6.200 Midterm

Fall 2024

Name: **Answers**

Kerberos/Athena Username:

5 questions 2 hours

- Please **WAIT** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5″ × 11″ sheet of handwritten notes (both sides) as a reference. This sheet much be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices (including computers, calculators, phones, etc.).
- If you have questions, please **come to us at the front** to ask them.
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.
- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam.
- **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

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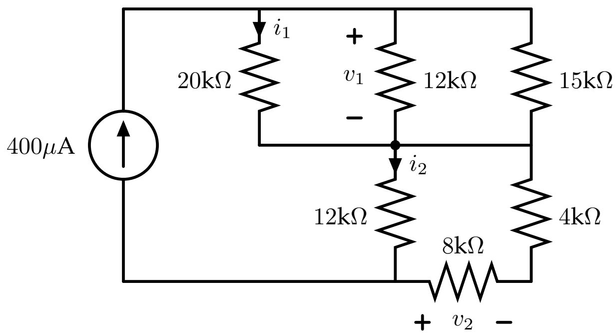
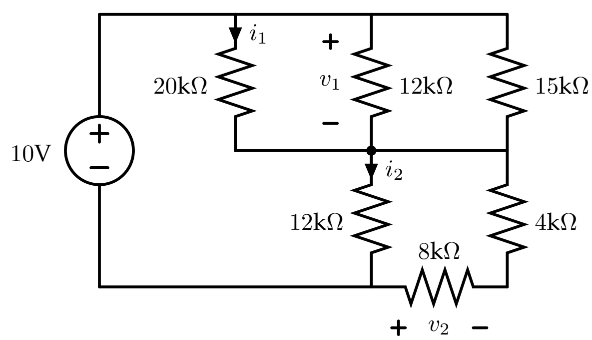
1 Sourcery

Consider the following three circuits, which are all slight variants of each other. The resistors are configured the same way in all three circuits; the only difference between them is the sources.

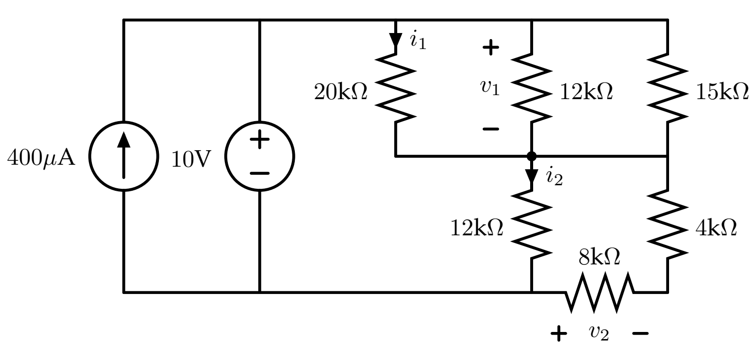
Variant 1 (Current Source)



Variant 2 (Voltage Source)



Variant 3 (Both Sources)



Answer the questions about these circuits on the facing page. Each of your answers should be a single number (or simplified fraction), with appropriate units.

1.1 Variant 1

Solve for the following values (including units) in variant 1 (with only the current source connected):

$v_1 = \boxed{2\text{V}}$ $v_2 = \boxed{-1.6\text{V}}$ $i_1 = \boxed{100\mu\text{A}}$ $i_2 = \boxed{200\mu\text{A}}$

The top three resistors are in parallel, so they form a current divider. The fraction of the current that flows through the left-most resistor is $\frac{1/20}{1/20+1/12+1/15} = 1/4$, so $i_1 = 100\mu\text{A}$.

The bottom three resistors also form a current divider (with $12\text{k}\Omega$ on one side and $4\text{k}\Omega + 8\text{k}\Omega$ on the other side). This cuts our input current in half, so $i_2 = 200\mu\text{A}$.

We can approach finding v_1 a couple of different ways. The simplest, perhaps, is to collapse the three top resistors into a single equivalent resistance of $5\text{k}\Omega$. Then we have $v_1 = 400\mu\text{A} \times 5\text{k}\Omega = 2\text{V}$.

Then we know that the current flowing through the right-hand branch of the bottom current divider is $200\mu\text{A}$, so $v_2 = -200\mu\text{A} \times 8\text{k}\Omega = -1.6\text{V}$ (note the direction in which v_2 is defined, which accounts for the negative sign here).

1.2 Variant 2

Solve for the following values (including units) in variant 2 (with only the voltage source connected):

$v_1 = \boxed{\frac{50}{11}\text{V}}$ $v_2 = \boxed{-\frac{40}{11}\text{V}}$ $i_1 = \boxed{\frac{5}{22}\text{mA}}$ $i_2 = \boxed{\frac{5}{11}\text{mA}}$

A good place to start is by simplifying the circuit down using resistor combinations, replacing the top three resistors with a single $5\text{k}\Omega$ resistance and the bottom three with a $6\text{k}\Omega$ resistance. Those two are in series, so we have a voltage divider, which we can use to solve for v_1 right away (it is just the voltage drop across the top resistor in our divider): $v_1 = \frac{5}{11} \times 10\text{V}$.

The drop across the bottom resistor in that image is $\frac{60}{11}\text{V}$, but that voltage undergoes another voltage division (the $4\text{k}\Omega$ and $8\text{k}\Omega$ resistors form a voltage divider) that cuts that voltage by a factor of $2/3$ to get v_2. We also have to pay careful attention to signs again, so $v_2 = -\frac{2}{3} \times \frac{60}{11}\text{V} = -\frac{40}{11}\text{V}$.

Once we know these voltage drops, then by Ohm's Law, $i_1 = \frac{50}{11}\text{V}/20\text{k}\Omega = \frac{5}{22}\text{mA}$. And also by Ohm's Law, $i_2 = \frac{60}{11}\text{V}/12\text{k}\Omega = \frac{5}{11}\text{mA}$.

