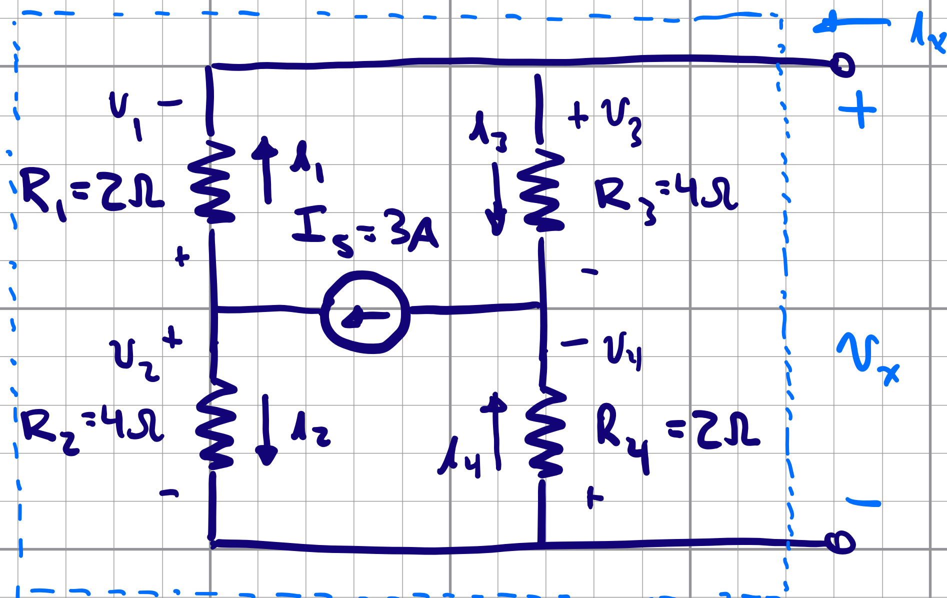
These also match the Thévenin model parameters we calculated directly for this circuit.

We can use Thévenin and Norton models to capture the behavior of all kinds of practical devices and systems.

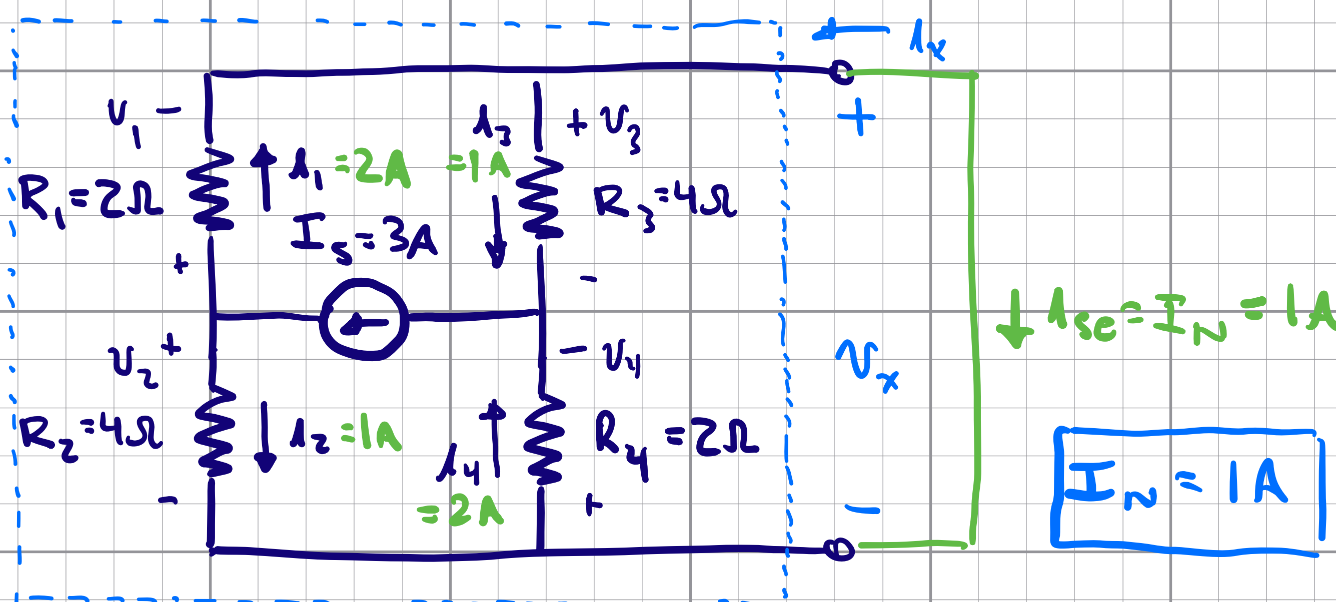
$\boxed{\text{See demo of modeling a battery with a Thévenin equivalent}}$

