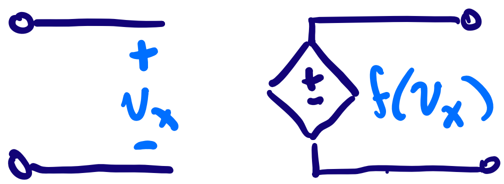
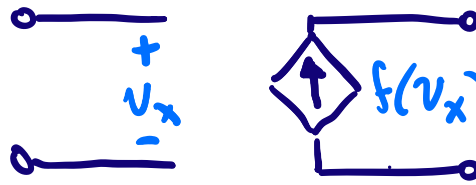
Circuits   Dependent sources and two-port networks

Reminder: Dependent sources measure a (v or i) at one place in a circuit, and impose a (v or i) at another place in the circuit that depends on the measured value.

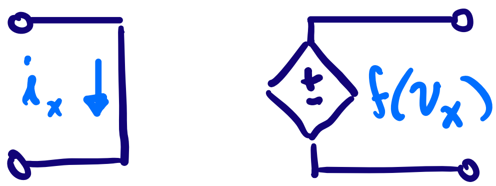
Voltage-controlled voltage source (VCVS)



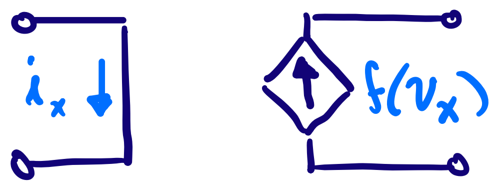
Voltage-controlled current source (VCCS)



current-controlled voltage source (CCVS)



Current-controlled current source (CCCS)



- for a <u>linear</u> dependent source $f(x) = K \cdot x$
- Dependent sources are used to model all kinds of practical devices (we'll consider some examples).

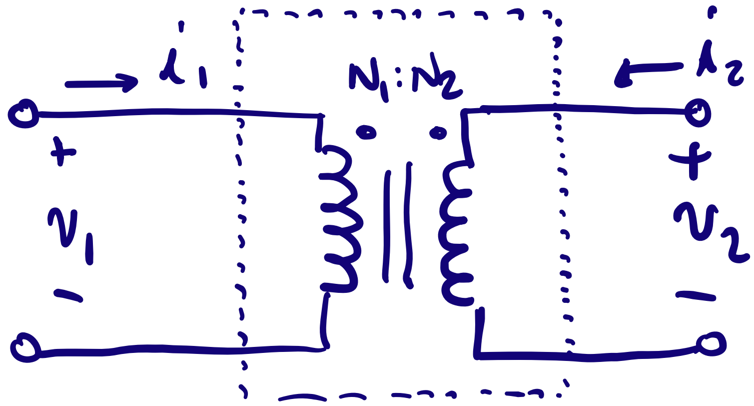
The Transformer:

One important practical circuit device is the magnetic transformer, originally developed by Michael Faraday

Transformers have many practical applications, including providing galvanic isolation (e.g. for safety) and for losslessly scaling ac voltage + current levels.

⇒ wall adapters for computers, phones, etc. all have transformers, and there are transformers all along the street to convert high "distribution" voltages down to levels for use in buildings

The <u>ideal transformer</u> captures the essential, idealized behavior of a real magnetic transformer, and is also sometimes used to model other kinds of devices



Ideal Transformer

$$\left.\begin{aligned} v_2 &= \frac{N_2}{N_1} \cdot v_1 \\ i_2 &= -\frac{N_1}{N_2} i_1 \end{aligned}\right\}$$

Circuit relations

Note the "dot" convention. The dots indicate the relative polarities of voltages of the voltages and currents at the two ports:

→ If current flows "into" one dot, a proportional current flows "out of" the other dot.

→ The relative polarity of the proportional voltages at the two ports of the transformer are indicated by the dots (being the same if the voltages are shown the same way with respect to the dots.)

Note that the transformer can "isolate" parts of a circuit as there is no direct flow of current from one port of the transformer to the other, and the two "sides" of the transformer can be at very different potentials with respect to a reference node such as gnd.

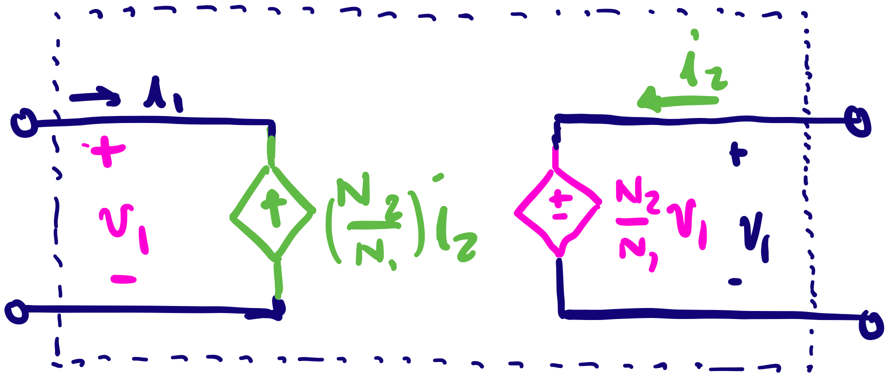
The ideal transformer provides a scaling of voltage at one port with respect to the other, and an opposite scaling in current. We thus get voltage + current <u>scaling</u> between the two sides, with no energy storage or loss:

$$v_1 \cdot i_1 = \left(\frac{N_1}{N_2} \cdot v_2\right) \cdot \left(-\frac{N_2}{N_1} \cdot i_2\right) = -v_2 \cdot i_2$$

Power in one port flows out the other! very useful for transforming power among voltage levels.

The ideal transformer is sometimes used as a two-port circuit element. However, can we model its behavior with circuit elements we already have?

Yes! Use dependent sources!



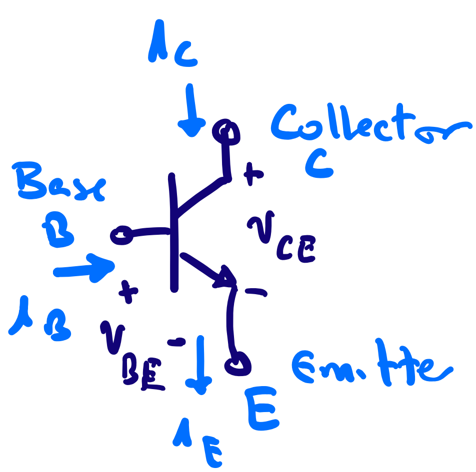
The dependent sources give us:

$$\left.\begin{aligned} v_2 &= \frac{N_2}{N_1} v_1 \\ i_1 &= -\frac{N_2}{N_1} i_2 \end{aligned}\right\} \checkmark$$

So we can use dependent sources to model the behavior of devices like transformers.

The behavior of two-port devices like transistors (and vacuum tubes) can also be modeled with dependent sources.

For example, consider the bipolar junction transistor, or BJT ("npn" flavor). This is a 3-terminal device that is used as a 2 port device (with one terminal in common between the two ports.)



the "active mode" equations, correct over some range

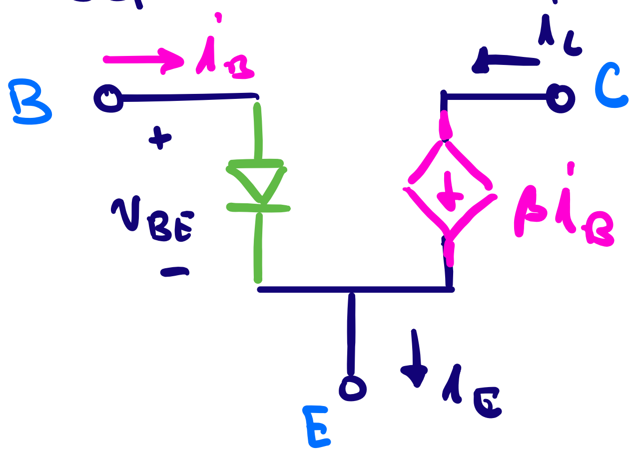
$$i_B = I_S\left(e^{v_{BE}/V_T} - 1\right)$$

$$i_C = \beta\, i_B$$

where $V_T \simeq 26\text{ mV}$ @ room temp
β is a device parameter (e.g. $\beta = 100$)
I_S is a device parameter.