1.3 Variant 3

Finally, solve for the following values (including units) in variant 3 (with both sources connected):

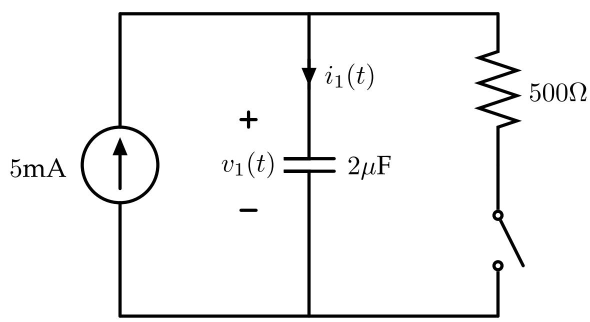
$v_1 = \boxed{\frac{50}{11}\text{V}}$ $v_2 = \boxed{-\frac{40}{11}\text{V}}$ $i_1 = \boxed{\frac{5}{22}\text{mA}}$ $i_2 = \boxed{\frac{5}{11}\text{mA}}$

The presence of the current source makes no difference when the voltage source is also connected. One way to see this is by superposition: with only the voltage source active, we have exactly the circuit from part 2; but with only the current source active, we have a short in parallel with the whole resistor combination on the right, so none of the current will actually flow through any of the resistors. Thus, the answer here is the same as in part 2.

2 Time Dynamics

For each of the two circuits that follow, sketch the indicated values as functions of time on the facing pages.

2.1 Part 1



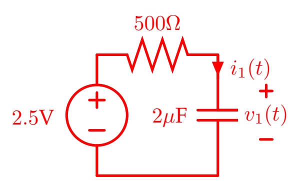
The switch is closed for all time $t < 0$, such that by time 0 it is operating in a steady state.

At time $t = 0$, the switch opens and remains open for 3ms.

At time $t = 3$ms, the switch closes again and remains closed forever.

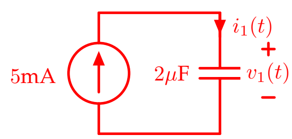
On the axes on the facing page, sketch $v_1(t)$ and $i_1(t)$ as functions of t, labeling all key values, slopes, asymptotes, and time constants.

With the switch closed, our circuit looks like the following (after Thévenizing the circuit around it):



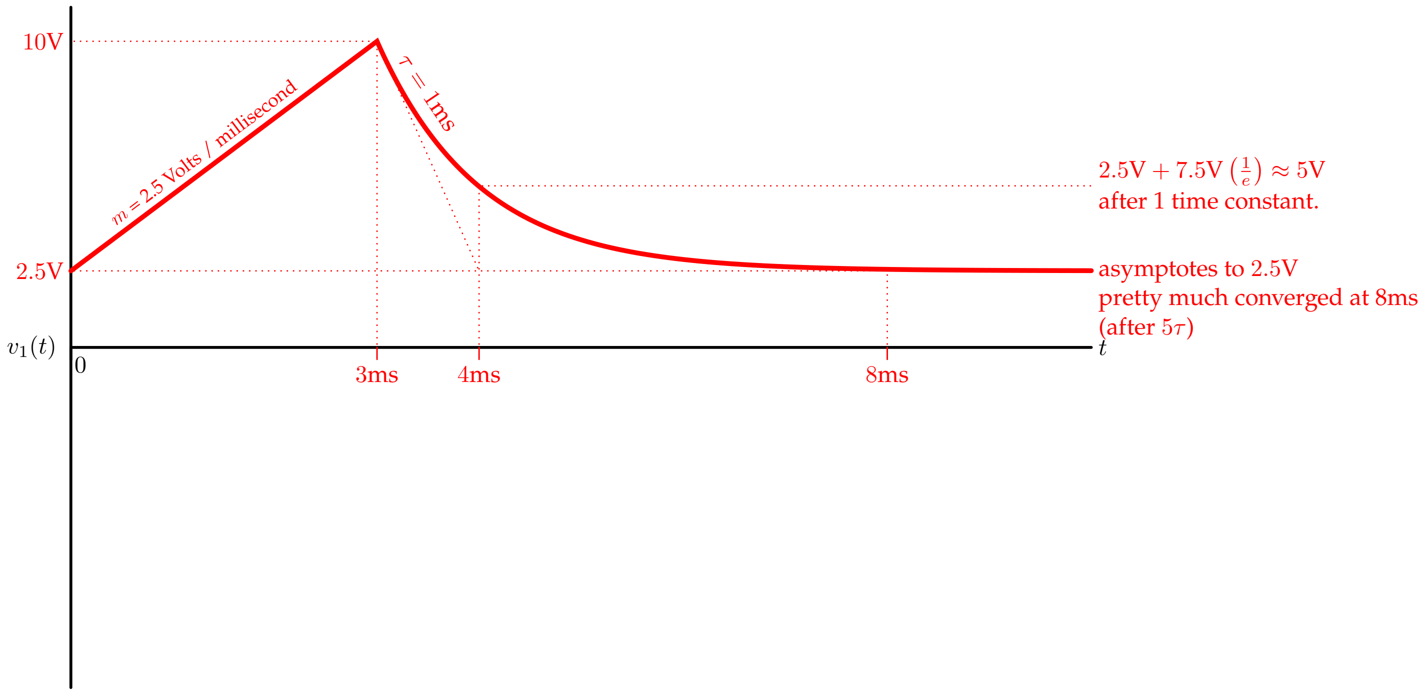
In this configuration, v_1 will converge to 2.5V following our usual exponential curve, with a time constant of $500\Omega \times 2\mu\text{F} = 1\text{ms}$. Since we had been sitting in this configuration for a long time, we know that $v_1(0) = 2.5\text{V}$.

