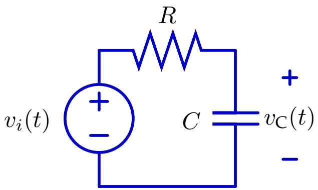
6.200 Circuits and Electronics

Week 11 Recitation:
Filtering, Bode Plots

Our Favorite Circuit

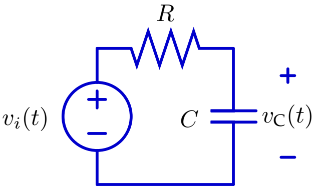


We started by looking at the "step response" of the circuit, i.e., its output when the input was of the form $v_i(t) = \begin{cases} V_1 & \text{if } t < t_0 \\ V_2 & \text{otherwise} \end{cases}$

Through a number of different methods, we found that:

$$\text{for } t \geq t_0, v_C(t) = V_1 + (V_2 - V_1)\left(1 - e^{-\frac{(t-t_0)}{RC}}\right)$$

Our Favorite Circuit Again



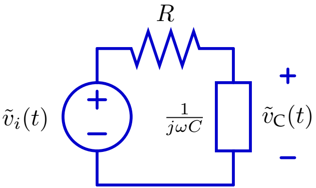
We later characterized its output when the input was of the form $v_i(t) = V\cos(\omega t + \phi_i)$.

In that case, we found:

$$v_C(t) = Be^{\frac{-t}{RC}} + \frac{V}{\sqrt{1 + (\omega RC)^2}}\cos(\omega t + \phi_i - \tan^{-1}(\omega RC))$$

If $t >> RC$, the first term can be ignored and our output looks like a (scaled and shifted) sinusoid at the same frequency as the input.

Our Favorite Circuit Yet Again



We then found that by treating the input instead as a complex exponential, we can simplify the math greatly. If $\tilde{v}_i(t) = \tilde{V}e^{j\omega t}$, then:

$$\tilde{v}_C(t) = \left(\frac{1}{j\omega RC + 1}\right)\tilde{V}e^{j\omega t}$$

And $\mathrm{Re}(\tilde{v}_C(t))$ is the solution to our original problem (with real-valued sinusoidal input).

Frequency Response

Last time, we introduced the *frequency response* as a way to characterize this circuit. Specifically:

$$\frac{\tilde{v}_{\text{C}}}{\tilde{v}_i} = \tilde{H}(\omega) = \frac{1}{1 + j\omega RC}$$

When the input to the system is a sinusoid at frequency ω_i and with phase shift ϕ_i, i.e., $v_i(t) = V\cos(\omega_i t + \phi_i)$, the output will be a sinusoid at the same frequency, scaled and shifted by values based on the frequency response:

$$v_{\text{C}}(t) = |\tilde{H}(\omega_i)| V\cos(\omega_i t + \phi_i + \angle\tilde{H}(\omega_i))$$

Bode Plots

A useful way to visualize the frequency response is a *Bode plot* (BOH-dee), which is actually a pair of plots: one showing how the gain (magnitude of $\tilde{H}$) changes with ω, and one showing how the phase offset (angle of $\tilde{H}$) changes with ω.

Here is a Bode plot for the circuit from the previous slides, for which $\tilde{H}(\omega) = \frac{1}{1+j\omega RC}$: