Circuits

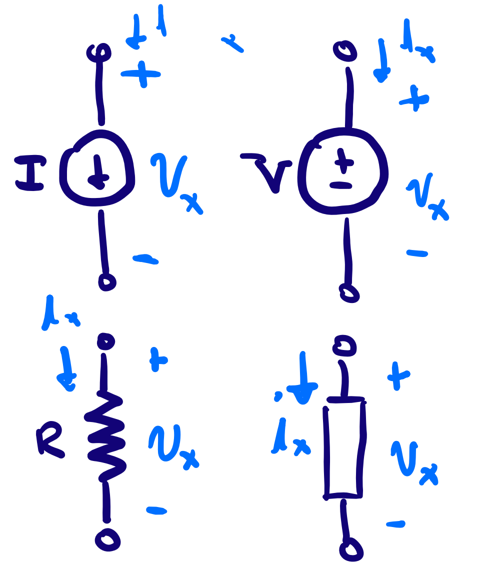
Dependent Sources

Up to now we have seen two types of circuit components:

- Independent Sources: impose a voltage or a current that does not depend on other constraints

  (These are system inputs)

- Components (like resistors) that impose a relationship between their own terminal voltage and current

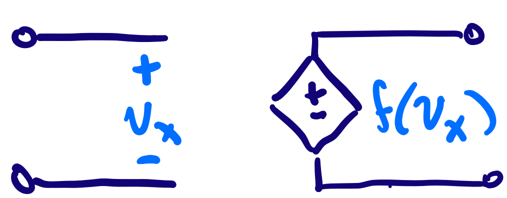


$V_x = i_x R$ $i_x = f(V_x)$ or $V_x = g(i_x)$

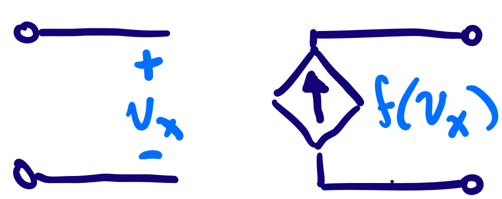
Dependent Sources are another important category of circuit elements where the voltage or current at one place in the circuit determines the voltage or current at another place.

Four basic types:

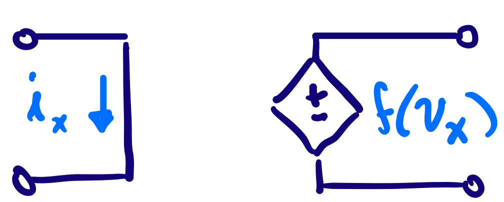
Voltage-controlled voltage source (VCVS)



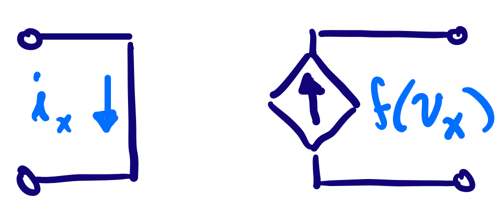
Voltage-controlled current source (VCCS)



current-controlled voltage source (CCVS)



Current-controlled current source (CCCS)



Circuits

Dependent Sources (2)

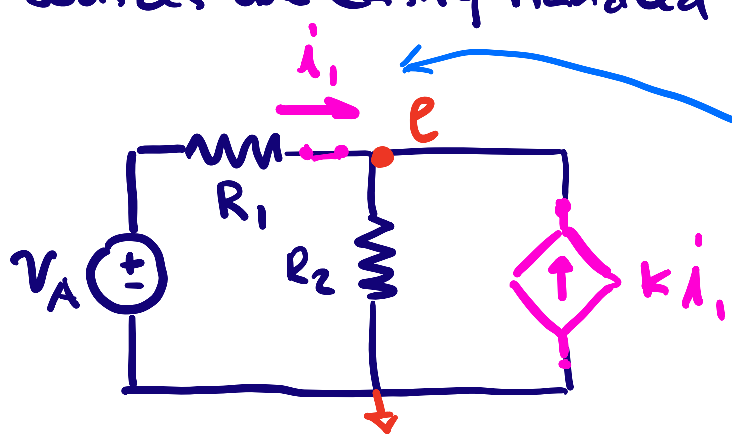
We can think of dependent sources as "two-port" devices, where a "port" is a terminal pair. The "control port" <u>measures</u> a (v or i) without disturbing it, while the "output port" <u>imposes</u> a (v or i) at its terminals that is a function of the measured variable.