{See demo}

Over some range, we can model the i-v characteristics of a BJT with a diode (a nonlinear resistor) and a dependent source:

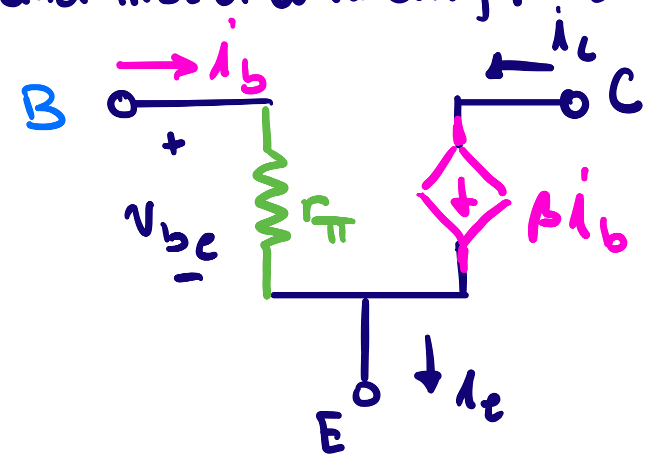


$$i_B = I_S(e^{v_{BE}/v_T} - 1)$$
$$= f(v_{BE})$$

(diode is a nonlinear resistor)

- current-controlled current source

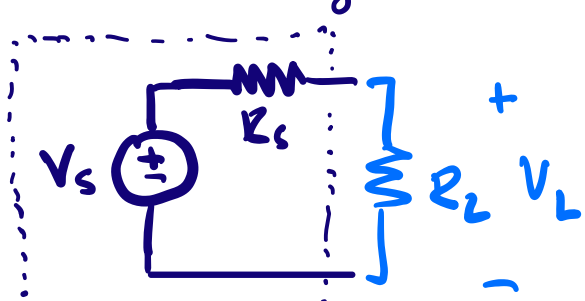
In some situations we can ignore the nonlinear behavior and model with only linear elements:



- $i_b = v_{be}/r_\pi$ (linear resistor)
- linear CCCS

How might we use this characteristic?

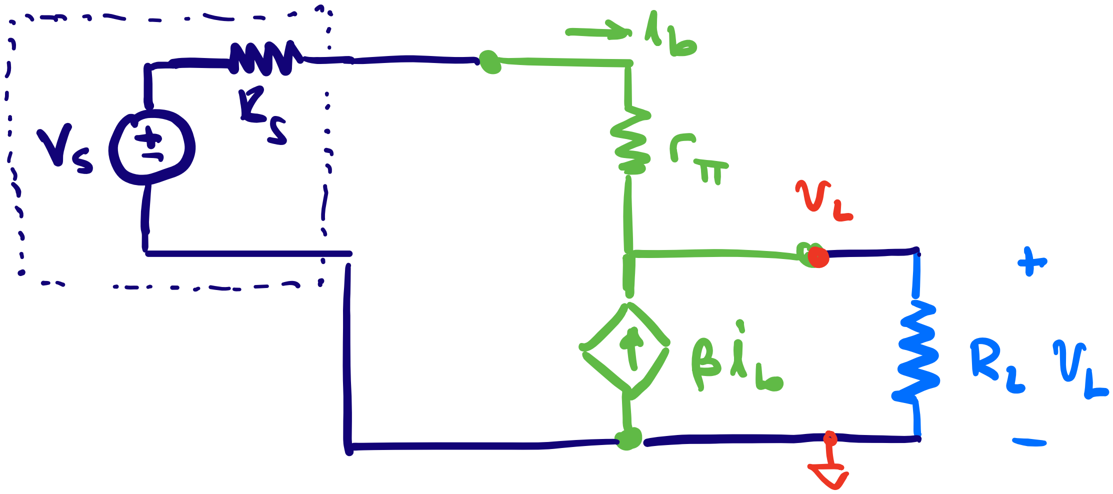
We've seen before that if we have a source with output resistance and connect it to a resistive load, then voltage division can ruin our day



$$V_L = V_S \cdot \frac{R_L}{R_L + R_S}$$

If $R_L \ll R_S$, $V_L \ll V_S$
(we don't get V_S at the output)

We could use a transistor circuit to help reduce loading, e.g.:



What is v_L here?

