

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science

6.200 – Circuits & Electronics  
Spring 2024

Quiz #1

13 March 2024

Name: \_\_\_\_\_ Solutions

MIT Kerberos Username: \_\_\_\_\_

Recitation Time: 11 12 1

- There are 15 pages in this quiz, including this cover page.
- Please put your name and Kerberos ID in the spaces provided above, and circle the time of your recitation.
- Please do not remove any pages from this quiz.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. *When finished with each part, clearly write your answer for that part into the corresponding answer box or graph.*
- Make sure all work is on pages with QR codes, and **do not write on the QR codes.**
- *All numerical answers require proper units.*
- *In order to guarantee receipt of full credit, all answers should be justified by supporting math and/or explanations.*
- This is a closed-book quiz, but calculators and a single two-sided page of notes are allowed.
- Good luck!



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2009-10-15

Page 2

Atlanta, Georgia

October 15, 2009

Dear Dr. [Name]

I am pleased to inform you that your application for the position of [Title] has been reviewed and your qualifications have been found to be excellent.

We are pleased to offer you the position of [Title] at the [Level] grade, effective [Date].

Your starting salary will be \$[Amount] per year, plus a [Percentage] percentage of your salary in the form of a [Type] benefit.

The [Type] benefit is a [Type] benefit that is designed to provide you with a [Type] benefit.

Your [Type] benefit is a [Type] benefit that is designed to provide you with a [Type] benefit.

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02,0,0022

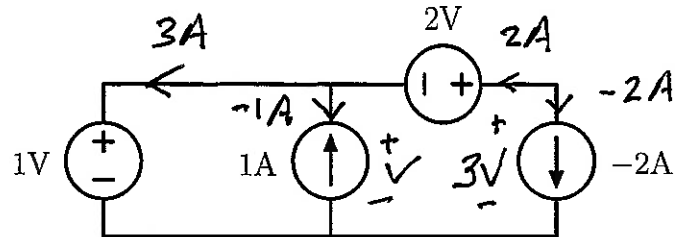


02,1,0022

**Problem 1: Miscellany – 35%**

All parts of this problem are independent of the other parts.

- (1A) Determine the power sourced (provided) by each of the four sources in the circuit shown below. If a source sinks (receives) power, then consider its sourced power to be negative. Proper units are required.



1 V Source: $-3\text{ W}$	1 A Source: $1\text{ W}$
2 V Source: $-4\text{ W}$	-2 A Source: $6\text{ W}$

See the voltages and currents labeled for each device according to the passive sign convention. Sourced power is then  $-v_i$ .



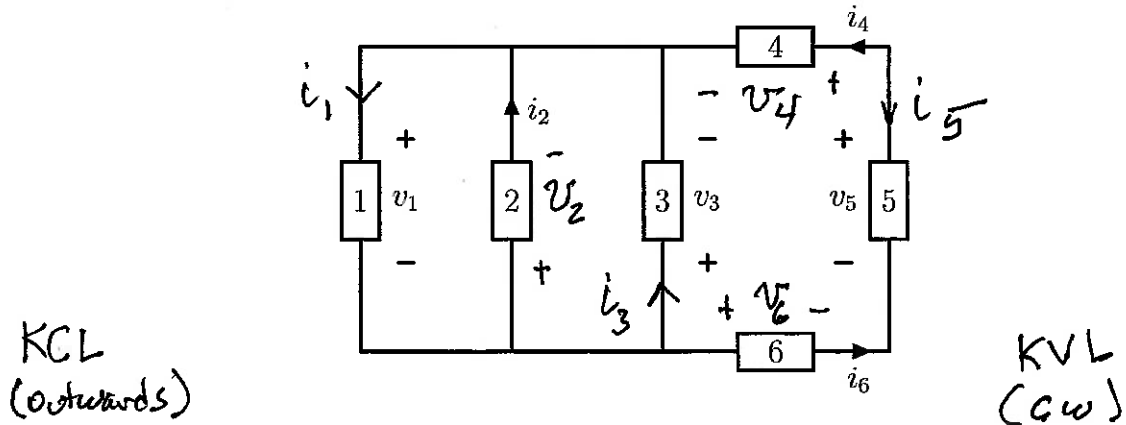


04,0,0022



04,1,0022

- (1B) The circuit shown below comprises six branches with partially labeled branch voltages and currents. Add the missing labels to the circuit diagram according to the passive sign convention. A brute-force analysis of the circuit would require the use of six equations obtained from KCL and KVL. Provide six such equations that are necessary and sufficient to carry out a brute-force analysis.



