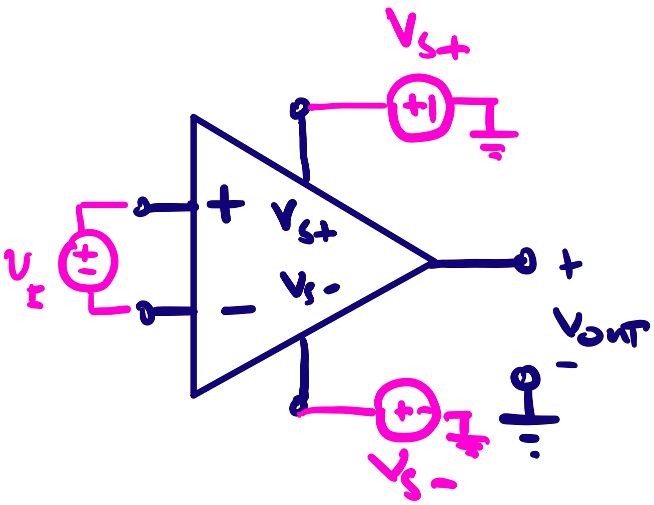
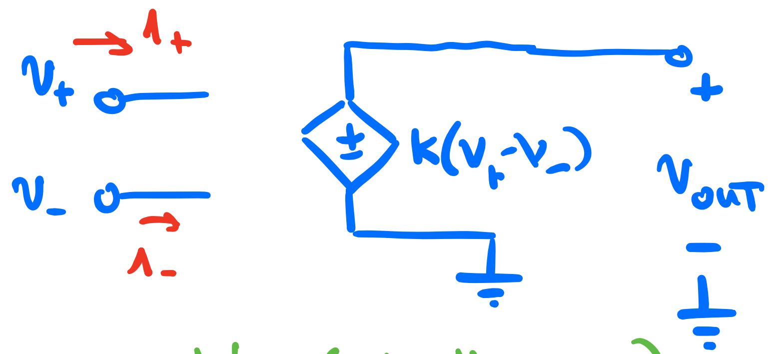
Circuits Ideal Op Amp

The symbol for an op-amp:



model As:
("linear" region only)



- K is big (ideally → ∞)
- This model only applies in the "linear" region, where $V_{s-} < V_{out} < V_{s+}$ (otherwise will saturate)

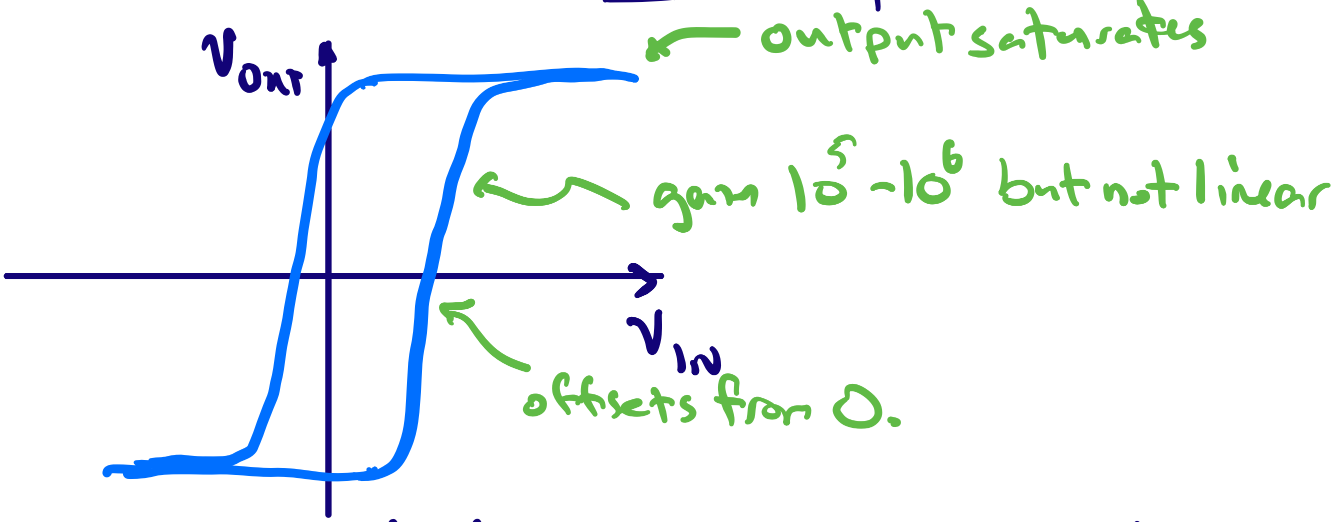
An ideal op amp has:

- linear gain K, with $K \to \infty$
- no input port current ($i_+ = 0$, $i_- = 0$)
- no output resistance
- no limits on output voltage

} "ideal" VCVS

- infinite bandwidth (instantaneous response)

Demo: Behavior of a real op amp

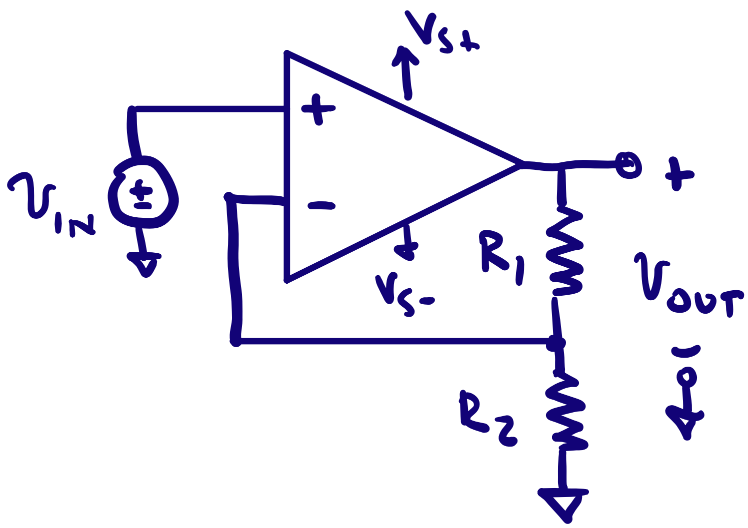


Real op amp: Nonlinear, temperature dependent, time dependent, part-to-part variations

But: Gain is LARGE

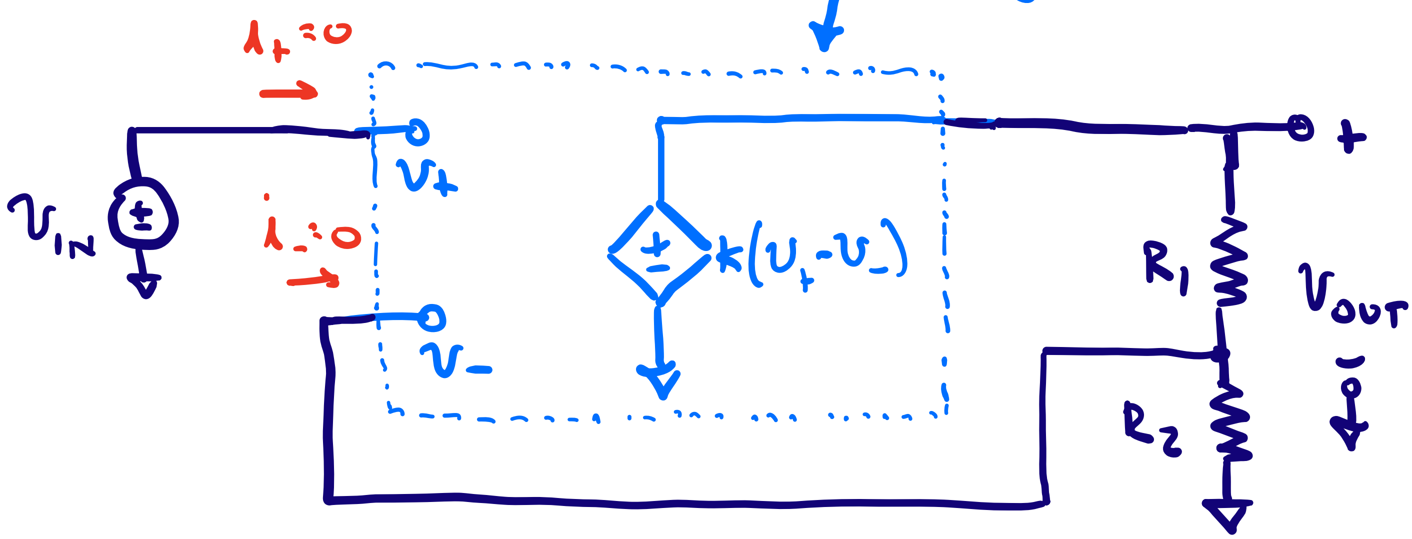
How can we make use of such a nonideal device?
Use NEGATIVE FEEDBACK to trade gain for more ideal behavior