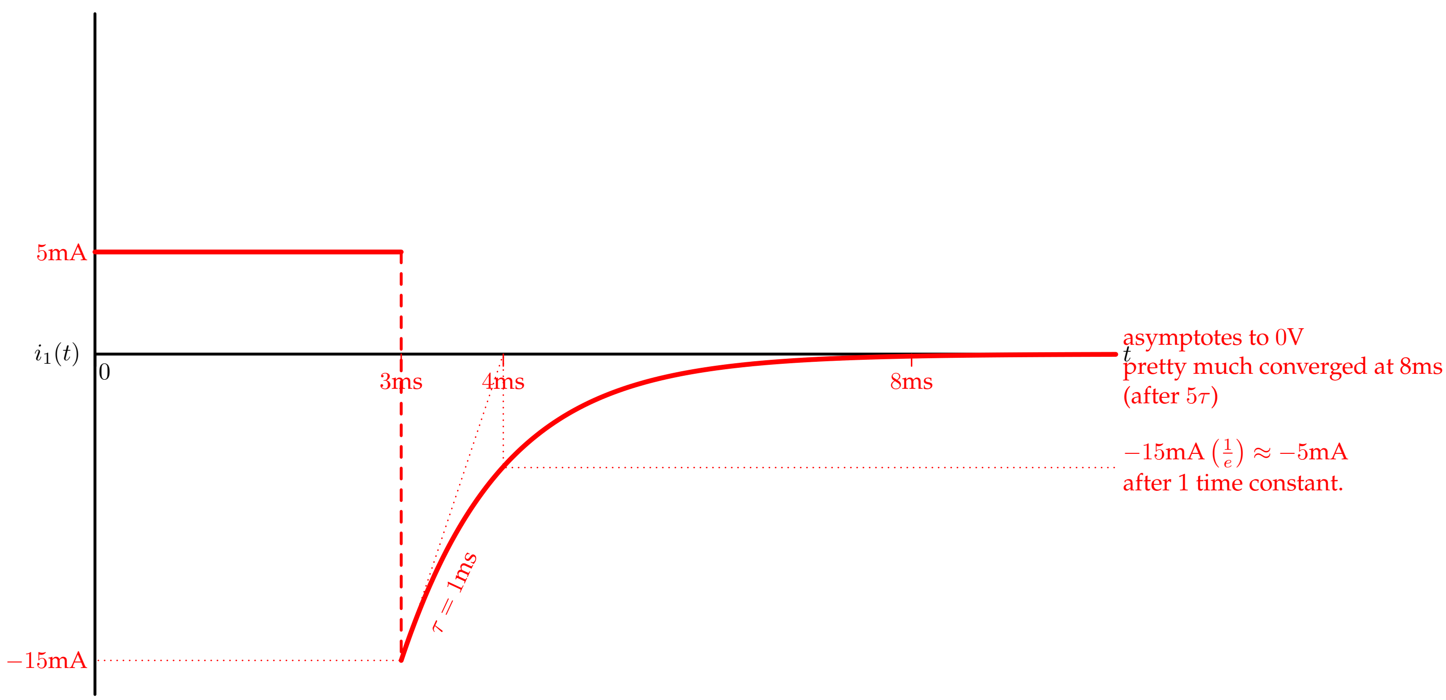
With the switch open, our circuit looks like the following instead:



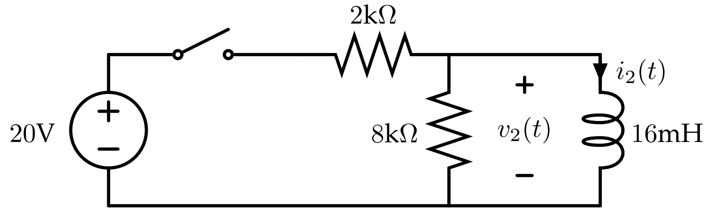
Here $i_1(t)$ is constant, so $v_1(t)$ will be changing linearly, with a slope $\frac{\text{d}}{\text{d}t}v_1(t) = \frac{i_1(t)}{C} = \frac{5\text{mA}}{2\mu\text{F}} = 2500\frac{\text{V}}{\text{s}} = 2.5\frac{\text{V}}{\text{ms}}$. Since the circuit is in this configuration for 3ms, the capacitor's voltage increases by a total of 7.5V during that time.

Putting this all together gives us the graphs on the facing page.


10V
2.5V
$v_1(t)$
0
$m = 2.5$ Volts / millisecond
$\tau = 1$ms
$2.5\text{V} + 7.5\text{V}\left(\frac{1}{e}\right) \approx 5\text{V}$
after 1 time constant.
asymptotes to 2.5V
pretty much converged at 8ms
(after 5τ)
t
3ms
4ms
8ms


5mA
$i_1(t)$
0
-15mA
3ms
4ms
8ms
t
$\tau = 1$ms
asymptotes to 0V
pretty much converged at 8ms
(after 5τ)
$-15\text{mA}\left(\frac{1}{e}\right) \approx -5\text{mA}$
after 1 time constant.

2.2 Part 2



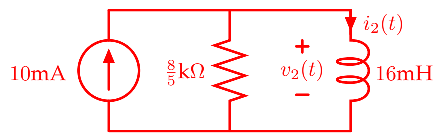
Before time 0, the switch is open and $i_2(0_-) = 0$.

The switch is then closed from $0 \leq t < 50\mu$s.

At $t = 50\mu$s, the switch is opened again, and it remains open forever.

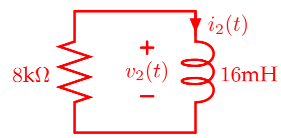
On the axes on the facing page, sketch $v_2(t)$ and $i_2(t)$ as functions of t, labeling all key values, slopes, asymptotes, and time constants.

With the switch closed, our circuit looks like the following (after Nortonizing the circuit around it):