Eqn #1: $i_1 - i_2 - i_3 - i_4 = 0$	Eqn #2: $v_1 + v_2 = 0$
Eqn #3: $i_4 + i_5 = 0$	Eqn #4: $-v_2 + v_3 = 0$
Eqn #5: $-i_5 - i_6 = 0$	Eqn #6: $-v_3 + v_4 - v_5 + v_6 = 0$

could also have

$$-i_1 + i_2 + i_3 + i_6 = 0$$

could also have

$$v_1 + v_4 - v_5 + v_6 = 0$$

among others



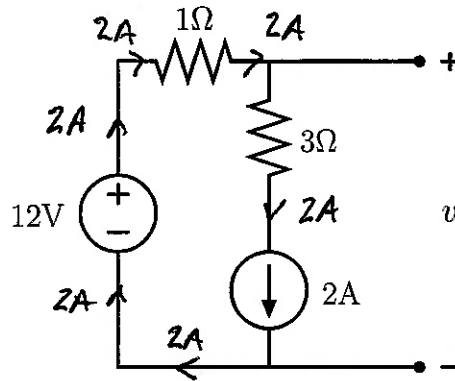


06,0,0022



06,1,0022

- (1C) Determine the voltage  $v$  in the circuit shown below. *Proper units are required.* If the voltage cannot be determined, enter "N/A" in the answer box.



$v = 10 \text{ V}$

The current in the left-hand loop is 2A as indicated. Thus  $v = 12 \text{ V} - 2\text{A} \cdot 1\Omega = 10 \text{ V}$ .





17.10

1. A 100 ohm resistor is connected in series with a 200 ohm resistor. The combination is connected to a 300V DC source. Calculate the current flowing through the circuit.

17.11



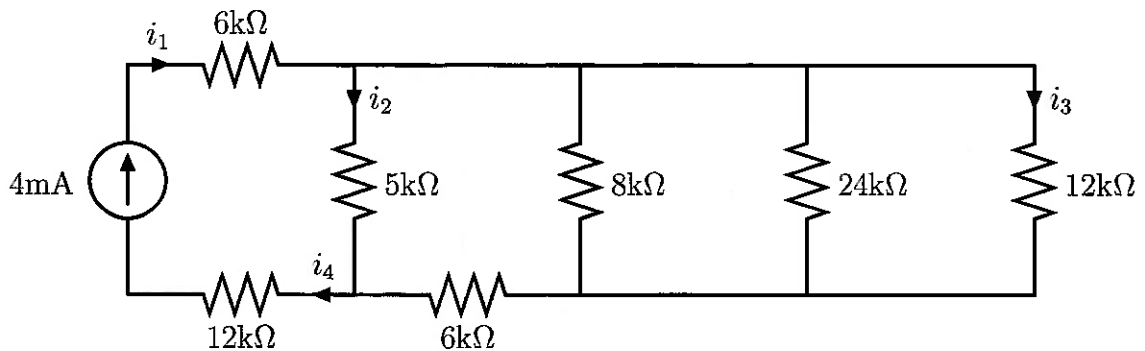
08,0,0022



08,1,0022



(1D) The circuit shown below has four labeled currents. Determine numerical values for all four currents. *Proper units are required.*



$i_1 = 4 \text{ mA}$	$i_2 = \frac{8}{3} \text{ mA}$
$i_3 = \frac{4}{9} \text{ mA}$	$i_4 = 4 \text{ mA}$

By inspection / KCL  $i_1 = i_4 = 4 \text{ mA}$ .

Next reduce the four rightmost resistors as follows.

$$\begin{array}{c} \circ \\ \text{---} \\ \text{---} \\ \text{---} \\ \circ \end{array} \left\{ \begin{array}{l} 8 \text{ k}\Omega \\ 24 \text{ k}\Omega \\ 12 \text{ k}\Omega \end{array} \right. = \frac{1}{\frac{1}{8 \text{ k}\Omega} + \frac{1}{24 \text{ k}\Omega} + \frac{1}{12 \text{ k}\Omega}} = \frac{24 \text{ k}\Omega}{3+1+2} = 4 \text{ k}\Omega$$

then  $\begin{array}{c} \circ \\ \text{---} \\ \text{---} \\ \text{---} \\ \circ \end{array} \left\{ \begin{array}{l} 4 \text{ k}\Omega \\ 6 \text{ k}\Omega \end{array} \right. = \frac{10}{5+10} \text{ k}\Omega$ . Finally use current dividers

$$i_2 = \frac{10}{5+10} \cdot 4 \text{ mA} = \frac{8}{3} \text{ mA}$$

$$\begin{aligned} i_3 &= \frac{\frac{1}{12}}{\frac{1}{8} + \frac{1}{24} + \frac{1}{12}} \cdot \frac{4}{3} \text{ mA} \\ &= \frac{2}{3+1+2} \cdot \frac{4}{3} \text{ mA} = \frac{4}{9} \text{ mA} \end{aligned}$$



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Handwritten text below the diagram, possibly describing the circuit components or providing calculations.