"Noninverting" connection with negative feedback (feedback to the negative sensing terminal)



Feed back a portion of the output to the "−" terminal of the op amp

model of op amp in its "linear" region



Analyzing: $v_+ = v_{IN}$

$$v_- = \frac{R_2}{R_1 + R_2} \cdot V_{OUT} \quad (\text{since } i_- = 0)$$

$$V_{OUT} = k(v_+ - v_-)$$

$$\therefore \; V_{OUT} = K\left(V_{IN} - \frac{R_2}{R_1+R_2} V_{OUT}\right)$$

$$V_{OUT}\left(1 + K\frac{R_2}{R_1+R_2}\right) = K V_{IN}$$

$$V_{OUT} = \frac{K}{1+K\frac{R_2}{R_1+R_2}} V_{IN} = \frac{1}{\frac{1}{K} + \frac{R_2}{R_1+R_2}} V_{IN}$$

If $K \gg \frac{R_1+R_2}{R_2}$ $\Rightarrow$ $\boxed{V_{OUT} \simeq \frac{R_1+R_2}{R_2} V_{IN}}$

In this case (big enough K) we get a gain G:

$$\frac{V_{OUT}}{V_{IN}} = G = 1 + \frac{R_1}{R_2}$$

See Demo!

- The gain G only depends on a resistor ratio
  - $\Rightarrow$ This holds so long as $G \ll K$
- It doesn't matter if "open loop" gain K is nonlinear, temperature dependent, time dependent etc.
  - ★ $\Rightarrow$ we traded a large but uncertain gain K for a smaller but precise gain G!
  - $\Rightarrow$ Sensitivities, nonlinearities have been suppressed!
- This is achieved through the power of negative feedback

$\Rightarrow$ If $V_{IN} = V_+$ varies, the large open-loop gain K drives the V_- pin to track close to the V_+ pin

$$V_+ - V_- = \frac{V_{OUT}}{K} \simeq \frac{G V_{IN}}{K} \Rightarrow V_+ - V_- \ll V_{IN}, V_{OUT}$$

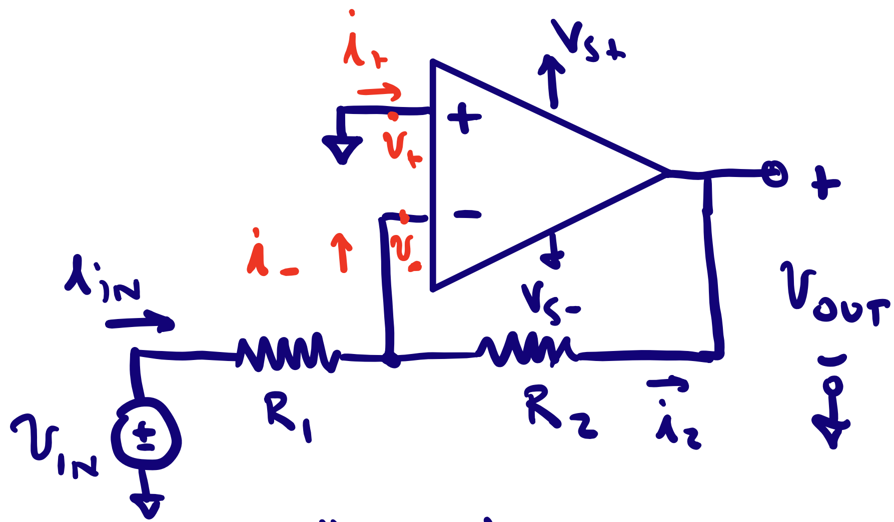
as $K \to \infty$ ("ideal" op amp) $V_+ - V_- \to 0$

