6.200 Midterm

Fall 2023

Name: **Answers**

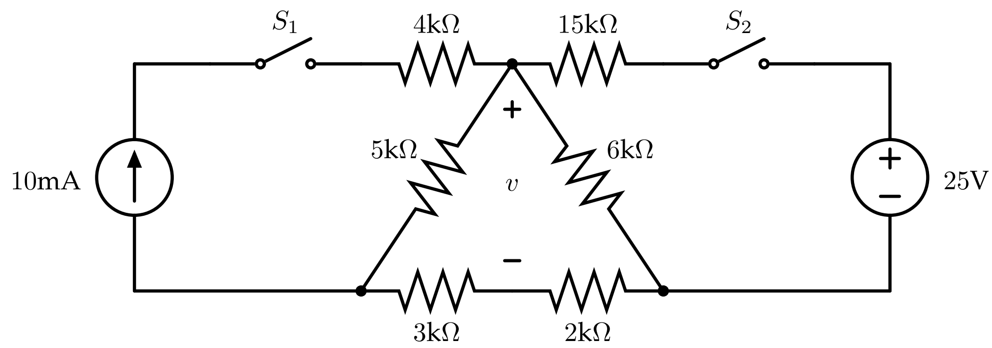
Kerberos/Athena Username:

5 questions | 1 hour and 50 minutes

- Please **WAIT** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5″ × 11″ sheet of handwritten notes (both sides) as a reference. This sheet much be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices (including computers, calculators, phones, etc.).
- If you have questions, please **come to us at the front** to ask them.
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit. **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

1 Sources and Switches

Consider the following circuit, with two switches labeled S_1 and S_2:



1.1 Part 1

Find v when S_1 is closed (shorted) and S_2 is open, including units:

$v =$ 25V

1.2 Part 2

Find v when S_2 is closed (shorted) and S_1 is open, including units:

$v =$ 4V

1.3 Part 3

You're just about to solve the circuit when both switches are closed (shorted) when your roommate suggests that, by superposition, the result for the circuit when both switches are closed will simply be the sum of your solutions from the previous two parts.

Are they correct? (circle one): YES / **NO**

Briefly explain why / why not:

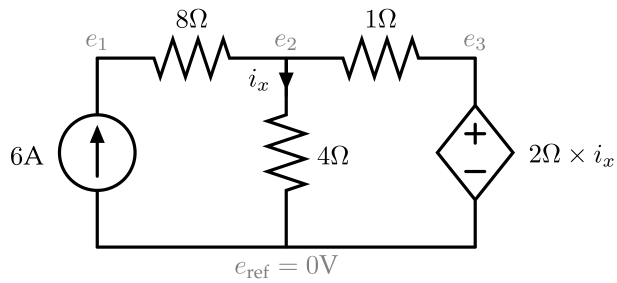
While it is true that we could solve the circuit using superposition, our answer for part 2 involved effectively replacing the voltage source with an *open*, which is not what we would do when using superposition to solve for the whole circuit with both switches closed (we would need to replace it with a short instead).

While it is not necessary to solve the circuit to receive full credit for this problem, for those who are interested, when both switches are closed is $v = 26\text{V}$.

2 Dependent Sources

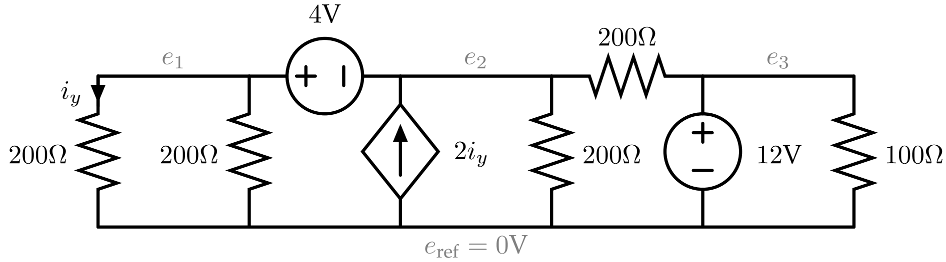
Solve for each of the indicated node potentials in each of the following circuits.