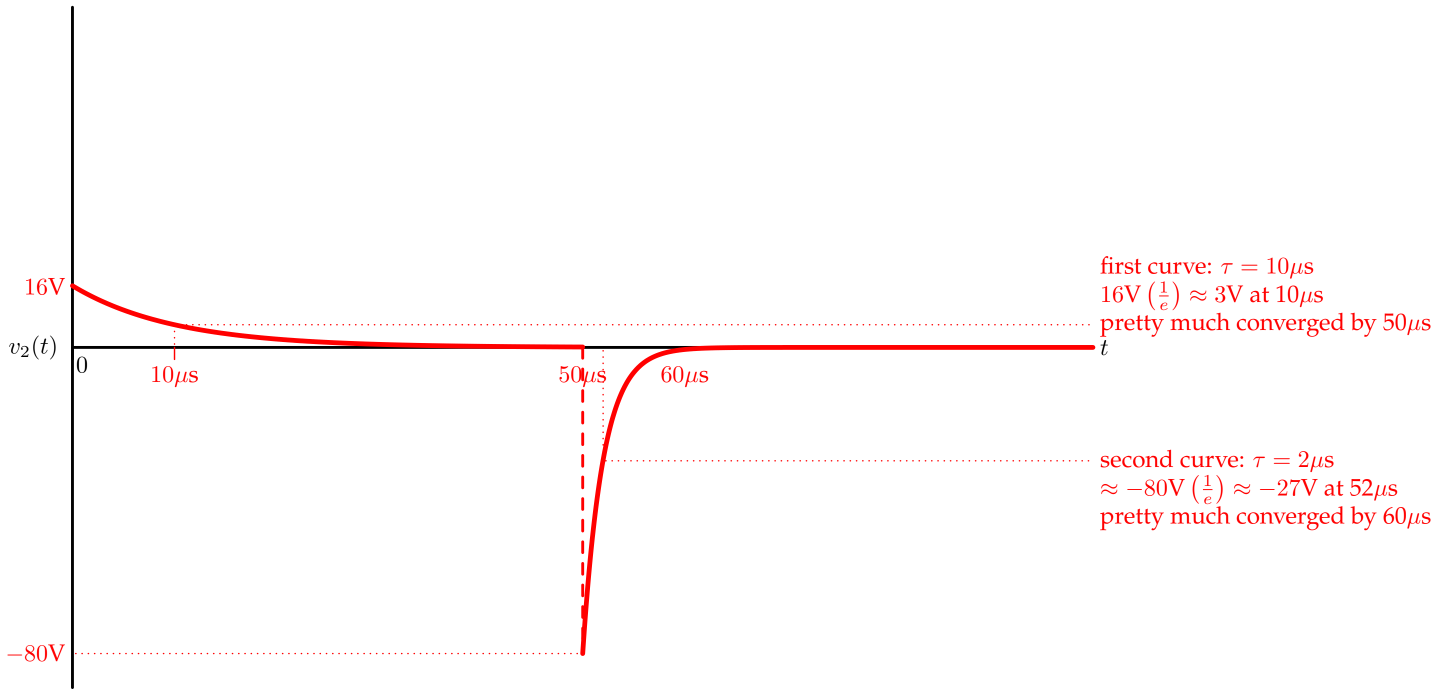
In this configuration, i_1 will converge to 10mA following our usual exponential curve, with a time constant of $16\text{mH}/(8/5\text{k}\Omega) = 10\mu\text{s}$.

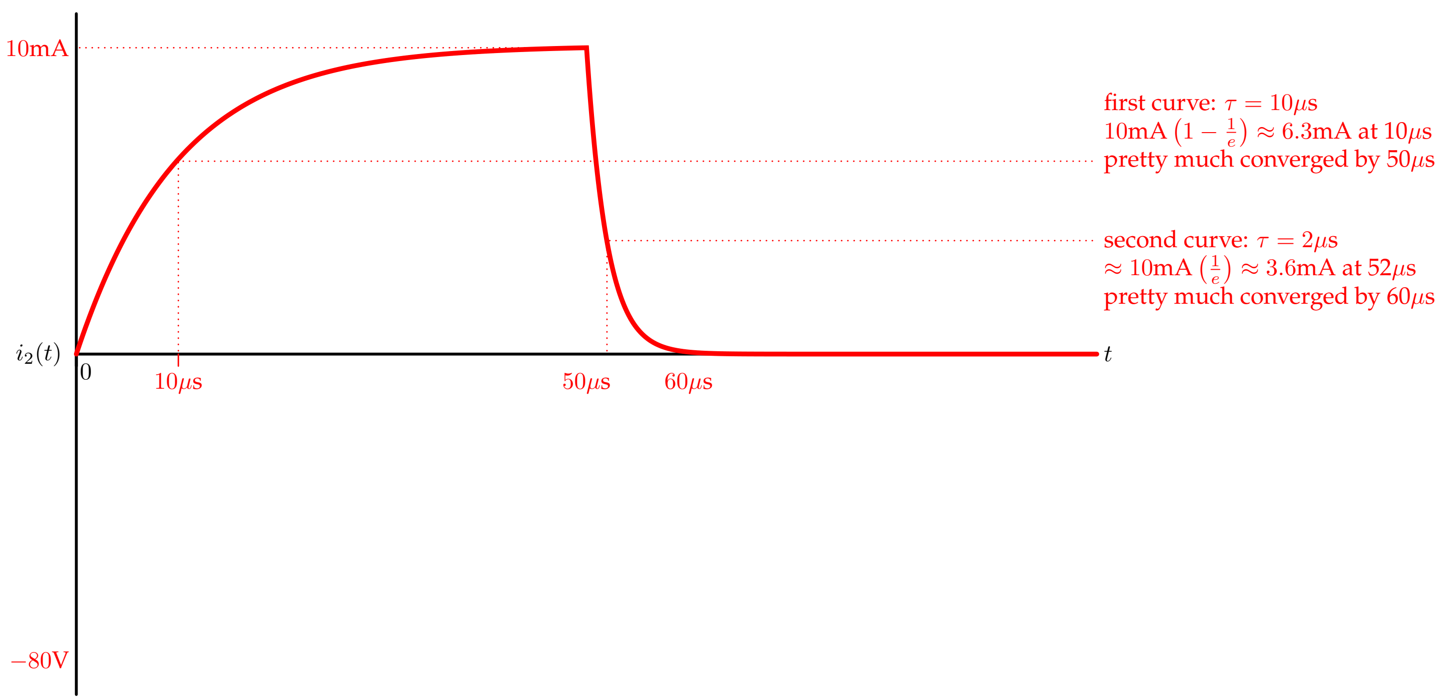
With the switch open, our circuit looks like the following instead:



In this configuration, i_1 will converge to 0 following our usual exponential curve, with a time constant of $16\text{mH}/8\text{k}\Omega = 2\mu\text{s}$ (a much faster response, which should be visible on our graphs).

Putting this all together gives us the graphs on the facing page.

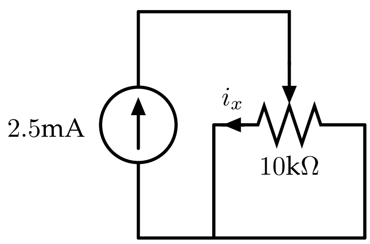

16V
$v_2(t)$
0
10μs
50μs
60μs
t
-80V
first curve: $\tau = 10\mu$s
16V $\left(\frac{1}{e}\right) \approx$ 3V at 10μs
pretty much converged by 50μs
second curve: $\tau = 2\mu$s
≈ -80V $\left(\frac{1}{e}\right) \approx -27$V at 52μs
pretty much converged by 60μs

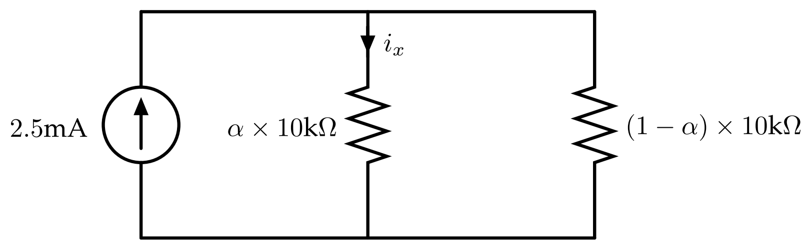

10mA
$i_2(t)$
0
10μs
50μs
60μs
t
-80V
first curve: $\tau = 10\mu$s
10mA $\left(1 - \frac{1}{e}\right) \approx$ 6.3mA at 10μs
pretty much converged by 50μs
second curve: $\tau = 2\mu$s
$\approx$ 10mA $\left(\frac{1}{e}\right) \approx$ 3.6mA at 52μs
pretty much converged by 60μs

Worksheet (intentionally blank)

3 Potentiometers

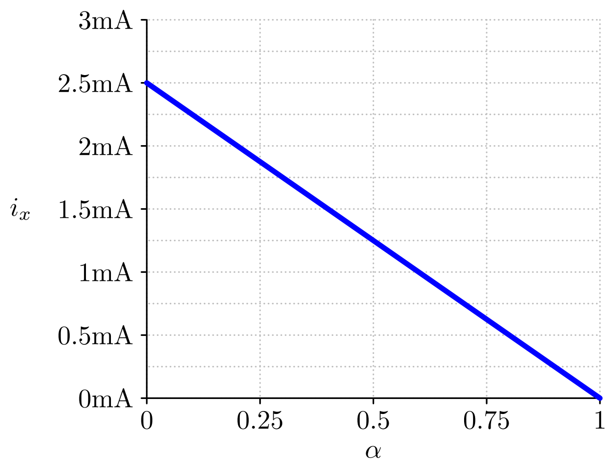
Problems from problem sets 2 and 4 explored using potentiometers as variable voltage dividers. In this problem, we'll explore using a potentiometer in a different way, as a current divider instead. We've drawn our starting circuit two different ways below, one that shows the potentiometer as a single device, and one that shows both individual resistors inside of the potentiometer (recall that α is a number in the range $0 \leq \alpha \leq 1$ that indicates how far the potentiometer as been turned):





As in those problem set questions, here we'll look at how a variable within the circuit changes as we turn the potentiometer (i.e., as α changes).

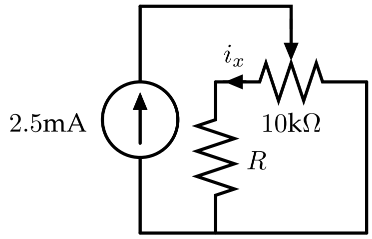
In the configuration shown above, the relationship between α and i_x is indicated on the following graph:



The following two pages show three variants of this circuit, each with an additional resistor added. For each variant, the value of R may be 1Ω, or $1\text{k}\Omega$, or $1\text{M}\Omega$.

Additionally, the last page of this exam (page 29, which you may remove from this handout) contains several graphs of functions relating α and i_x. For each variant and resistance value, indicate which of the graphs best represents the relationship from that circuit. If the resulting graph would be indistinguishable from that of the original circuit, write `same` in the box instead.

3.1 Variant 1



When $R = 1\Omega$, what graph shows the relationship between i_x and α? same

When $R = 1\text{k}\Omega$, what graph shows the relationship between i_x and α? B

When $R = 1\text{M}\Omega$, what graph shows the relationship between i_x and α? D

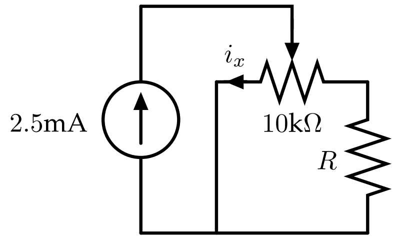
$i_x = \frac{(1-\alpha)10\text{k}\Omega}{10\text{k}\Omega + R} \times 2.5\text{mA}$ (note that unlike the p-set problems, this relationship is a line)

When $R = 1\Omega$, its effect on the graph is negligible (too small to be seen on this scale), so the graph is practically identical to the original.

When $R = 1\text{M}\Omega$, the effect is that very little current wants to flow down the left branch, so i_x sits near 0 for the whole range.

When $R = 1\text{k}\Omega$, we can find endpoint values to characterize our line. When $\alpha = 0$, we have $i_x = 2.5\text{mA} \times 10/11 \approx 2.25\text{mA}$. When $\alpha = 1$, we have $i_x = 0$.

3.2 Variant 2



When $R = 1\Omega$, what graph shows the relationship between i_x and α? same

When $R = 1\text{k}\Omega$, what graph shows the relationship between i_x and α? C

When $R = 1\text{M}\Omega$, what graph shows the relationship between i_x and α? E

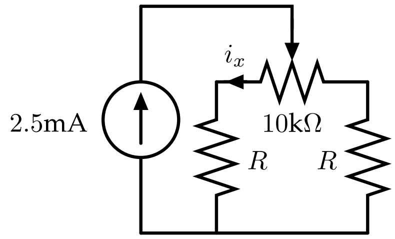
$i_x = \dfrac{(1-\alpha)10\text{k}\Omega + R}{10\text{k}\Omega + R} \times 2.5\text{mA}$ (as with the first variant, this relationship is a line)

Just like when $R = 1\Omega$, its effect on the graph is negligible (too small to be seen on this scale), so the graph is practically identical to the original.

When $R = 1\text{M}\Omega$, the effect is that very little current wants to flow down the right branch, so i_x sits near 2.5mA for the whole range.

When $R = 1\text{k}\Omega$, we can find endpoint values to characterize our line. When $\alpha = 0$, we have $i_x = 2.5\text{mA}$. When $\alpha = 1$, we have $i_x = 2.5\text{mA} \times \frac{1}{11} \approx 0.25\text{mA}$.