The large gain K drives V_- to be $\simeq V_+$. This is a common feature of negative feedback connections (where the output is fed back through the large gain with negative amplification.)

To simplify analysis under negative feedback, we often use the "ideal op amp" approximation of $K \to \infty$, such that $v_- \approx v_+$ (since any difference between v_+ and v_- would lead to $v_{OUT} \to \infty$. After analysis we can check that the output is not saturated.

$$\boxed{\text{Key analysis steps / assumptions: } \quad v_+ \approx v_-, \; i_+ \approx 0, \; i_- \approx 0}$$

Example: "inverting amplifier"



Analyze with ideal op amp model:

① Negative feedback makes $v_- = v_+ = 0$

② $\therefore i_{IN} = \frac{1}{R_1} v_{IN}$

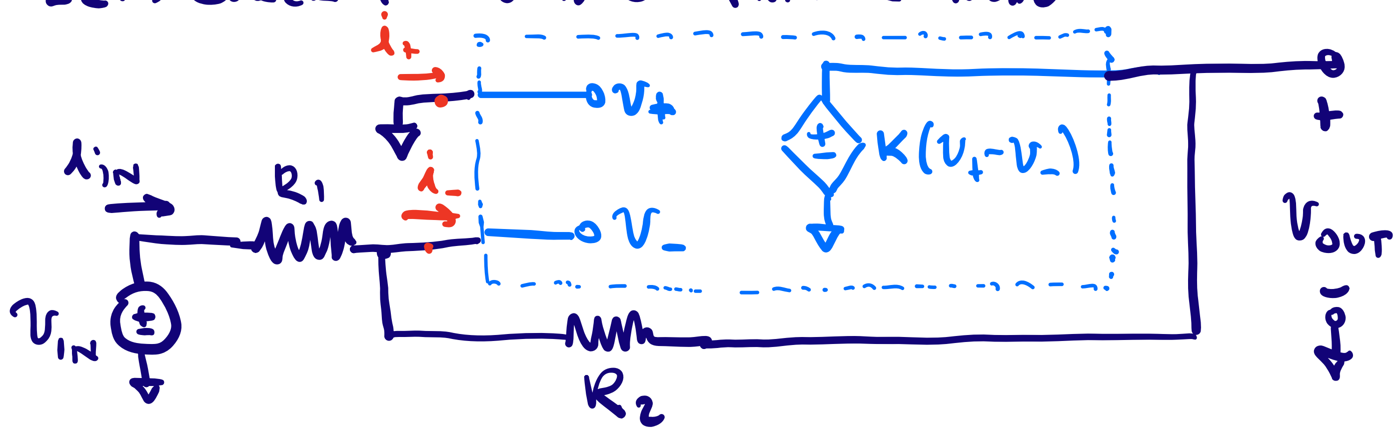
③ $i_- = 0 \therefore i_2 = i_1$

④ $v_{OUT} = 0 - i_2 \cdot R_2$

We thus get:

$$\boxed{v_{OUT} = -\frac{R_2}{R_1} v_{IN}, \text{ or } G = \frac{v_{OUT}}{v_{IN}} = -\frac{R_2}{R_1}}$$

Let's check this with a "finite K" model



Result: $\dfrac{v_{out}}{v_{IN}} \simeq -\dfrac{R_2}{R_1}$ for $K \gg \dfrac{R_2}{R_1}$