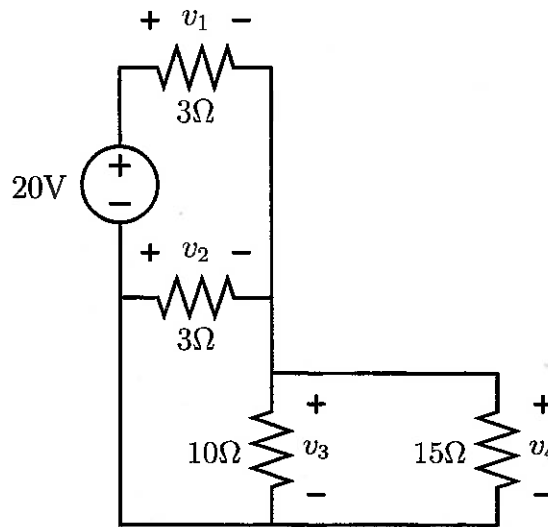
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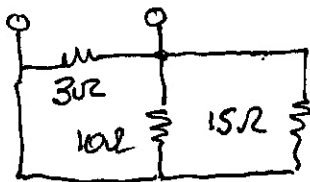


(1E) The circuit shown below has four labeled voltages. Determine numerical values for all four voltages. *Proper units are required.*



$v_1 =$	12 V	$v_2 =$	-8 V
$v_3 =$	8 V	$v_4 =$	8 V

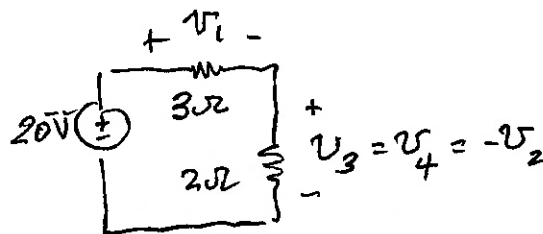
Reduce the bottom three resistors as follows



$$= \frac{1}{\frac{1}{3} + \frac{1}{10} + \frac{1}{15}} = \frac{30 \Omega}{10 + 3 + 2} = 2 \Omega$$

All in parallel

resulting in



$$v_1 = \frac{3}{5} 20 \text{ V}$$

$$v_3 = \dots = \frac{2}{5} 20 \text{ V}$$





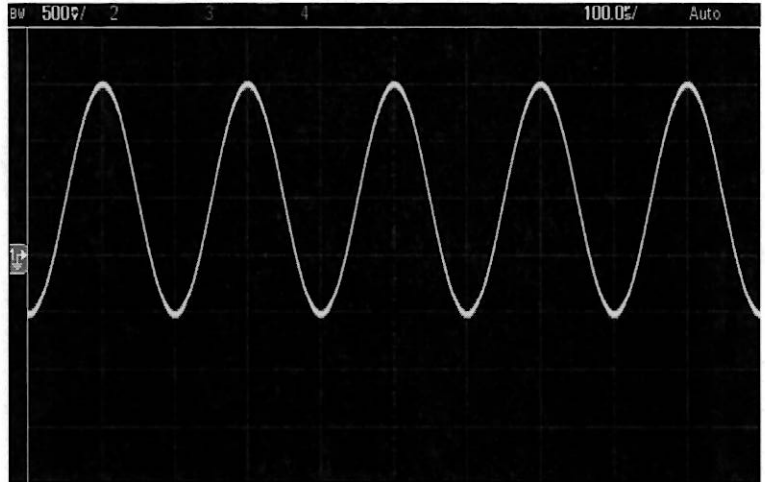
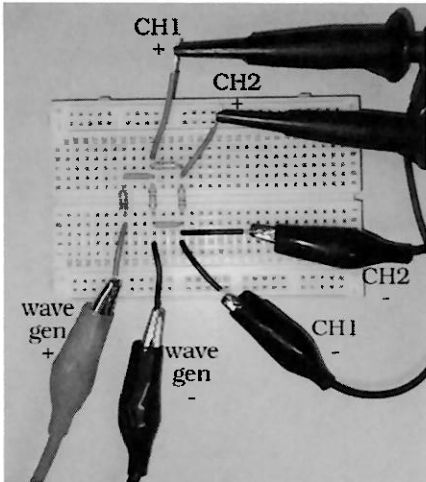
12,0,0022