3.3 Variant 3



When $R = 1\Omega$, what graph shows the relationship between i_x and α? same

When $R = 1\text{k}\Omega$, what graph shows the relationship between i_x and α? A

When $R = 1\text{M}\Omega$, what graph shows the relationship between i_x and α? F

$i_x = \dfrac{(1-\alpha)10\text{k}\Omega + R}{10\text{k}\Omega + 2R} \times 2.5\text{mA}$ (note that unlike the p-set problems, this relationship is a line)

When $R = 1\Omega$, its effect on the graph is negligible (too small to be seen on this scale), so the graph is practically identical to the original.

When $R = 1\text{M}\Omega$, the effect is that the resistance inside the pot doesn't matter all that much; we've pretty much just got two $\approx 1\text{M}\Omega$ resistors making up our current divider, so about half of the current will flow down each branch (regardless of α).

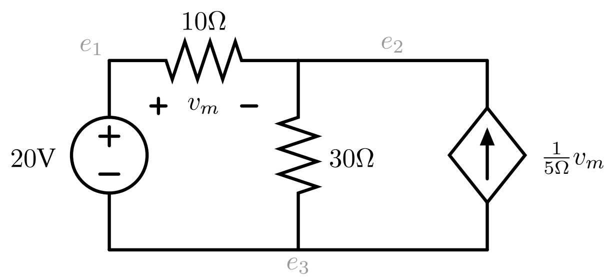
When $R = 1\text{k}\Omega$, we can find endpoint values to characterize our line. When $\alpha = 0$, we have $i_x = 2.5\text{mA} \times 11/12 \approx 2.3\text{mA}$. When $\alpha = 1$, we have $i_x = 2.5\text{mA} \times 1/12 \approx 0.2\text{mA}$.

Worksheet (intentionally blank)

4 Dependent Sources

4.1 Circuit 1

Consider the circuit below:



Choose one of the labeled nodes to be your 0V reference point and solve for the other node potentials relative to that potential. Enter your answers (including units) in the boxes below:

$e_1 = 20\text{V}$

$e_2 = 18\text{V}$

$e_3 = 0\text{V}$

If we choose $e_3 = 0\text{V}$, then $e_1 = 20\text{V}$ because of the voltage source.

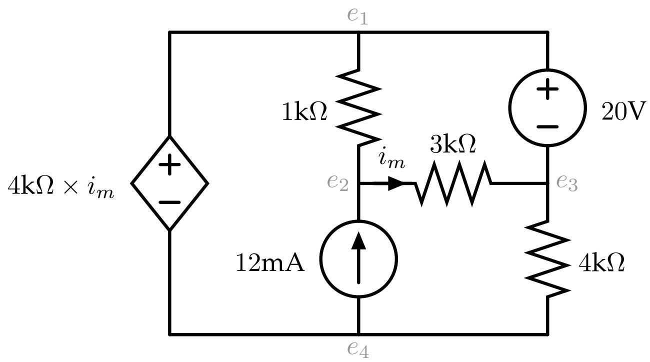
To find e_2, we write KCL at that node, taking into account that $v_m = 20\text{V} - e_2$:

$$\frac{20\text{V} - e_2}{10\Omega} + \frac{20\text{V} - e_2}{5\Omega} = \frac{e_2}{30\Omega}$$

Solving, we find $10e_2 = 180\text{V}$

4.2 Circuit 2

Now consider the circuit below:



Choose one of the labeled nodes to be your 0V reference point and solve for the other node potentials relative to that potential. Enter your answers (including units) in the boxes below:

$e_1 = 32\text{V}$

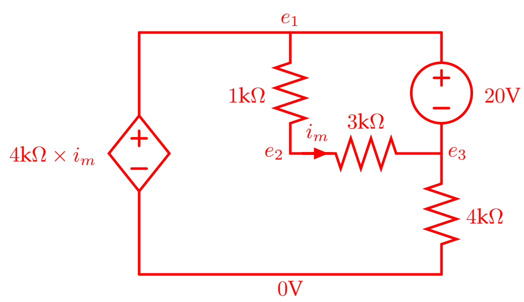
$e_2 = 36\text{V}$

$e_3 = 12\text{V}$

$e_4 = 0\text{V}$

There are multiple approaches we could take here. Superposition is going to be the easiest. In order to apply superposition here, we'll have to solve two circuits. We'll do the analysis below with e_4 as our reference, but you could have chosen any of the nodes.

Let's start by zeroing out the current source:



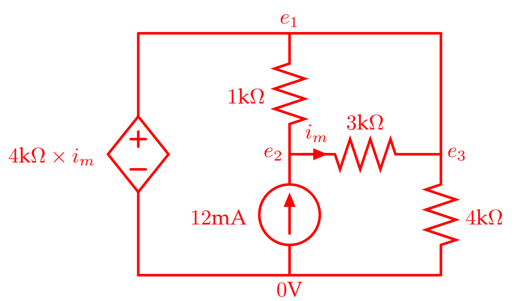
We can start by noticing that the 20V source is connected to two resistors in series ($4\text{k}\Omega$ total resistance), so $i_m = 5\text{mA}$.

From there, via the dependent voltage source, $e_1 = 20\text{V}$ relative to e_4.

Then, the 20V source says that $e_1 - e_3 = 20\text{V}$, so we must have $e_3 = 0$.

Finally, we can find e_2 using Ohm's law on the $3\text{k}\Omega$ resistor. $e_2 - 0 = 3\text{k}\Omega \times 5\text{mA} = 15\text{V}$.

Having solved for all of those node potentials, we then make ourselves another circuit to solve by zeroing out the independent voltage source:



Here, the $1\text{k}\Omega$ and $3\text{k}\Omega$ resistors are in parallel; they form a current divider, so $i_m = \frac{1}{4}(12\text{mA}) = 3\text{mA}$.

The dependent voltage source, then, says that $e_1 = 4\text{k}\Omega \times 3\text{mA} = 12\text{V}$.

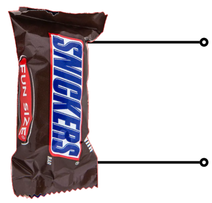
Because of the short $e_3 = e_1 = 12\text{V}$.

Then we can again use Ohm's law on the $3\text{k}\Omega$ resistor to find: $e_2 = (3\text{mA})(3\text{k}\Omega) + 12\text{V} = 21\text{V}$.

