Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

# 6.200 – Circuits & Electronics
Spring 2024

## Quiz #1

13 March 2024

**Name:** Solutions

**MIT Kerberos Username:** ____________________

**Recitation Time:** 11 12 1

- There are 15 pages in this quiz, including this cover page.
- Please put your name and Kerberos ID in the spaces provided above, and circle the time of your recitation.
- Please do not remove any pages from this quiz.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. *When finished with each part, clearly write your answer for that part into the corresponding answer box or graph.*
- Make sure all work is on pages with QR codes, and **do not write on the QR codes.**
- *All numerical answers require proper units.*
- *In order to guarantee receipt of full credit, all answers should be justified by supporting math and/or explanations.*
- This is a closed-book quiz, but calculators and a single two-sided page of notes are allowed.
- Good luck!

01,0,0022

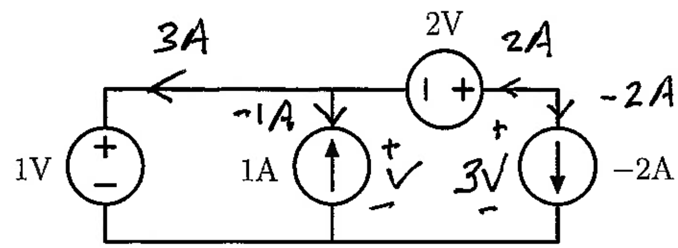


01,1,0022

## Problem 1: Miscellany – 35%

All parts of this problem are independent of the other parts.

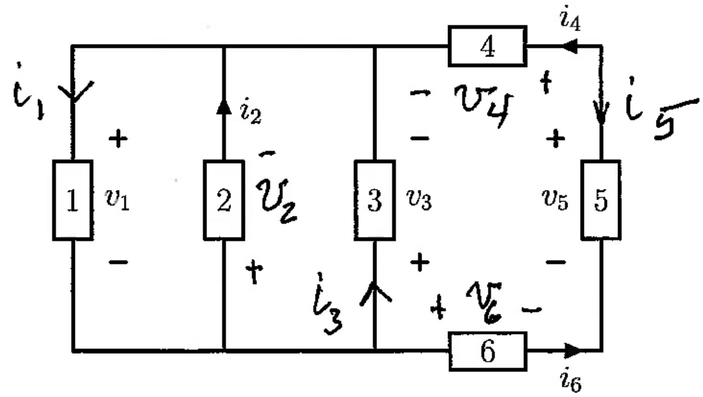
(1A) Determine the power sourced (provided) by each of the four sources in the circuit shown below. If a source sinks (receives) power, then consider its sourced power to be negative. *Proper units are required.*




| | |
|---|---|
| 1 V Source: $-3$ W | 1 A Source: $1$ W |
| 2 V Source: $-4$ W | -2 A Source: $6$ W |

See the voltages and currents labeled for each device according to the passive sign convention. Sourced power is then $-vi$.

(1B) The circuit shown below comprises comprises six branches with partially labeled branch voltages and currents. Add the missing labels to the circuit diagram according to the passive sign convention. A brute-force analysis of the circuit would require the use of six equations obtained from KCL and KVL. Provide six such equations that are necessary and sufficient to carry out a brute-force analysis.




KCL (outwards)

KVL (cw)

| | |
|---|---|
| Eqn #1: $i_1 - i_2 - i_3 - i_4 = 0$ | Eqn #2: $v_1 + v_2 = 0$ |
| Eqn #3: $i_4 + i_5 = 0$ | Eqn #4: $-v_2 + v_3 = 0$ |
| Eqn #5: $-i_5 - i_6 = 0$ | Eqn #6: $-v_3 + v_4 - v_5 + v_6 = 0$ |

Could also have

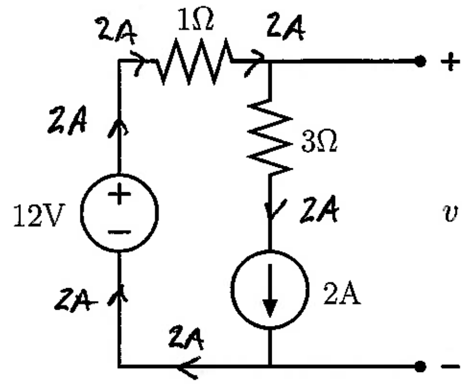
$-i_1 + i_2 + i_3 + i_6 = 0$

Could also have

$v_1 + v_4 - v_5 + v_6 = 0$

among others

(1C) Determine the voltage $v$ in the circuit shown below. *Proper units are required.* If the voltage cannot be determined, enter "N/A" in the answer box.

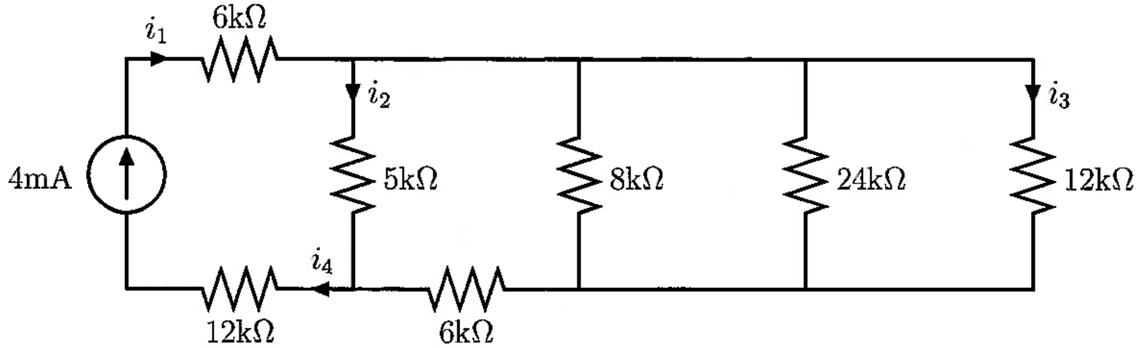



$v =$ 10 V

The current in the left-hand loop is 2A as indicated. Thus $v = 12\,V - 2A \cdot 1\Omega$

$= 10\,V.$

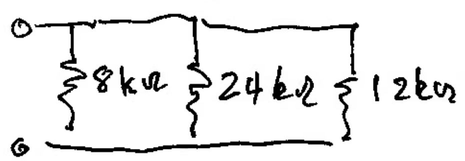
(1D) The circuit shown below has four labeled currents. Determine numerical values for all four currents. *Proper units are required.*




| | | | |
|---|---|---|---|
| $i_1 =$ | $4\ mA$ | $i_2 =$ | $\frac{8}{3}\ mA$ |
| $i_3 =$ | $\frac{4}{9}\ mA$ | $i_4 =$ | $4\ mA$ |

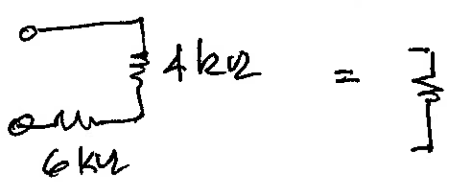
By inspection / KCL $i_1 = i_4 = 4\ mA$.

Next reduce the four rightmost resistors as follows.




$$= \frac{1}{\frac{1}{8k\Omega} + \frac{1}{24k\Omega} + \frac{1}{12k\Omega}} = \frac{24k\Omega}{3+1+2} = 4k\Omega$$

then



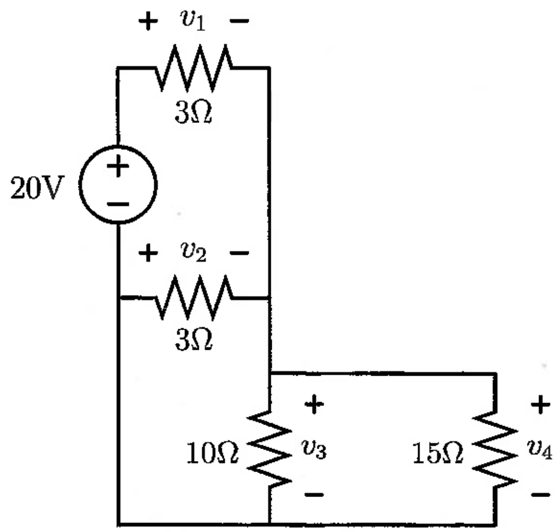

Finally use current dividers

$$i_2 = \frac{10}{5+10} \cdot 4\,mA = \frac{8}{3}\,mA$$

$$i_3 = \frac{\frac{1}{12}}{\frac{1}{8}+\frac{1}{24}+\frac{1}{12}} \cdot \frac{4}{3}\,mA$$

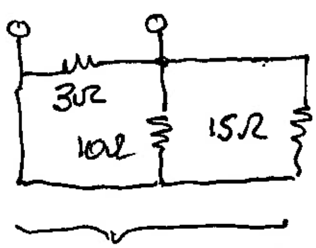
$$= \frac{2}{3+1+2} \cdot \frac{4}{3}\,mA = \frac{4}{9}\,mA$$

(1E) The circuit shown below has four labeled voltages. Determine numerical values for all four voltages. *Proper units are required.*




| | |
|---|---|
| $v_1 =$ 12 V | $v_2 =$ −8 V |
| $v_3 =$ 8V | $v_4 =$ 8 V |

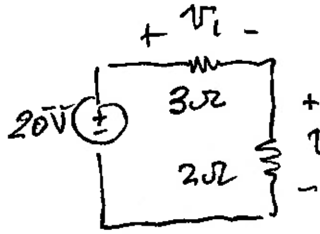
Reduce the bottom three resistors as follows




$$= \frac{1}{\frac{1}{3} + \frac{1}{10} + \frac{1}{15}} = \frac{30\ \Omega}{10 + 3 + 2} = 2\ \Omega$$

All in parallel

resulting in



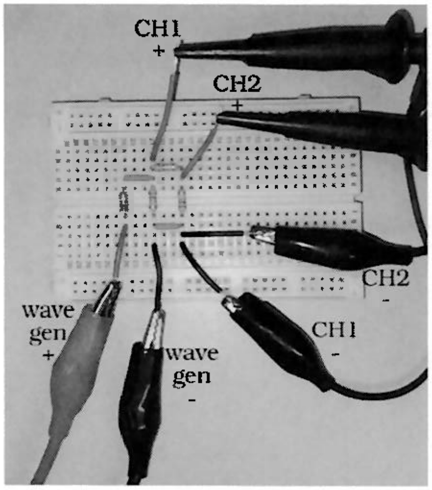

$v_3 = v_4 = -v_2$

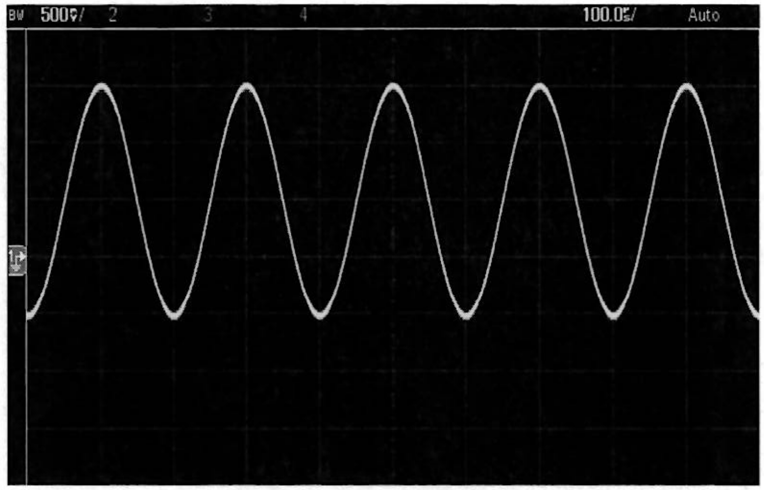
$v_1 = \frac{3}{5} 20\ \text{V}$

$v_3 = \ldots = \frac{2}{5} 20\ \text{V}$

## Problem 2: Measurements – 15%

The top figure below shows a photograph of a circuit built on a protoboard. The resistance of each of the four resistors in the circuit is 22 kΩ. The oscilloscope image shows the voltage measured on Channel 1.



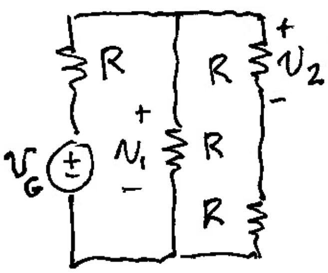



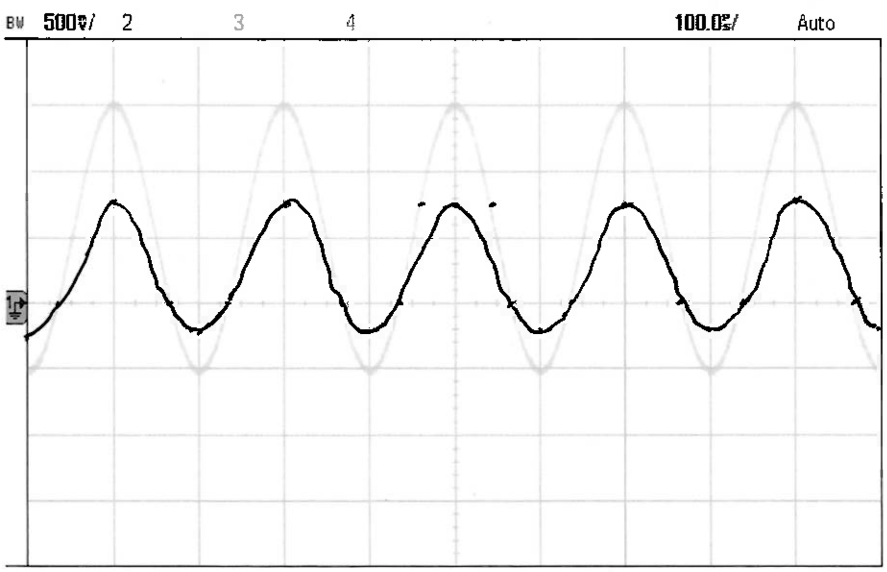

$500 \frac{mV}{Div} * 4\ Div = 2\ V_{pp}$ $100 \frac{\mu s}{Div} * 2\ Div = 200\ \mu s$

Given this measurement, determine the amplitude and frequency of the sine wave produced by the signal generator. *Proper units are required.*

| | |
|---|---|
| Amplitude: $5\ V_{pp} = \frac{5}{2}\ V_{pk}$ | Frequency: $\frac{1}{200\ \mu s}$ $5\ kHz$ |

In addition, on the axes given below, sketch the waveform measured by Channel 2. The voltage from Channel 1 has been reproduced in light grey for reference.



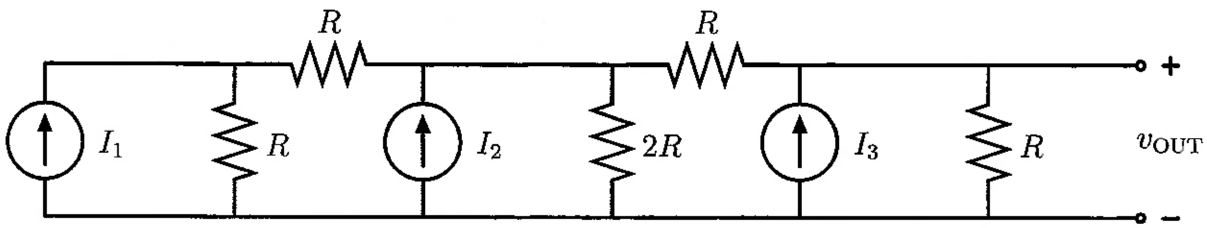

← Note the zero location!

$v_2 = \frac{1}{2} v_1$

$v_1 = \frac{2}{5} v_G$

## Problem 3: Multiple Sources – 14%

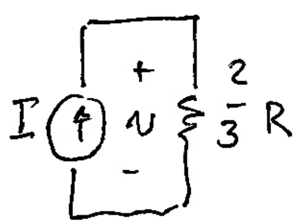
This problem concerns the circuit shown below, comprising three current sources and five resistors. The circuit has one port labeled with the voltage $v_{OUT}$.




(3A) Determine $v_{OUT}$ in terms of $I_1$, $I_2$, $I_3$ and $R$.

$$v_{OUT} = \frac{2R}{3}\left(I_3 + \frac{1}{2} I_2 + \frac{1}{4} I_1\right)$$

Consider each source independently. In this case, each source acts locally as shown;
"I" ⇒ $I_1$ or $I_2$ or $I_3$.

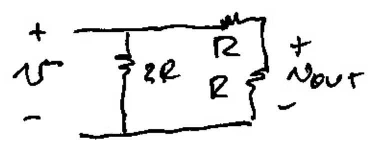



Therefore, each source produces a voltage across itself equal to $v = \frac{2}{3} RI$.

The self voltages can then be added by superposition at $v_{OUT}$ after appropriate voltage division for each.

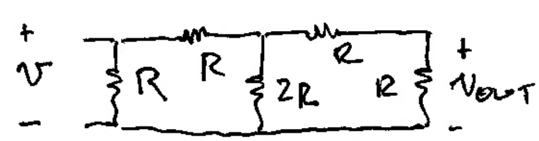
$I_3$ direction produces a $v_{OUT}$ contribution; $v_{OUT} = \frac{2}{3} R I_3$

$I_2$ ⇒




⇒ $v_{OUT} = \frac{1}{2} \cdot \frac{2}{3} R I_2$

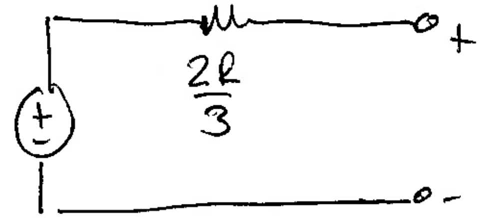
$I_1$ ⇒



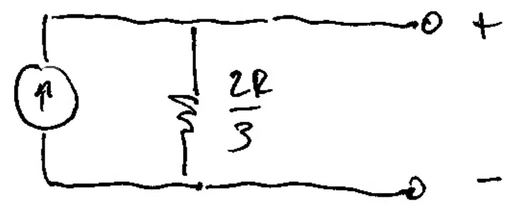

⇒ $v_{OUT} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3} R I_1$

(3B) Draw and clearly label the Thévenin and Norton equivalents for the original circuit when viewed from the port labeled $v_{\mathrm{OUT}}$.

Thévenin: $\frac{2R}{3}\left(I_3 + \frac{1}{2} I_2 + \frac{1}{4} I_1\right)$

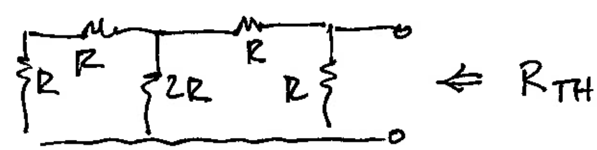



Norton: $I_3 + \frac{1}{2} I_2 + I_1$




$$v_{TH} = \text{Open Circuit Voltage} = \frac{2R}{3}\left(I_3 + \frac{1}{2} I_2 + \frac{1}{4} I_1\right)$$

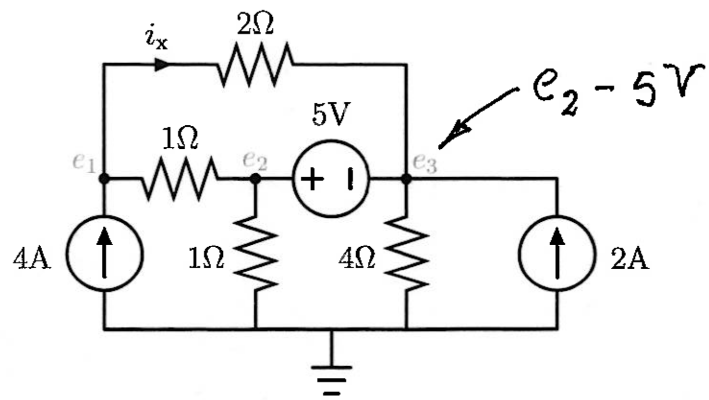
$$R_{TH} = \frac{2}{3} R \quad \text{from}$$




$$I_N = \frac{v_{TH}}{R_{TH}} = I_3 + \frac{1}{2} I_2 = \frac{1}{4} I_1$$

## Problem 4: Node Analysis – 20%

Consider the network shown below. The objective of this problem is to use nodal analysis to determine the current $i_x$ flowing through the $2\Omega$ resistor. To do so, follow the steps outlined below. *To receive partial credit, please clearly explain your approach including the equations you are using for part (4A).*




(4A) Write two node equations that can be solved for the unknown node voltages $e_1$ and $e_2$. *Proper units are required.*

| | |
|---|---|
| Equation 1: | $\frac{e_1 - (e_2 - 5V)}{2\Omega} + \frac{e_1 - e_2}{1\Omega} - 4A = 0$ |
| Equation 2: | $\frac{e_2 - e_1}{1\Omega} + \frac{e_2}{1\Omega} + \frac{e_2 - 5V}{4\Omega} + \frac{(e_2 - 5V) - e_1}{2\Omega} - 2A = 0$ |

The floating voltage source requires the use of a super node, for example $e_3 \rightarrow e_2 - 5V$. Then, write a KCL for $e_1$ (Eqn 1) and $e_2$ (Eqn 2).

(4B) Determine the values of $e_1$, $e_2$ and $e_3$. *Proper units are required.*

| | |
|---|---|
| $e_1$: | 6.8 V |
| $e_2$: | 5.8 V |
| $e_3$: | 0.8 V |

$$e_1\left(\frac{1}{2\Omega}+\frac{1}{1\Omega}\right)-e_2\left(\frac{1}{2\Omega}+\frac{1}{1\Omega}\right) = 4A - \frac{5V}{2\Omega} \quad \leftarrow \text{Eqn 1}$$

$$-e_1\left(\frac{1}{1\Omega}+\frac{1}{2\Omega}\right)+e_2\left(\frac{1}{1\Omega}+\frac{1}{1\Omega}+\frac{1}{4\Omega}+\frac{1}{2\Omega}\right) = \frac{5V}{4\Omega}+\frac{5V}{2\Omega}+2A \quad \leftarrow \text{Eqn 2}$$

$$\underbrace{\begin{bmatrix} 1.5 & -1.5 \\ -1.5 & 2.75 \end{bmatrix}}_{\text{Ohms}^{-1}} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1.5 \\ 5.75 \end{bmatrix}}_{\text{Amps}}$$ ... next, multiply both rows by 4

$$\begin{bmatrix} 6 & -6 \\ -6 & 11 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 23 \end{bmatrix}$$ ... next, invert the equations

$$\frac{\begin{bmatrix} 11 & 6 \\ 6 & 6 \end{bmatrix}}{30} \begin{bmatrix} 6 \\ 23 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 6.8 \\ 5.8 \end{bmatrix} V \Rightarrow e_3 = 0.8V$$

(4C) Determine the current $i_x$ flowing through the $2\Omega$ resistor. *Proper units are required.*

*If you were unable to determine numerical values for* $e_1$, $e_2$ *and* $e_3$, *then provide symbolic expressions for partial credit.*

$i_x$: 3 A

$$i_x = \frac{e_1 - e_3}{2\Omega} = \frac{6V}{2\Omega} = 3A$$

23,0,0022



23,1,0022

(4D) Determine the power dissipated in the $2\Omega$ resistor. *Proper units are required.*

*If you were unable to determine numerical values for* $e_1$, $e_2$ *and* $e_3$, *then provide symbolic expressions for partial credit.*

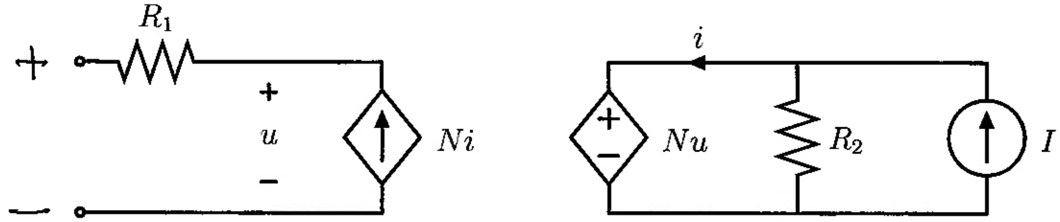
Power: 18 W

$$\text{Power} = \underset{e_1 - e_3}{\underset{\uparrow}{6V}} \cdot \underset{i_x}{\underset{\uparrow}{3A}}$$

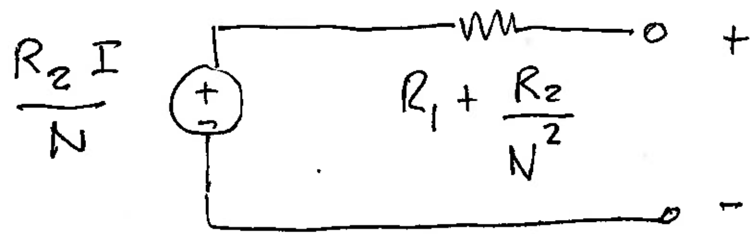
## Problem 5: Dependent Sources – 16%

~~With one exception,~~ All parts of this problem are independent of the other parts. ~~The one exception is that Parts (C) and (D) consider the same circuit~~.

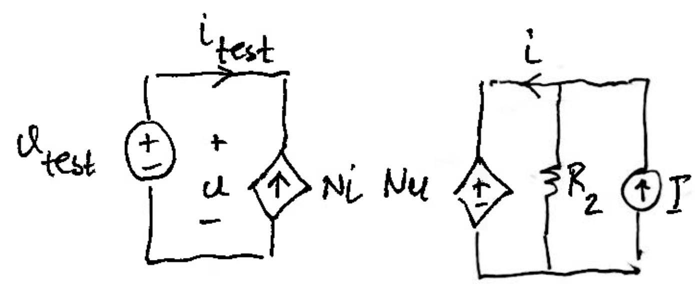
(5A) Determine the Thévenin equivalent, as viewed from the open port, of the circuit shown below. Make sure to clearly label the open port in the Thévenin equivalent.




Thévenin:




Conside $u$ to be a test voltage "$u_{test}$" and compute "$i_{test}$" as shown to the right

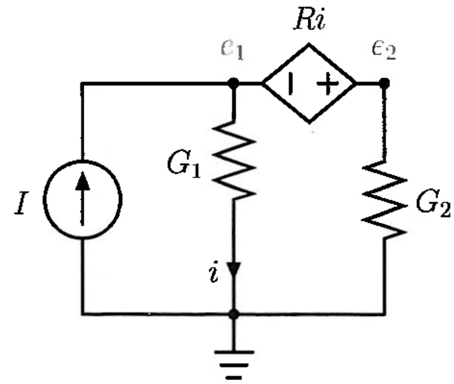



In this case $i = I - \frac{Nu_{test}}{R_2} \Rightarrow i_{test} = -N\left(I - \frac{Nu_{test}}{R_2}\right)$.

This relation has a $V_{TH}$ of $\frac{R_2 I}{N}$ and an $R_{TH}$ of $\frac{R_2}{N^2}$.

Finally, add in $R_1$ to get the complete $R_{TH}$

(5B) Determine the two unknown node voltages $e_1$ and $e_2$ in the circuit shown below. Note that the two resistors in the circuit are labeled with their *conductances* as opposed to their resistances.




| | |
|---|---|
| $e_1 =$ | $\dfrac{I}{G_1 + (1+RG_1)G_2}$ |
| $e_2 =$ | $\dfrac{I}{G_1 + (1+RG_1)G_2}(1+RG_1)$ |

The use of a supernode is appropriate with

$e_2 = e_1 + Ri = e_1 + Re_1G_1 = e_1(1+RG_1)$.

Node method $\Rightarrow$ $I = e_1G_1 + e_2G_2 = e_1(G_1 + (1+RG_1)G_2)$.

$\Rightarrow e_1 = I/(G_1 + (1+RG_1)G_2)$