A <u>linear</u> dependent source has the control law form $f(x) = kx$, where x is the (v or i) measured and $f(x)$ is the (v or i) imposed at the output.

Dependent sources have a huge range of uses and are used in modeling all kinds of practical devices. For example, they are handy in modeling transducers, amplifiers, transformers, etc.

- In practice, we often just indicate the circuit variable that the dependent source depends upon, rather than explicitly drawing the measurement port.
- Dependent sources are easily handled in nodal analysis.

example:



we can think of measured current i_1 as just the current through R_1

$$(V_A - e)\frac{1}{R_1} - e\frac{1}{R_2} + \underbrace{k\cdot(V_A - e)\frac{1}{R_1}}_{\text{Node-voltage-based expression for } i_1} = 0$$

Rearranging: $\frac{(k+1)}{R_1} \cdot e + \frac{1}{R_2} e = \frac{(k+1) V_A}{R_1}$

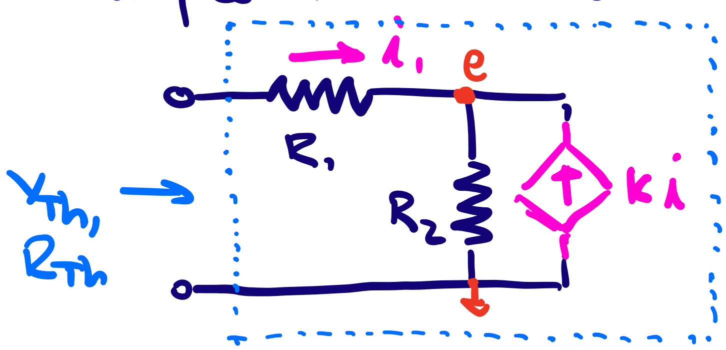
$$\left[(k+1)R_2 + R_1\right] e = (k+1)R_2 \cdot V_A$$

$$\boxed{e = \frac{(k+1)R_2}{R_1 + (k+1)R_2} \cdot V_A}$$

The dependent source increases the "effective resistance" of R_2 by a factor $(k+1)$

- Note: in cases where we have a "floating" dependent voltage source, we can use our "supernode" trick on the nodes spanned by the dependent source.

Example: Find the Thévenin equivalent for the circuit:



V_{Th} = "open circuit" v = ⊕ :

@ open ckt, $i_1 = 0$, $ki_1 = 0$
∴ $e = 0$ ∴ $V_{Th} = 0$

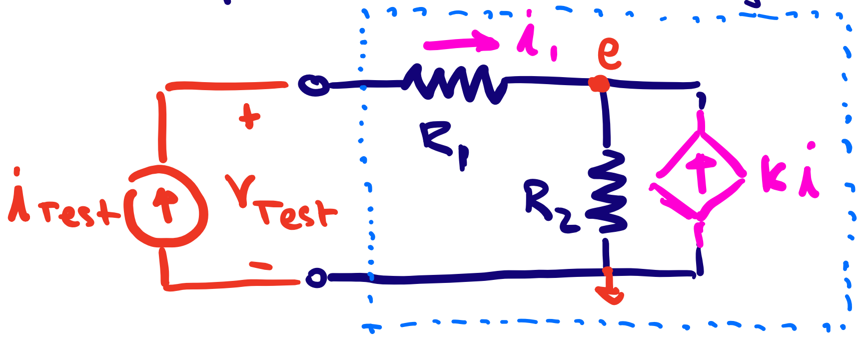
To find R_{Th}: kill <u>INDEPENDENT</u> sources + find resistance looking into the port