The answers for the original circuit come from summing up the corresponding node voltages from each of these solutions.

5 Trick-or-Circuit

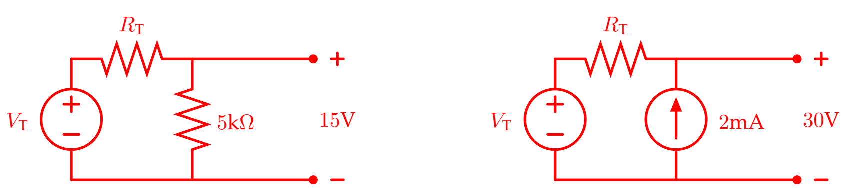
After a night of trick-or-treating, you discover that your neighbor has given you what looks like a fun-size Snickers candy bar. But the presence of two metal terminals on it make you suspect that they may be playing a prank on you, giving you what looks like a Snickers bar but is actually a circuit consisting of only linear components (classic!). To test your theory, you connect two wires to those terminals, like so:



You feel vindicated in your earlier assessment when you make the following measurements after hooking up some additional components to the prank "candy bar;" there's clearly more in there than just nougat, caramel, peanuts, and milk chocolate!

You decide to investigate a little further. For each of the connections on the facing page, indicate what voltage would be measured so as to be consistent with your measurements from above.

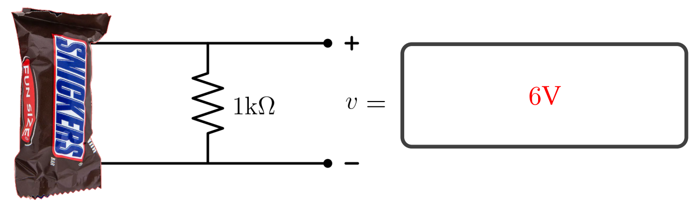
Because the circuit consists only of linear components, we can model it with either a Thévenin or a Norton. Thus, we can redraw these circuits as:

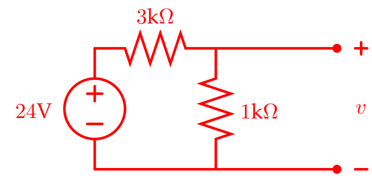


Each of these gives us an equation:

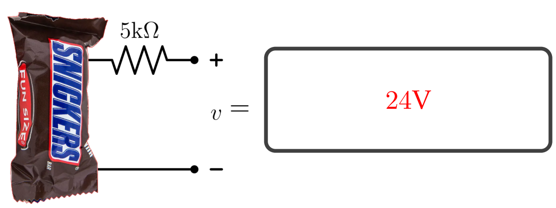
$$\frac{5\text{k}\Omega}{5\text{k}\Omega + R_T} V_T = 15\text{V} \qquad\qquad V_T + (2\text{mA})R_T = 30\text{V}$$

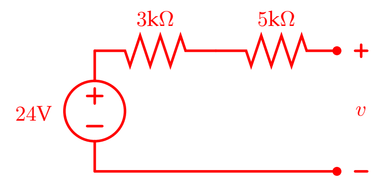
Solving this system of equations, we find $V_T = 24\text{V}$ and $R_T = 3\text{k}\Omega$.



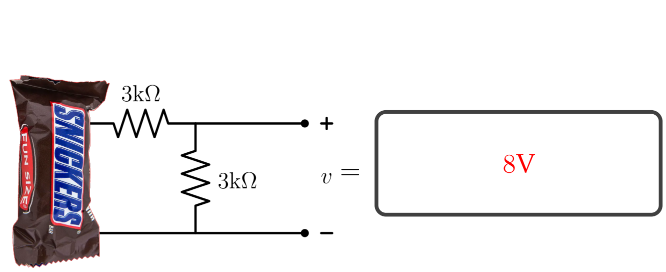


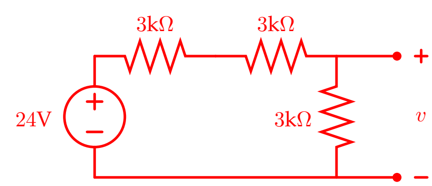
This is a simple voltage divider that cuts our Thévenin voltage by a factor of 4.



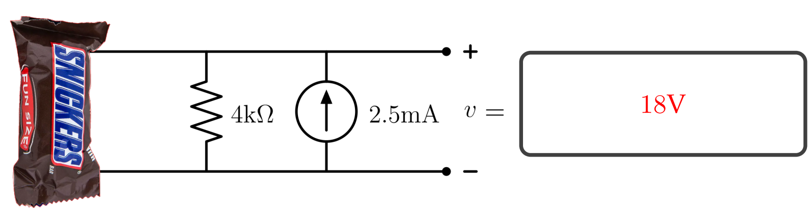


No current flows through the resistors, so v is just our Thévenin voltage.

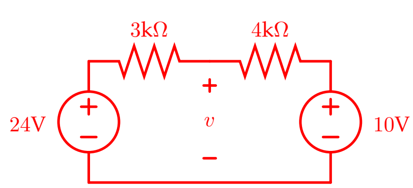




Here we have a voltage divider again, but we have to be careful since we're only measuring the drop across the bottom 3kΩ resistor; so this voltage divider cuts our Thévenin voltage by a factor of 3.

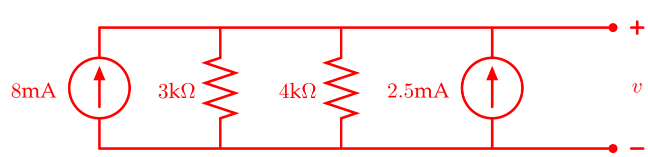


Here we have a couple of reasonable options. We could solve directly using the node method, but an easier approach involves either Thévenizing these two external components or Nortonizing what's inside the candy bar, which simplifies the process:



voltage divider

$$v = 10\text{V} + \left(\tfrac{4}{7}\right)(24\text{V} - 10\text{V})$$
$$= 10\text{V} + \left(\tfrac{4}{7}\right)14\text{V} = 18\text{V}$$



everything in parallel

$$v = (8\text{mA} + 2.5\text{mA})\,(3\text{k}\Omega \,||\, 4\text{k}\Omega)$$
$$= (10.5\text{mA})\left(\tfrac{12}{7}\text{k}\Omega\right) = 18\text{V}$$