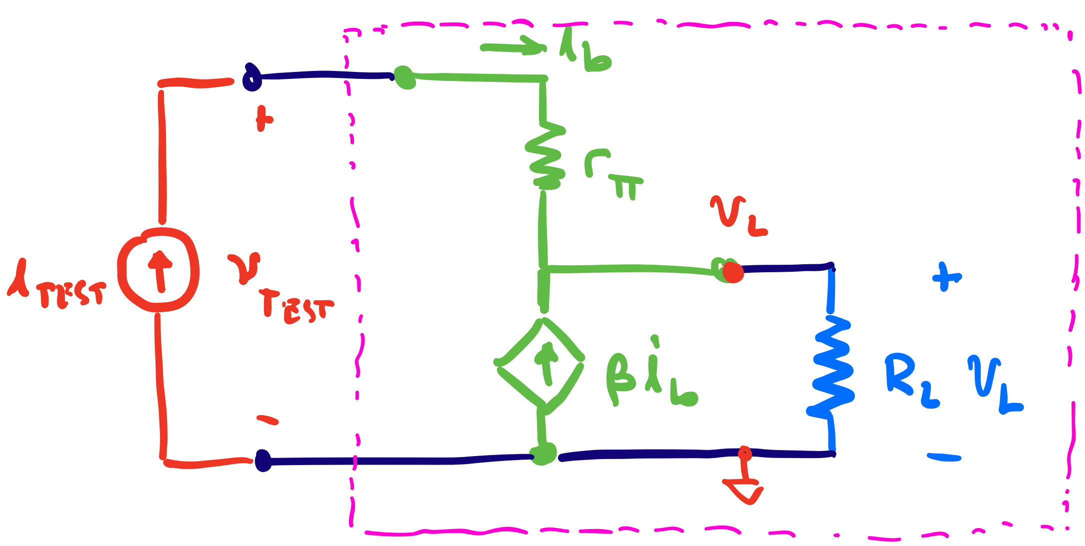
$$(V_s - v_L)\frac{1}{R_s + r_\pi} + \beta(V_s - v_L)\frac{1}{R_s + r_\pi} - v_L \cdot \frac{1}{R_L} = 0$$

$$v_L \cdot \left[\frac{(\beta+1)}{R_s + r_\pi} + \frac{1}{R_L}\right] = V_s \cdot \frac{(\beta+1)}{R_s + R_\pi}$$

$$\boxed{v_L = V_s \cdot \frac{(\beta+1) R_L}{(\beta+1) R_L + R_s + r_\pi}}$$

If $\beta \uparrow\uparrow$ then $v_L \to V_s$:)

What equivalent Thévenin resistance R_{Th} is seen looking into the new network? ($V_{Th} = 0$, since no indep. sources)



$$v_L = (\beta+1)\, i_{TEST} R_L$$

$$\therefore\ v_{TEST} = i_{TEST}\, r_\pi + (\beta+1)\, i_{TEST} R_L$$

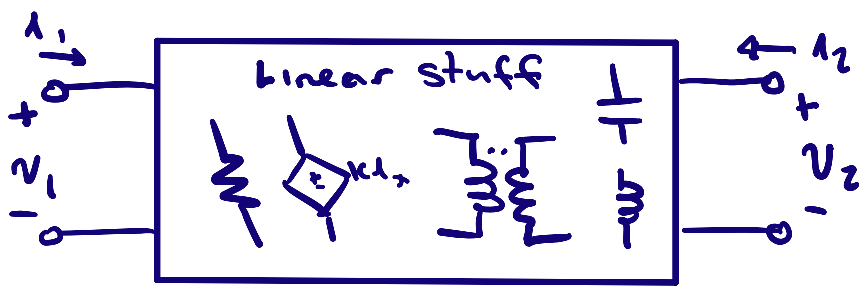
$$\boxed{R_{Th} = \frac{v_{TEST}}{i_{TEST}} = r_\pi + (\beta+1) R_L}$$

If $\beta \gg 1$, the resistance $R_{Th} \gg R_L$

So the dependent source effect of the transistor circuit increases the effective resistive loading and increases the voltage at the load!

