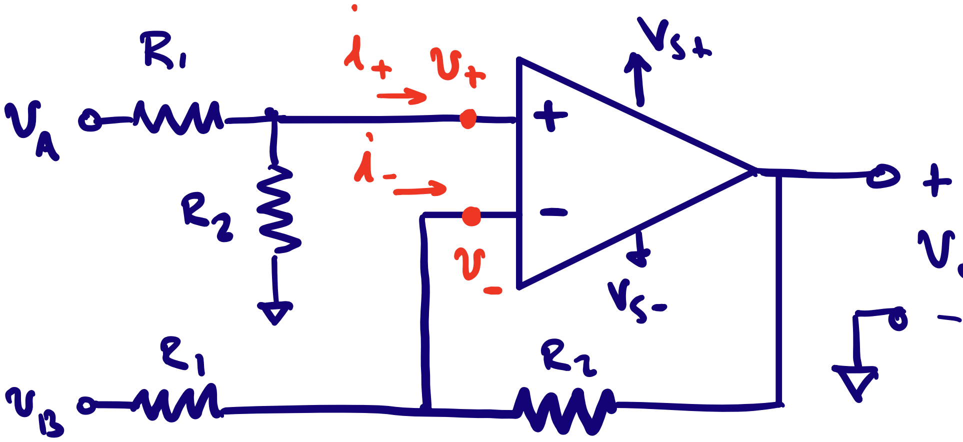
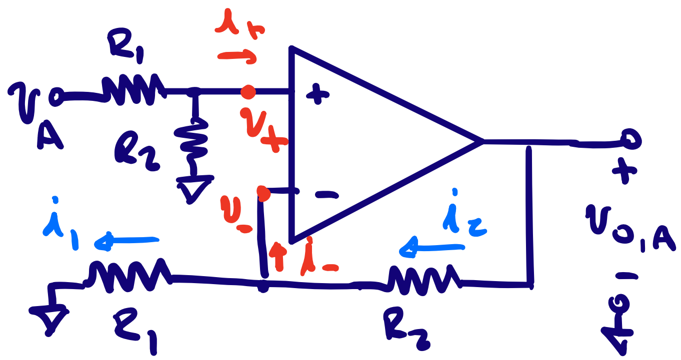
Consider the following circuit:

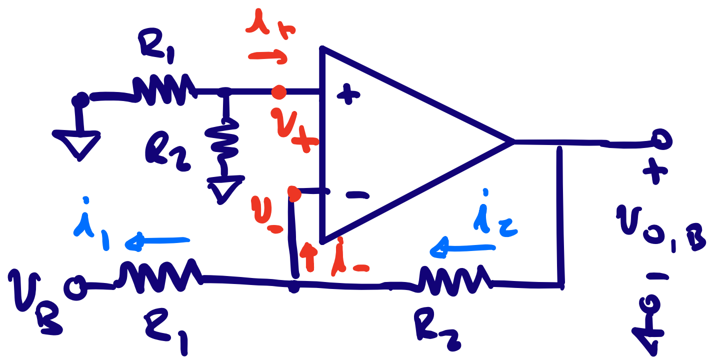


ideal op amp ($K \to \infty$) is linear $\therefore$ we can analyze with superposition