12,1,0022

**Problem 2: Measurements – 15%**

The top figure below shows a photograph of a circuit built on a protoboard. The resistance of each of the four resistors in the circuit is  $22\text{ k}\Omega$ . The oscilloscope image shows the voltage measured on Channel 1.

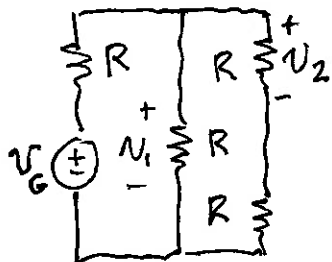


$$500 \frac{\mu\text{V}}{\text{Div}} \times 4 \text{ Div} = 2 \text{ V}_{\text{PP}} \quad 100 \frac{\mu\text{s}}{\text{Div}} \times 2 \text{ Div} = 200 \mu\text{s}$$

Given this measurement, determine the amplitude and frequency of the sine wave produced by the signal generator. *Proper units are required.*

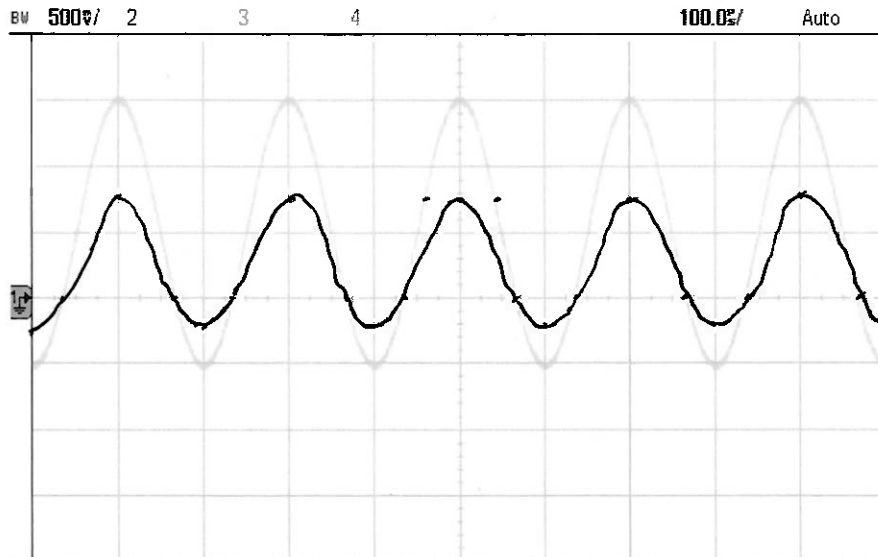
Amplitude: $5 \text{ V}_{\text{PP}} = \frac{5}{2} \text{ V}_{\text{Pk}}$	Frequency: $\frac{1}{200 \mu\text{s}} = 5 \text{ kHz}$
--	--

In addition, on the axes given below, sketch the waveform measured by Channel 2. The voltage from Channel 1 has been reproduced in light grey for reference.



$$V_2 = \frac{1}{2} V_1$$

$$V_1 = \frac{2}{5} V_G$$



← Note the zero location!



The first part of the problem is to find the Fourier transform of the function  $f(x) = \cos(x)$ . The second part is to find the Fourier transform of the function  $f(x) = \sin(x)$ .



The Fourier transform of  $f(x) = \cos(x)$  is  $F(\omega) = \pi [\delta(\omega - 1) + \delta(\omega + 1)]$ . The Fourier transform of  $f(x) = \sin(x)$  is  $F(\omega) = \pi [\delta(\omega - 1) - \delta(\omega + 1)]$ .

$$f(x) = \cos(x) \Rightarrow F(\omega) = \pi [\delta(\omega - 1) + \delta(\omega + 1)]$$

The Fourier transform of  $f(x) = \sin(x)$  is  $F(\omega) = \pi [\delta(\omega - 1) - \delta(\omega + 1)]$ .