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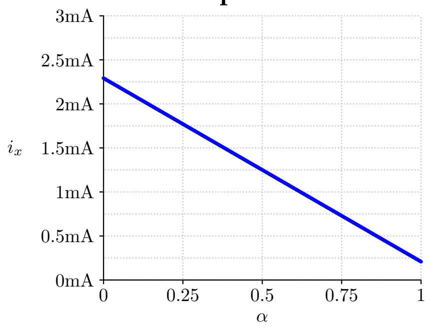
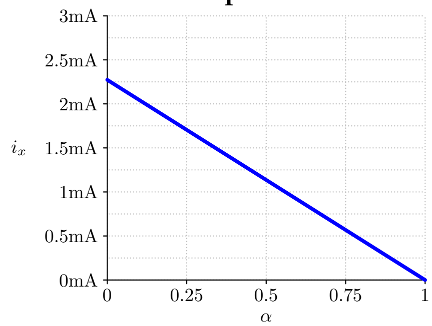
Graphs for Potentiometer Question

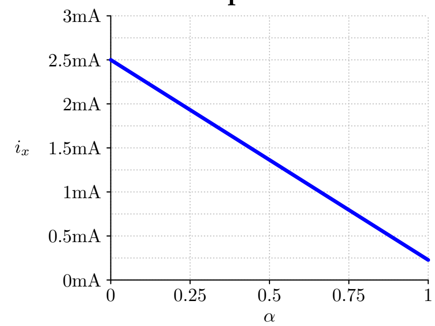
Graph A



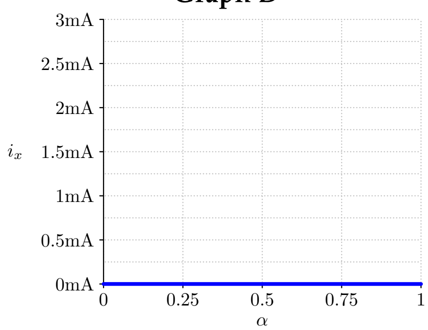
Graph B



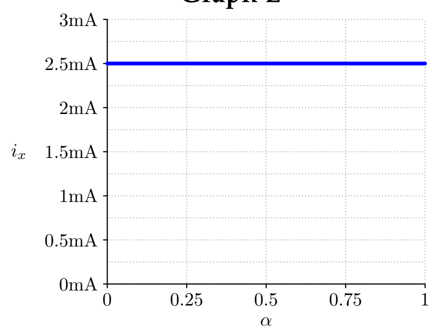
Graph C



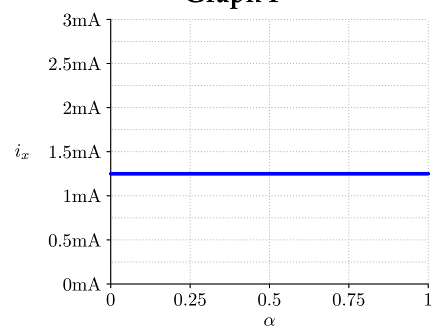
Graph D



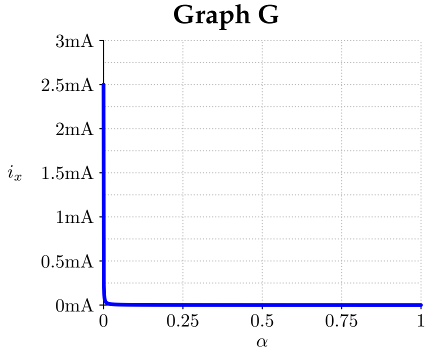
Graph E



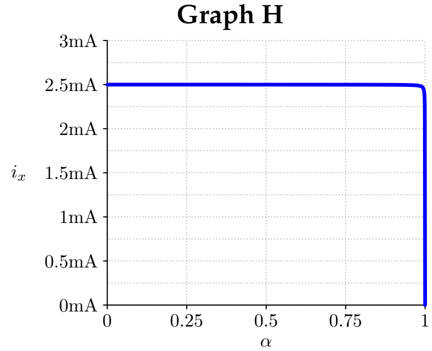
Graph F



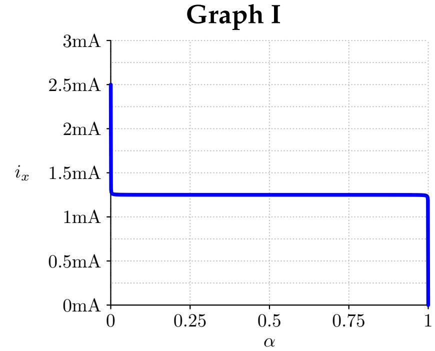
Graph G



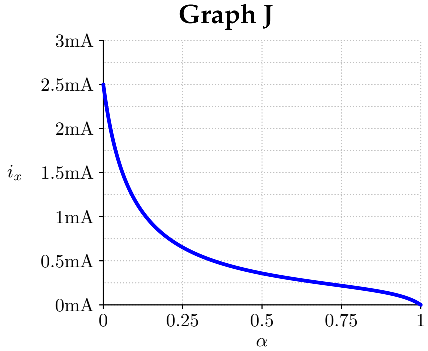
Graph H



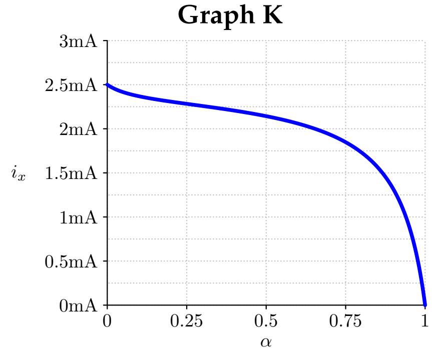
Graph I



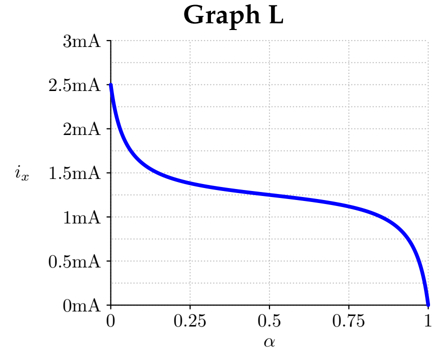
Graph J



Graph K



Graph L



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