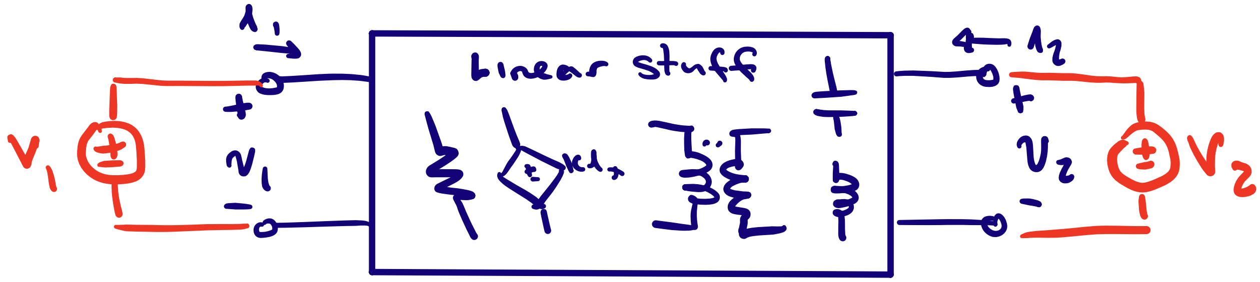
→ we will use a MOSFET transistor to similarly help us in a future lab.

We can also use dependent sources to help us model arbitrary linear two-port networks (with no independent sources inside):



Regardless of the number of internal components, we can find a simple representation of the system terminal behavior owing to linearity by superposition

e.g. apply independent voltage sources to each terminal and find the terminal currents by superposition.



Suppose we find 4 responses. Impose V_1, V_2 one at a time with the other shorted, and measure the specified responses

$$y_{11} = \frac{i_1}{V_1}\Big|_{V_2=0} \qquad y_{12} = \frac{i_1}{V_2}\Big|_{V_1=0}$$

$$y_{21} = \frac{i_2}{V_1}\Big|_{V_2=0} \qquad y_{22} = \frac{i_2}{V_2}\Big|_{V_1=0}$$

These ($y_{11}, y_{12}, y_{21}, y_{22}$) are called the "short circuit admittance parameters, and they characterize the network. (admittance is like a generalized conductance)

By superposition

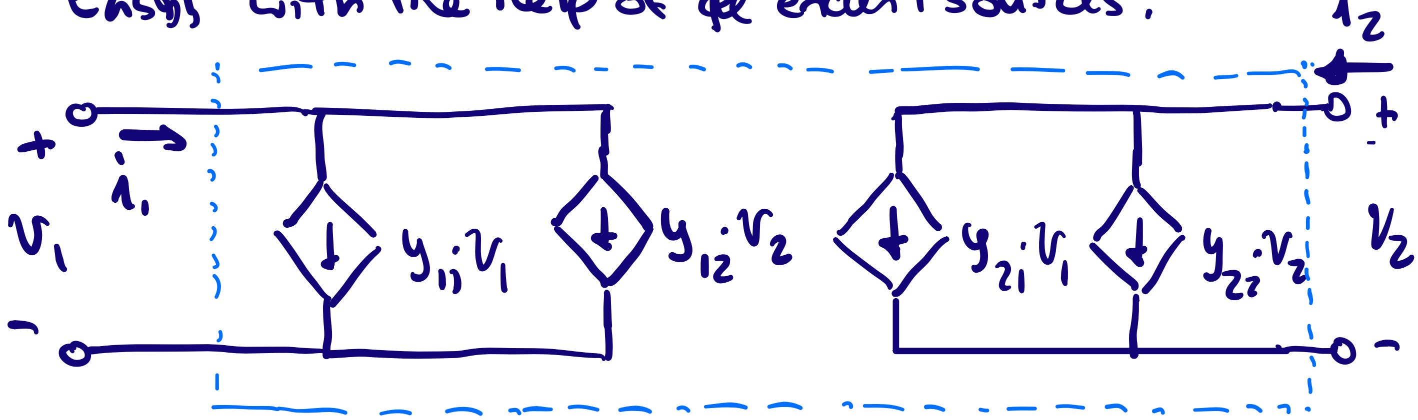
$$\begin{aligned} i_1 &= y_{11} \cdot V_1 + y_{12} \cdot V_2 \\ i_2 &= y_{21} \cdot V_1 + y_{22} \cdot V_2 \end{aligned} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

This is the "y parameter" representation of the linear 2-port system. It fully represents the i-v relations of the system.

Key idea: If we can (by measurement or analysis) find the 4 parameters, we can represent the terminal behavior of the system, regardless of its internal complexity.

How can we represent this in circuit terms?

Easily with the help of dependent sources!



4 dependent sources let us model any linear 2-port!

(can also use 2 conductances, and 2 dependent sources