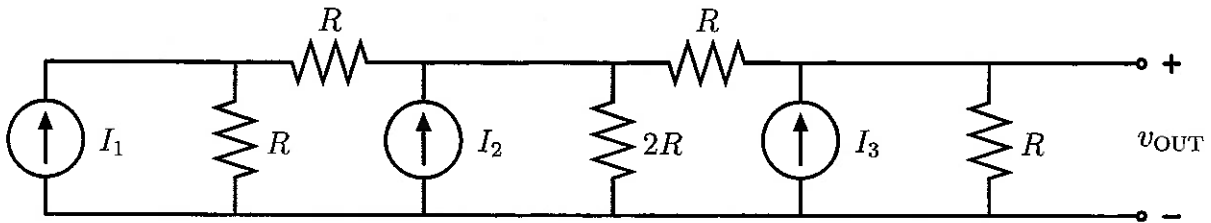


The Fourier transform of  $f(x) = \cos(x)$  is  $F(\omega) = \pi [\delta(\omega - 1) + \delta(\omega + 1)]$ . The Fourier transform of  $f(x) = \sin(x)$  is  $F(\omega) = \pi [\delta(\omega - 1) - \delta(\omega + 1)]$ .



**Problem 3: Multiple Sources – 14%**

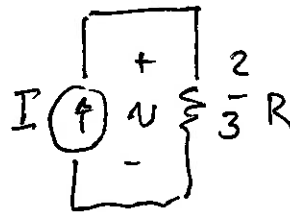
This problem concerns the circuit shown below, comprising three current sources and five resistors. The circuit has one port labeled with the voltage  $v_{OUT}$ .



(3A) Determine  $v_{OUT}$  in terms of  $I_1$ ,  $I_2$ ,  $I_3$  and  $R$ .

$$v_{OUT} = \frac{2R}{3} \left( I_3 + \frac{1}{2} I_2 + \frac{1}{4} I_1 \right)$$

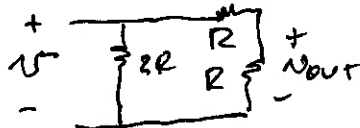
Consider each source independently. In this case, each source acts locally as shown; "I"  $\Rightarrow$   $I_1$ , or  $I_2$  or  $I_3$ .

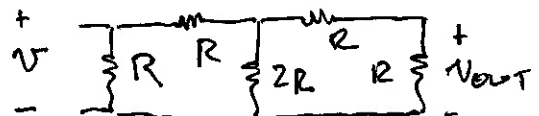


Therefore, each source produces a voltage across itself equal to  $v = \frac{2}{3} RI$ .

The self voltages can then be added by superposition at  $v_{out}$  after appropriate voltage division for each.

$I_3$  direction produces a  $v_{out}$  contribution;  $v_{out} = \frac{2}{3} RI_3$

$I_2 \Rightarrow$    $\Rightarrow v_{out} = \frac{1}{2} \cdot \frac{2}{3} RI_2$

$I_1 \Rightarrow$    $\Rightarrow v_{out} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3} RI_1$





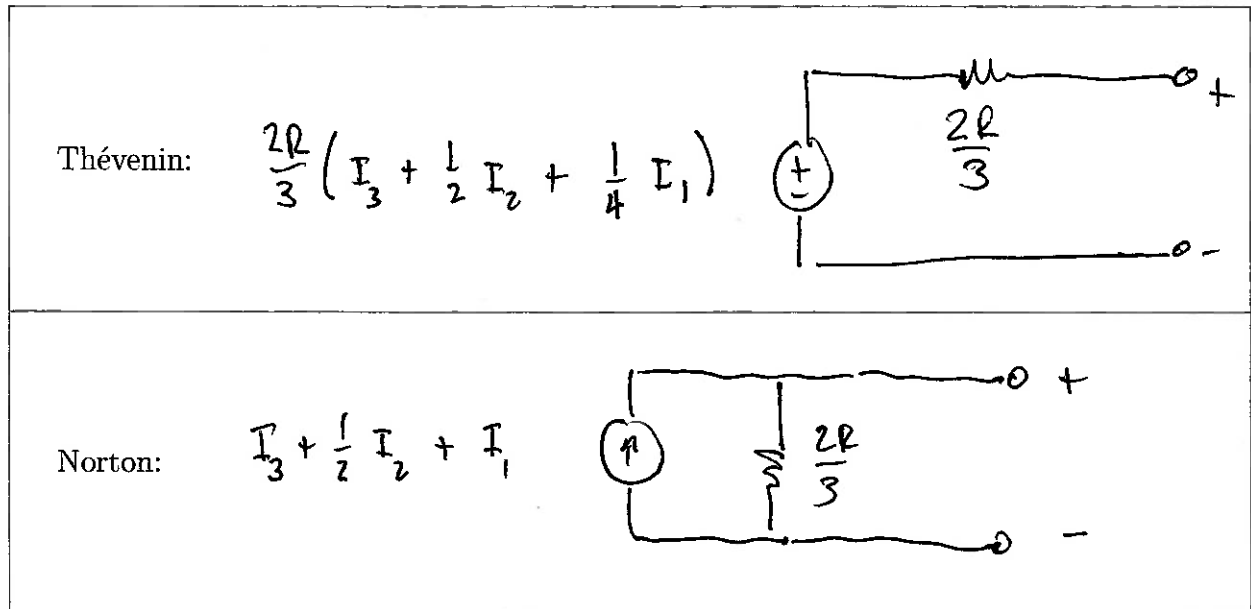
16,0,0022



16,1,0022



(3B) Draw and clearly label the Thévenin and Norton equivalents for the original circuit when viewed from the port labeled  $v_{OUT}$ .



$$V_{TH} = \text{Open Circuit Voltage} = \frac{2R}{3} \left( I_3 + \frac{1}{2} I_2 + \frac{1}{4} I_1 \right)$$

$$R_{TH} = \frac{2}{3} R \quad \text{from} \quad \begin{array}{c} \text{---} R \text{---} \\ | \quad | \quad | \\ R \quad 2R \quad R \\ | \quad | \quad | \\ \text{---} \end{array} \Leftrightarrow R_{TH}$$

$$I_N = \frac{V_{TH}}{R_{TH}} = I_3 + \frac{1}{2} I_2 = \frac{1}{4} I_1$$





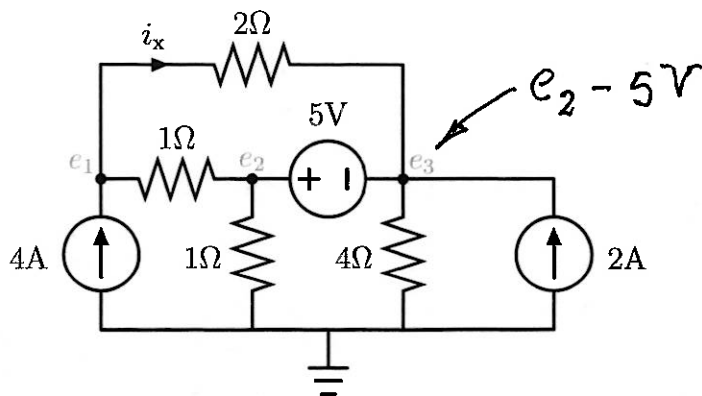
18,0,0022



18,1,0022

**Problem 4: Node Analysis – 20%**

Consider the network shown below. The objective of this problem is to use nodal analysis to determine the current  $i_x$  flowing through the  $2\Omega$  resistor. To do so, follow the steps outlined below. *To receive partial credit, please clearly explain your approach including the equations you are using for part (4A).*



- (4A) Write two node equations that can be solved for the unknown node voltages  $e_1$  and  $e_2$ . *Proper units are required.*

Equation 1:	$\frac{e_1 - (e_2 - 5V)}{2\Omega} + \frac{e_1 - e_2}{1\Omega} - 4A = 0$
Equation 2:	$\frac{e_2 - e_1}{1\Omega} + \frac{e_2}{1\Omega} + \frac{e_2 - 5V}{4\Omega} + \frac{(e_2 - 5V) - e_1}{2\Omega} - 2A = 0$

The floating voltage source requires the use of a supernode, for example  $e_3 \rightarrow e_2 - 5V$ . Then, write a KCL for  $e_1$  (Egn 1) and  $e_2$  (Egn 2).



Die Funktion  $f: \mathbb{R} \rightarrow \mathbb{R}$  ist durch  $f(x) = \frac{1}{2}x^2 - 3x + 4$  gegeben. Berechnen Sie die Nullstellen von  $f$ .



$$f(x) = \frac{1}{2}x^2 - 3x + 4 = 0$$

$$x^2 - 6x + 8 = 0$$

Die Nullstellen sind  $x_1 = 2$  und  $x_2 = 4$ .  
Die Nullstellen sind  $x_1 = 2$  und  $x_2 = 4$ .  
Die Nullstellen sind  $x_1 = 2$  und  $x_2 = 4$ .



(4B) Determine the values of  $e_1$ ,  $e_2$  and  $e_3$ . Proper units are required.

$e_1:$	6.8 V
$e_2:$	5.8 V
$e_3:$	0.8 V

$$e_1 \left( \frac{1}{2\Omega} + \frac{1}{1\Omega} \right) - e_2 \left( \frac{1}{2\Omega} + \frac{1}{1\Omega} \right) = 4A - \frac{5V}{2\Omega} \quad \leftarrow \text{Egn 1}$$

$$-e_1 \left( \frac{1}{1\Omega} + \frac{1}{2\Omega} \right) + e_2 \left( \frac{1}{1\Omega} + \frac{1}{1\Omega} + \frac{1}{4\Omega} + \frac{1}{2\Omega} \right) = \frac{5V}{4\Omega} + \frac{5V}{2\Omega} + 2A \quad \leftarrow \text{Egn 2}$$

$$\underbrace{\begin{bmatrix} 1.5 & -1.5 \\ -1.5 & 2.75 \end{bmatrix}}_{\text{Ohms}^{-1}} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1.5 \\ 5.75 \end{bmatrix}}_{\text{Amps}} \quad \dots \text{ next, multiply both rows by 4}$$

$$\begin{bmatrix} 6 & -6 \\ -6 & 11 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 23 \end{bmatrix} \quad \dots \text{ next, invert the equations}$$

$$\frac{\begin{bmatrix} 11 & 6 \\ 6 & 6 \end{bmatrix}}{30} \begin{bmatrix} 6 \\ 23 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 6.8 \\ 5.8 \end{bmatrix} \text{ V} \quad \Rightarrow e_3 = 0.8 \text{ V}$$



188

189

190

$$f(x) = \frac{1}{x^2} = x^{-2} \Rightarrow f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$$f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \left( \frac{1}{x^2} \right) = -\frac{2}{x^3} \quad \frac{d}{dx} \left( \sqrt{x} \right) = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \left( x^{-2} \right) = -2x^{-3} \quad \frac{d}{dx} \left( x^{1/2} \right) = \frac{1}{2}x^{-1/2}$$

$$\frac{d}{dx} \left( x^{-2} \right) = -2x^{-3} \quad \frac{d}{dx} \left( x^{1/2} \right) = \frac{1}{2}x^{-1/2}$$



(4C) Determine the current  $i_x$  flowing through the  $2\Omega$  resistor. *Proper units are required.*

*If you were unable to determine numerical values for  $e_1$ ,  $e_2$  and  $e_3$ , then provide symbolic expressions for partial credit.*

$$i_x: \quad 3 \text{ A}$$

$$i_x = \frac{e_1 - e_3}{2\Omega} = \frac{6\text{V}}{2\Omega} = 3 \text{ A}$$



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24,0,0022



24,1,0022



(4D) Determine the power dissipated in the  $2\Omega$  resistor. *Proper units are required.*

*If you were unable to determine numerical values for  $e_1$ ,  $e_2$  and  $e_3$ , then provide symbolic expressions for partial credit.*

Power: 18 W

$$\text{Power} = 6\text{V} \cdot 3\text{A}$$

$\uparrow$                      $\uparrow$

$e_1 - e_3$             $i_x$



(11) Determine the power spectrum of the signal  $x(t) = \cos(2\pi t)$ .

The power spectrum is given by  $P(f) = |X(f)|^2$ , where  $X(f)$  is the Fourier transform of  $x(t)$ .

$$X(f) = \int_{-\infty}^{\infty} \cos(2\pi t) e^{-j2\pi ft} dt$$

$$X(f) = \frac{1}{2} \int_{-\infty}^{\infty} (e^{j2\pi t} + e^{-j2\pi t}) e^{-j2\pi ft} dt$$
$$= \frac{1}{2} \left[ \int_{-\infty}^{\infty} e^{j2\pi(1-f)t} dt + \int_{-\infty}^{\infty} e^{-j2\pi(1+f)t} dt \right]$$
$$= \frac{1}{2} \left[ \delta(f-1) + \delta(f+1) \right]$$



26,0,0022

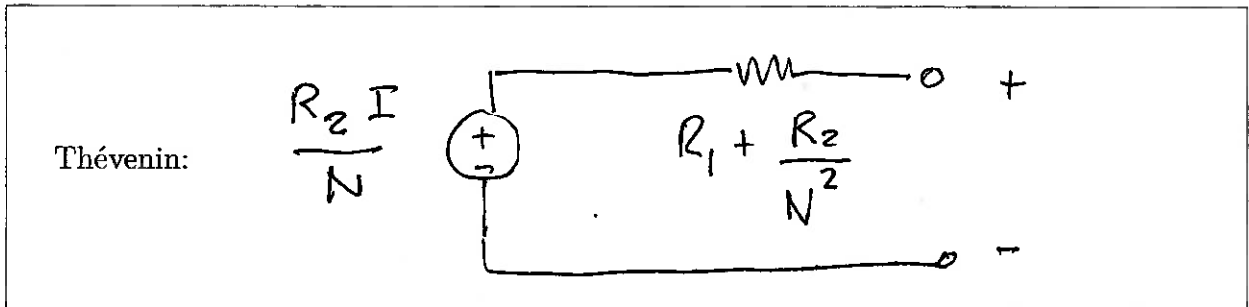
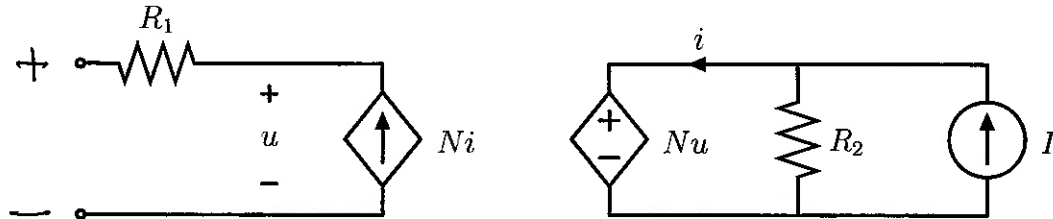


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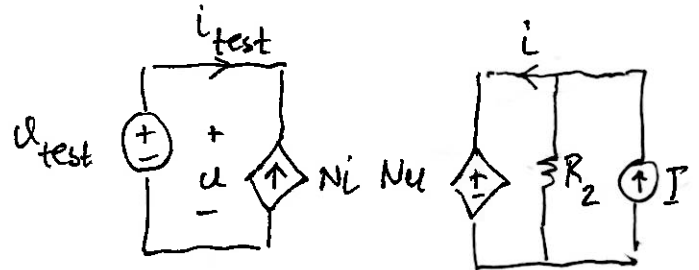
**Problem 5: Dependent Sources – 16%**

~~With one exception, all parts of this problem are independent of the other parts. The one exception is that Parts (C) and (D) consider the same circuit.~~

(5A) Determine the Thévenin equivalent, as viewed from the open port, of the circuit shown below. Make sure to clearly label the open port in the Thévenin equivalent.



Consider  $u$  to be a test voltage " $u_{test}$ " and compute " $i_{test}$ " as shown to the right



$$\text{In this case } i = I - \frac{Nu_{test}}{R_2} \Rightarrow i_{test} = -N \left( I - \frac{Nu_{test}}{R_2} \right)$$

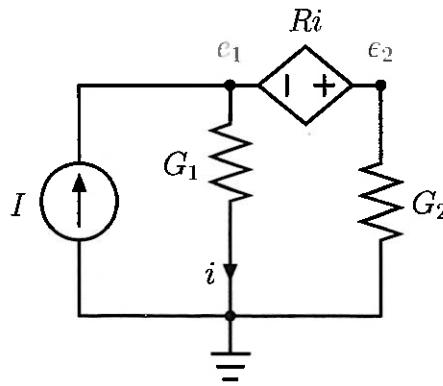
This relation has a  $V_{TH}$  of  $\frac{R_2 I}{N}$  and an  $R_{TH}$  of  $\frac{R_2}{N^2}$ .

Finally, add in  $R_1$  to get the complete  $R_{TH}$





- (5B) Determine the two unknown node voltages  $e_1$  and  $e_2$  in the circuit shown below. Note that the two resistors in the circuit are labeled with their *conductances* as opposed to their resistances.



$e_1 =$	$\frac{I}{G_1 + (1 + R G_1) G_2}$
$e_2 =$	$\frac{I}{G_1 + (1 + R G_1) G_2} (1 + R G_1)$

The use of a supernode is appropriate with  
 $e_2 = e_1 + Ri = e_1 + R e_1 G_1 = e_1 (1 + R G_1)$ .

Node method  $\Rightarrow I = e_1 G_1 + e_2 G_2 = e_1 (G_1 + (1 + R G_1) G_2)$ .

$\Rightarrow e_1 = I / (G_1 + (1 + R G_1) G_2)$





30,0,0022



30,1,0022