with $v_B = 0$

with $v_A = 0$





$i_+ = 0 \quad \therefore v_+ = \frac{R_2}{R_1+R_2} \cdot v_A$

$v_- = v_+ \quad \therefore i_1 = \frac{R_2}{R_1(R_1+R_2)} \cdot v_A$

$i_- = 0 \quad \therefore i_2 = i_1$

$v_{oA} = i_1(R_1+R_2) = \frac{R_2}{R_1} v_A$

$i_+ = 0 \quad \therefore v_+ = 0$

$v_- = v_+ \quad \therefore v_- = 0$

$i_- = 0 \quad \therefore i_1 = i_2 = -\frac{v_B}{R_1}$

$v_{oB} = v_- + i_2 R_2$

$= -\frac{R_2}{R_1} \cdot v_{oB}$

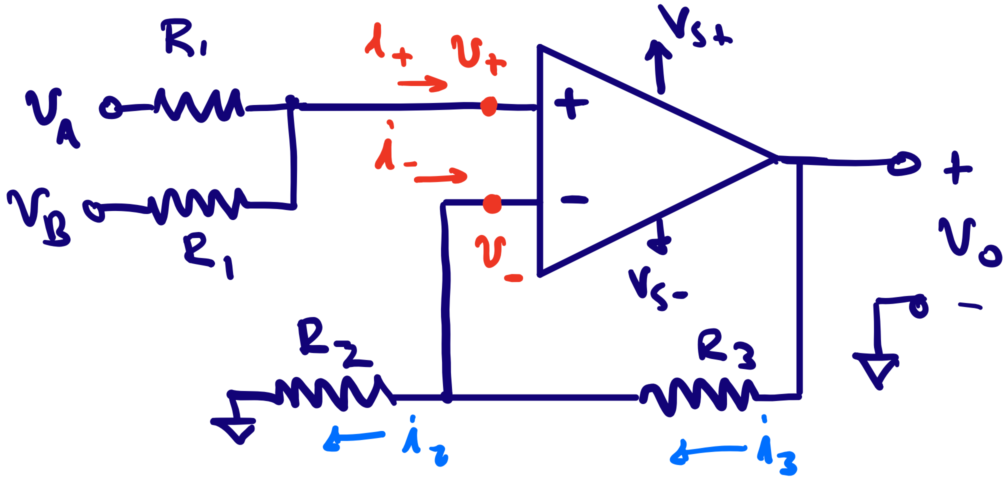
By superposition $v_o = v_{oA} + v_{oB}$

$\therefore \boxed{v_o = \frac{R_2}{R_1}(v_A - v_B)}$

This is a <u>differential amplifier</u>. It amplifies the difference between v_A, v_B by a gain R_2/R_1

Such amplifiers have many uses (see the demo!)

Another example: A "summing" amplifier:



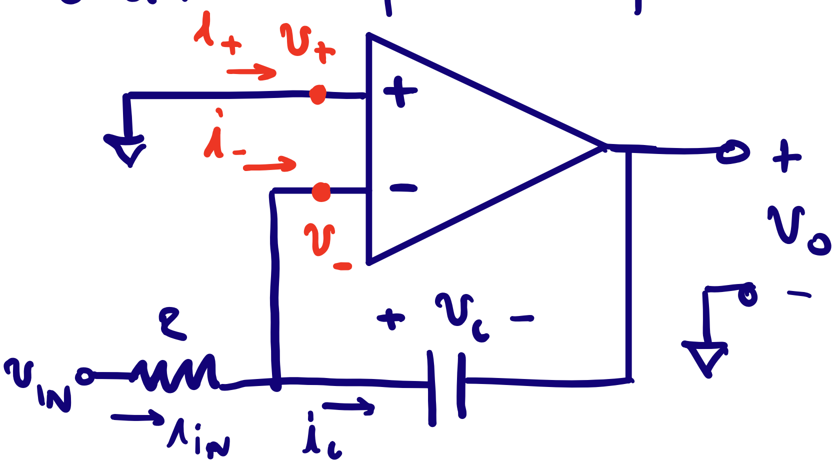
$i_+ = 0 \quad \therefore$ By superposition $\quad v_+ = \frac{1}{2}(v_A + v_B)$

$v_- = v_+ \, , i_- = 0 \quad \therefore \quad i_2 = i_3 = v_- \cdot \frac{1}{R_2}$

$$\therefore \boxed{v_o = \frac{R_2 + R_3}{R_2} \cdot \frac{1}{2}(v_A + v_B)}$$

We amplify the sum (or the average) of v_A, v_B

We can also implement important dynamic functions, e.g.