NOTES: (1) Do **NOT** kill dependent sources. (These are components, **NOT** system inputs, so we don't zero them for Thévenin, Norton, or superposition)

(2) Dependent sources make resistor combination difficult. Instead, use V_{Test}, I_{Test} method (see below)

Never "zero" or "kill" dependent sources! (e.g., in superposition) They are components, **NOT** independent sources / inputs

Find input resistance using v_{Test} – i_{Test} method:



- Apply test source i_{Test} (or V_{Test})
- measure response V_{Test} (or I_{Test})
- $R_{Th} = \dfrac{v_{Test}}{i_{Test}}$

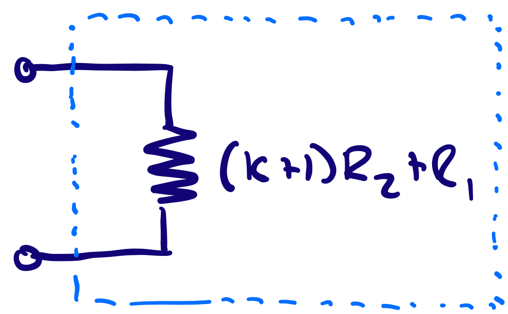
Driven by test source:

$$e = (k+1)\, i_{Test} \cdot R_2$$

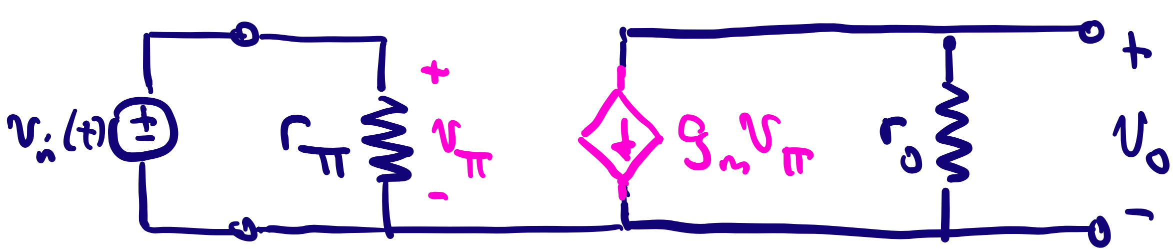
$$v_{Test} = e + i_{Test} R_1 = [(k+1)R_2 + R_1]\, i_{Test}$$

$$\therefore \boxed{R_{Th} = \frac{v_{Test}}{i_{Test}} = (k+1)R_2 + R_1}$$

Thévenin equivalent:



Example: Dependent source in an amplifier
(small-signal model of a "common-emitter" amplifier.)
⇒ Behavior of a transistor modeled using a VCCS and resistors.



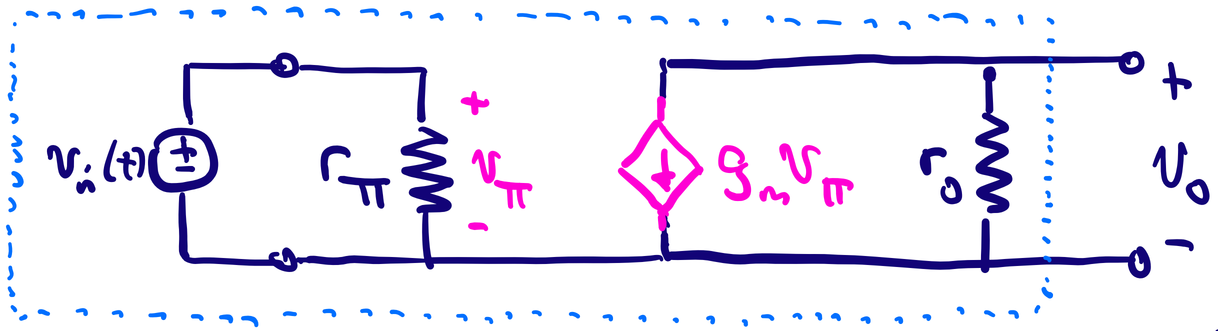
g_m is known as a "transconductance": ratio of current at one place to voltage at another. Units are ℧.