Whether we use a Thévenin or Norton model (or switch between them) is a matter of choice + convenience

Example: Let's find Norton and Thévenin equivalents for this circuit



Find I_N from the short-circuit current:



$$\boxed{I_N = 1A}$$

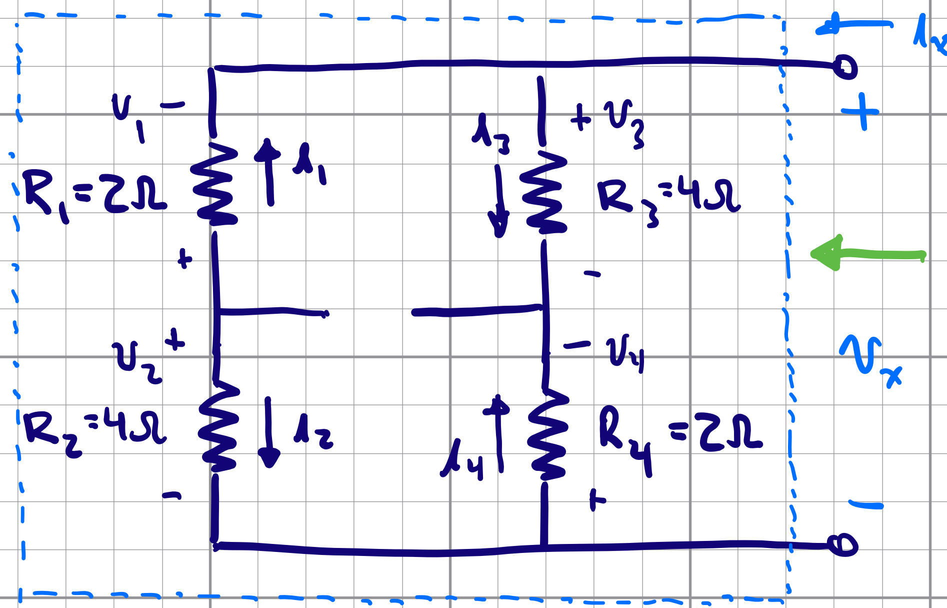
Under short circuit conditions R_1, R_2 share the same voltage as do R_3, R_4, so we can use current division to find the currents under short circuit

$$i_1 = 2 i_2 \quad \text{and} \quad i_1 + i_2 = 3\,A \quad \therefore\ i_1 = 2A,\ i_2 = 1A$$

$$i_4 = 2 i_3 \quad \text{and} \quad i_3 + i_4 = 3A \quad \therefore\ i_3 = 1A,\ i_4 = 2A$$

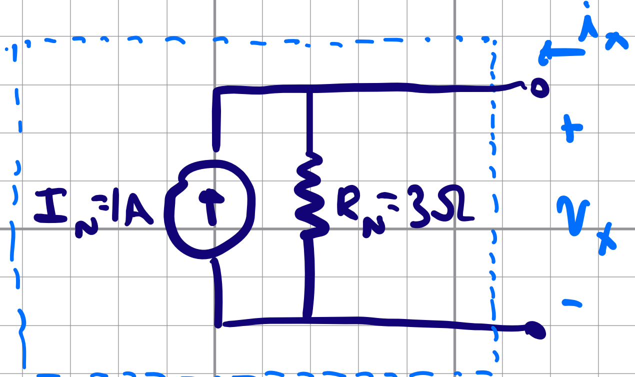
$$i_N = i_4 - i_2 \ (= i_1 - i_3) = 1A$$

Find R_N from input resistance with $I_s = 0$ (I_s "killed")

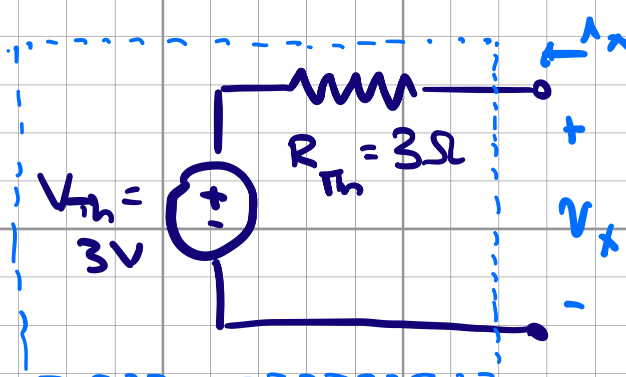


$$R_N = (R_1 + R_2) \,||\, (R_3 + R_4) = 6 \,||\, 6 = 3\Omega$$

$$\boxed{R_N = 3\Omega}$$



Norton model



Thévenin model

$$V_{Th} = I_N R_N = 3V$$

$$R_{Th} = R_N = 3\Omega$$