- $v_- = v_+ = 0 \quad \therefore$
  $i_{IN} = v_{IN}/R$
- $i_- = 0 \quad \therefore$
  $i_c = \frac{v_{IN}}{R} = C\frac{dv_c}{dt}$

$v_o = -v_c \quad \Rightarrow \quad v_{IN} = -RC\frac{dv_o}{dt}$

We've created an integrator!

$$\Rightarrow \boxed{v_o(t) = v_o(0) - \frac{1}{RC}\int_0^t v_{IN}(\tau)\,d\tau}$$

(output voltage is $\int$ on input voltage)

- Integrators have many uses: ① Find position from velocity
  ② Analog computers ...

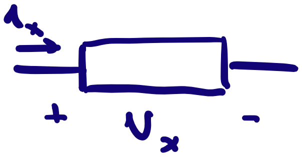
Note: by switching the resistor + capacitor, we could make a differentiator!

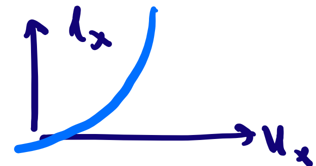
Could be used to find rate of change of a signal (e.g. compute velocity from a position signal.)

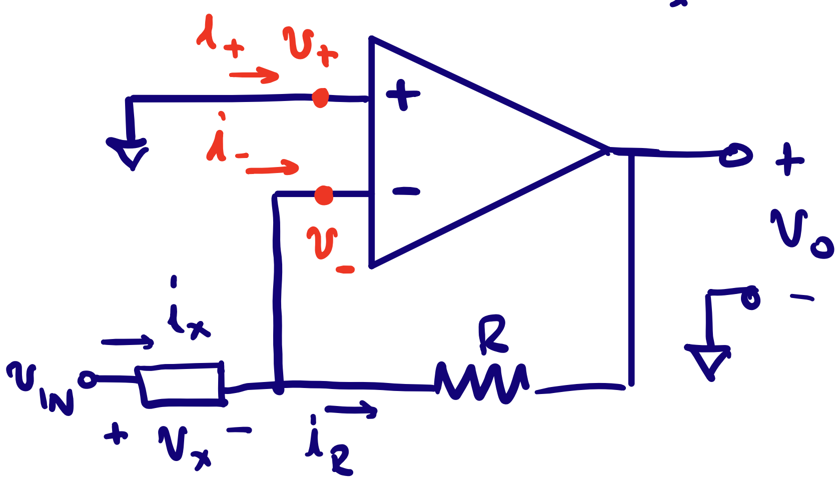
Op-Amp circuits are often used to realize useful nonlinear functions using nonlinear elements

Suppose we have a nonlinear resistive element:

$$i_x = f(v_x)$$





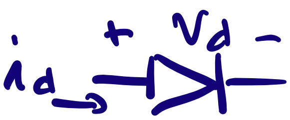


$$i_x = f(v_{IN})$$

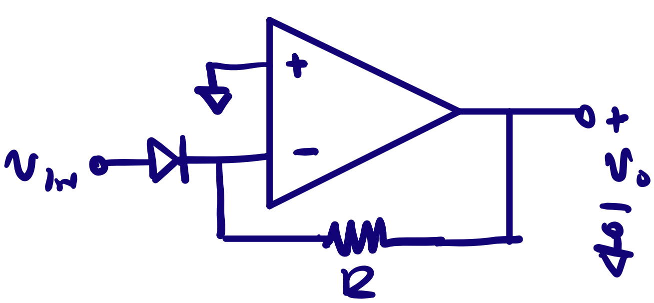
$$v_0 = 0 - i_x \cdot R$$

$$\boxed{v_o = -R f(v_{in})}$$

This can be useful for computation. For example, consider a diode:



$$i_d \simeq I_s\left[e^{v_d/v_T} - 1\right]$$ where $v_T \simeq 25\text{ mV}$ @ room temp