Find v_o as a function of v_i:

$$\boxed{v_o = -r_o g_m \cdot v_i}$$

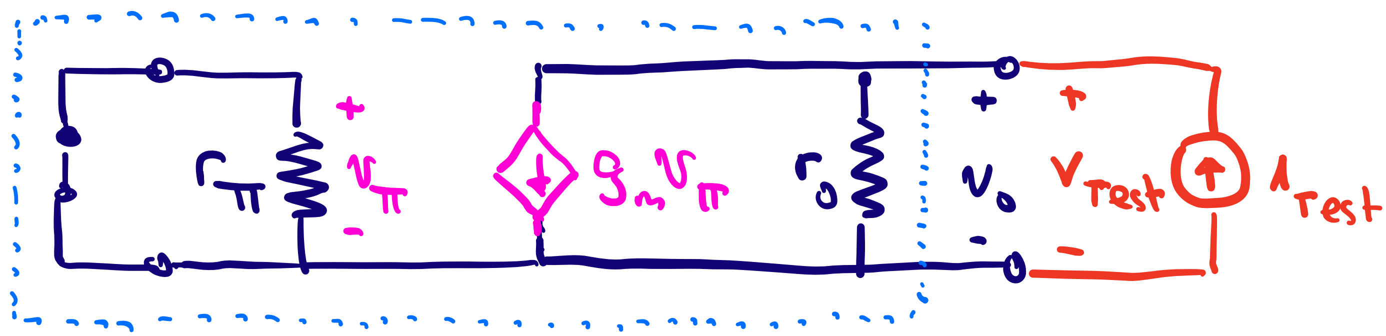
If $r_o g_m > 1$ we get voltage gain!

What is the Thévenin model looking into the output port?



open circuit voltage $V_{Th} = -r_o g_m v_i$ (already found)

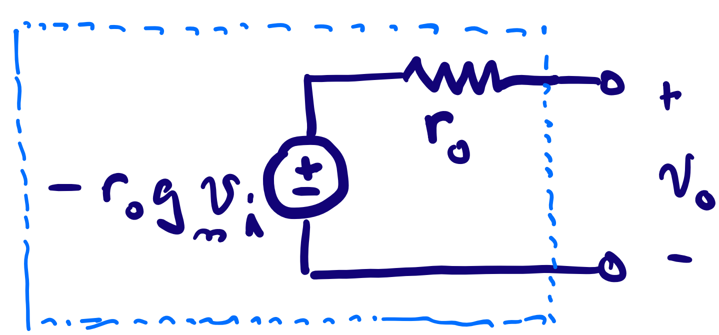
To find R_{Th} "output resistance", "kill" v_{in} and use a test source:



$v_\pi = 0 \therefore g_m v_\pi = 0 \therefore v_{Test} = i_{Test} \cdot r_o$

$R_{Th} = r_o$

Thévenin model from output port



- Note that in this circuit, what happens at the control port of the VCCS (v_π) affects the output port ($g_m v_\pi$), but the reverse is <u>not</u> true, so R_{Th} does not depend on impedances at the control port.

  ⇒ Dependent sources are "nonreciprocal" elements. This is a useful property!

- See demo example of a dependent source (a "Howland" current source as a VCCS.

- Attached page shows example of a dependent source (VCVS) used in a circuit for a thermal model.

Example: Circuit model with a dependent source

-Models heat transfer in an automotive generator

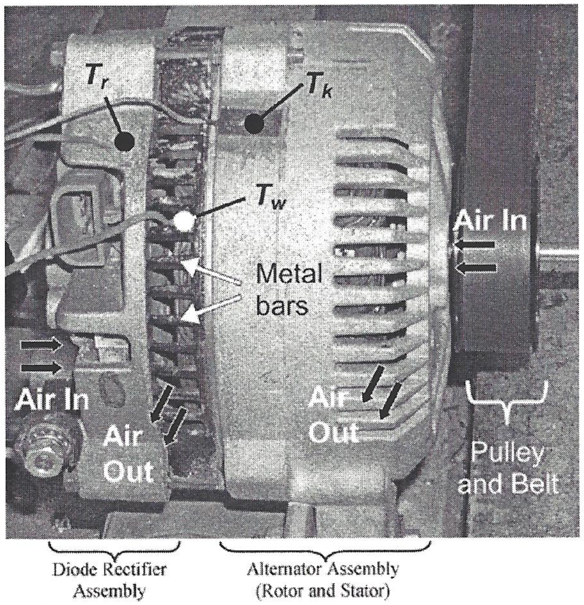


Fig. 1. Typical Lundell alternator.