Analysis for finite K (skip in lecture):

$$v_{out} = -K\, v_-$$

By superposition of v_{IN}, v_{out} (considering $i_- = 0$):

$$v_- = \frac{R_2}{R_1+R_2} v_{IN} + \frac{R_1}{R_1+R_2} v_{out}$$

$$\therefore \quad \frac{R_2}{R_1+R_2} v_{IN} + \frac{R_1}{R_1+R_2} v_{out} = -\frac{1}{K} v_{out}$$

$$v_{out} = -\left(\frac{1}{\frac{R_1}{R_1+R_2} + \frac{1}{K}}\right) \frac{R_2}{R_1+R_2} \cdot v_{IN}$$

$$v_{out} = -\frac{R_2}{R_1 + \frac{R_1+R_2}{K}} \cdot v_{IN}$$

$$\simeq -\frac{R_2}{R_1} v_{IN} \quad \text{for } K \gg \frac{R_2}{R_1} \quad \checkmark$$

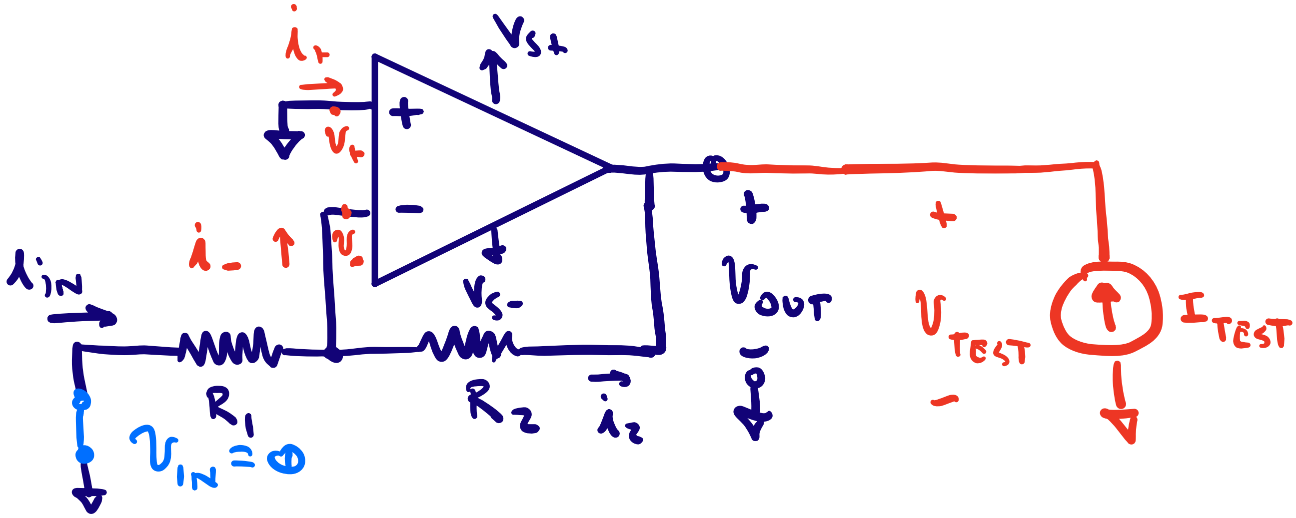
What "input resistance" is seen by v_{IN} for this inverting amplifier?

$$v_- \approx v_+ = 0 \quad \therefore \; i_{IN} \simeq v_{IN}/R_1$$

$$\Rightarrow \; R_{IN} = \frac{v_{IN}}{i_{IN}} = R_1$$

(a similar analysis for the "non inverting amplifier" gives $R_{in} \to \infty$ since $i_+ = i_{IN} \to 0$)

What is the "output resistance" of the inverting amplifier (with ideal op amp)?



$$v_- = v_+ = 0 \quad \therefore \; i_{IN} = 0$$

$$i_- = 0 \quad \therefore \; i_2 = 0$$

$$\therefore \; v_{OUT} = V_{TEST} = 0 - i_2 \cdot R_2 = 0$$

$$\therefore \quad R_{Th} = R_{OUT} = \frac{V_{TEST}}{I_{TEST}} = 0$$

(we would get a similar result for a noninverting amplifier with an ideal op amp.)