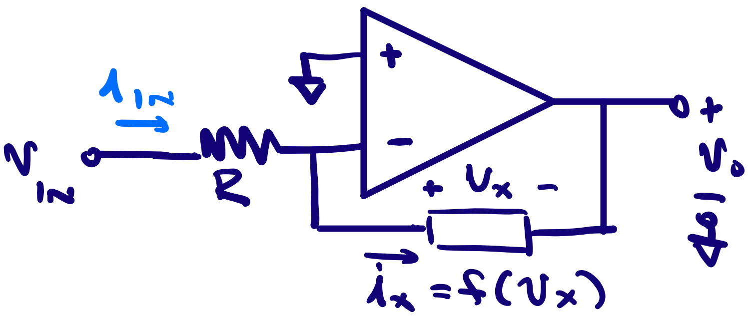
$$v_o \simeq -RI_s\left[e^{v_{IN}/v_T} - 1\right]$$

$$\boxed{v_o \simeq -RI_s e^{v_{IN}/v_T} \quad \text{for } v_{IN} \gg v_T}$$

we have an exponentiator circuit!

We can also <u>invert</u> nonlinearities with op-amp negative feedback.

Suppose we have a nonlinear resistor with a monotonic nonlinear relationship $i = f(v)$ such that the inverse function $f^{-1}(\cdot)$ exists and is unique. If we place this inside the feedback loop:

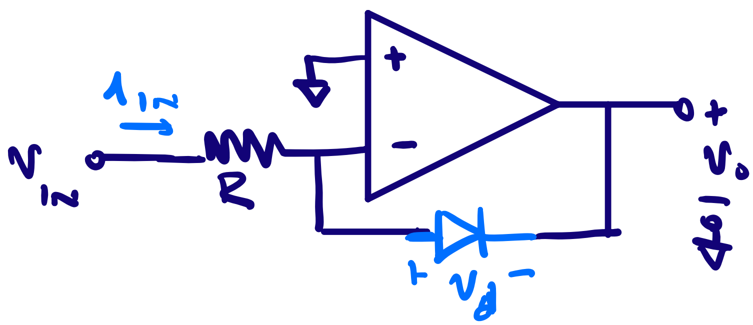


$$i_{IN} = \frac{1}{R} v_{IN}$$

$$\therefore \; i_{IN} = f(v_x)$$

$$\therefore \; \frac{1}{R} v_{IN} = f(-v_o)$$

$$\therefore \; \boxed{v_o = -f^{-1}\left(\frac{1}{R} \cdot v_{IN}\right)}$$



$$i_d = I_s\left[e^{v_d/v_T} - 1\right]$$

$$\simeq I_s\left[e^{v_d/v_T}\right]$$

$$\therefore \; v_d \simeq V_T \ln\left(\frac{i_d}{I_s}\right)$$

$$\boxed{v_o \simeq -V_T \ln\left(\frac{v_d}{I_s R}\right)}$$

We can take a logarithm!

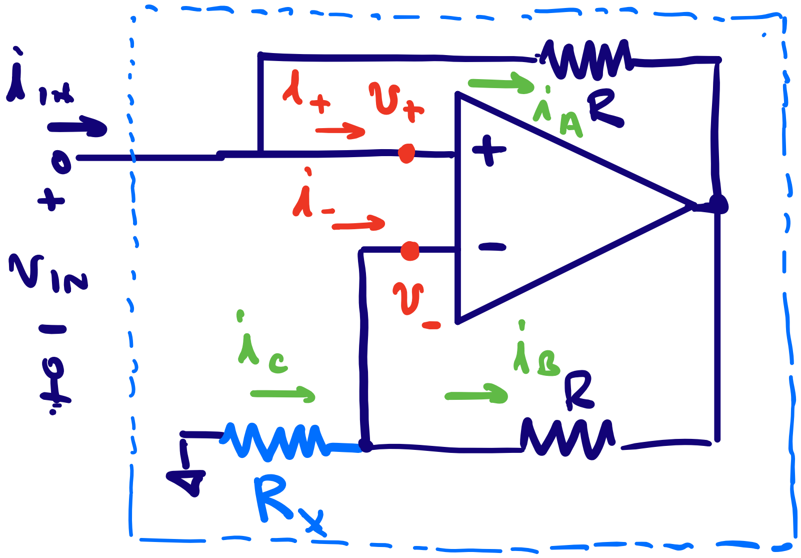
⇒ This can be used to build multipliers, dividers, ...