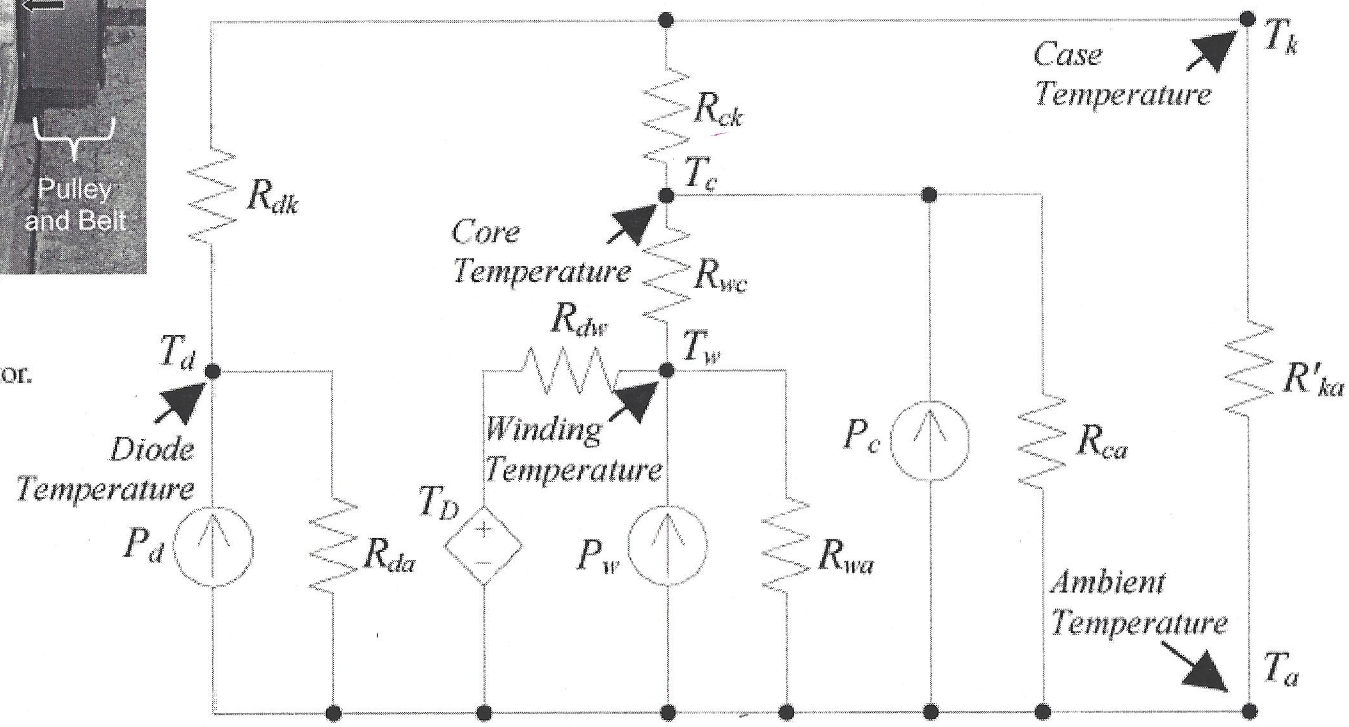


Fig. 8. Thermal model for Lundell alternators.

From: S.C. Tang, et.al., "Thermal Modeling of Lundell Alternators," *IEEE Transactions on Energy Conversion,* Vol. 20, No. 1, March 2005, pp. 25-36.