Op Amp Applications

We can also create useful effects with combinations of positive and negative feedback that yield stable operation in the "linear" region

Example: a "Negative Impedance Converter" (NIC)

Consider the Thévenin input resistance of this circuit



Since $v_+ = v_-$ and $i_+ = 0$

$$i_{IN} = i_A = i_B = i_C$$

$$v_{IN} = v_+ = v_- = -i_{IN} R_x$$

$$\therefore \boxed{\frac{v_{IN}}{i_{IN}} = -R_x}$$

So by placing R_x on one port of the NIC, we create an effective resistance $-R_x$ at the input port!

⇒ we can make the input look like a negative resistor?

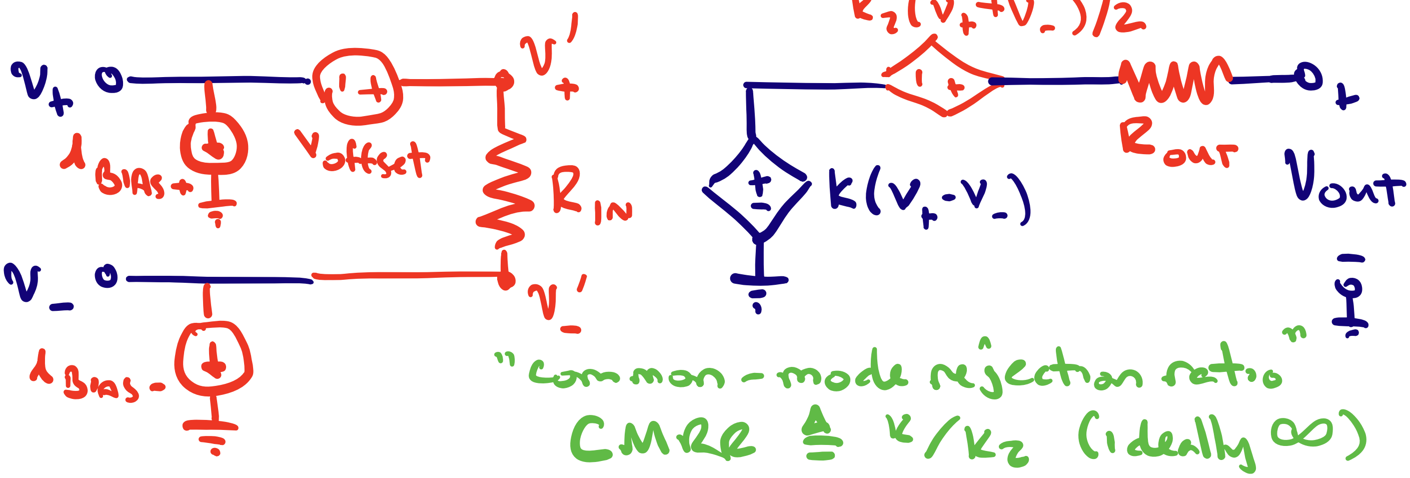
Where does the energy come from? → from the op amp power supplies (not shown).

There are many other useful functions we can realize by using op amps with feedback.

What about op amp nonidealities?

- Real op amp outputs are limited between supply rails
- Real op amps respond with limited speed

There are also other nonidealities that can be captured with added circuit elements:



"common-mode rejection ratio"

$CMRR \triangleq K/K_2$ (ideally ∞)

Typical Specs	Ideal	LM741	LF356
K	∞	$>10^5$	$>10^5$
CMRR	∞	>3200	$>10^4$
R_{OUT}	0	$<75\Omega$	0.1–40Ω (frequency dependent)
R_{IN}	∞	$>2M\Omega$	$>10^{12}\Omega$
V_{offset}	0	$<10mV$	$<10mV$
$i_{bias\,+/-}$	0	$<500nA$	$<100pA$

Achievable bandwidth also varies among op amps.