2.1 Circuit 1



$e_1 = 56\text{V}$ $e_2 = 8\text{V}$ $e_3 = 4\text{V}$

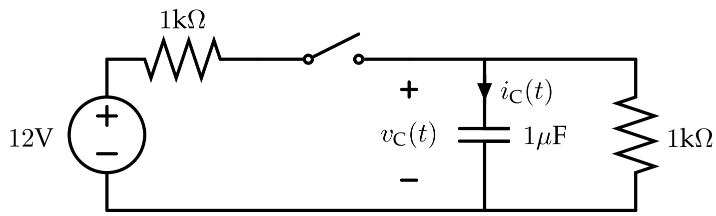
2.2 Circuit 2



$e_1 = 10\text{V}$ $e_2 = 6\text{V}$ $e_3 = 12\text{V}$

3 RC Graph

Consider the following circuit:



At time $t = 0^-$, the switch is open and the capacitor is fully discharged, i.e., $v_C(t = 0^-) = 0$V.

At time $t = 0$, the switch closes.

At time $t = 1$ms, the switch opens again and remains open forever.

Answer the questions below about this circuit. For each, you may leave your answer as a simple symbolic expression that can involve e, π, and/or the natural log function ln.

What is the maximum value that $v_C(t)$ reaches, and at what time does this maximum occur?

Maximum value: $6\left(1 - \frac{1}{e^2}\right)$ Volts at time $t = 1$ms

If you prefer, you may refer to this value as v_{max} in other answers below that depend on it (if any), rather than re-writing your expression for it.

What is the value of $v_C(t)$ at time $t = 2$ms?

$$\frac{v_{\text{max}}}{e} = 6\text{V} \times \left(1 - \frac{1}{e^2}\right)\left(\frac{1}{e}\right)$$

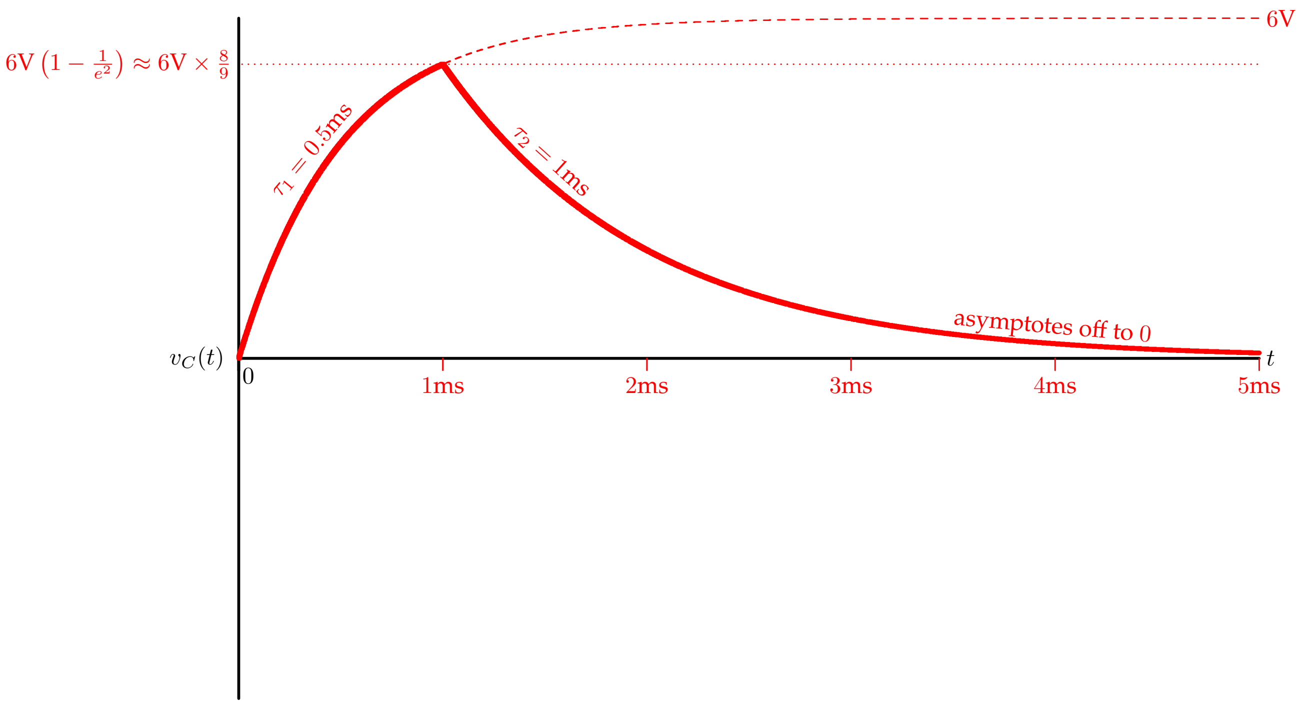
What is the maximum value that $i_C(t)$ reaches, and at what time does this maximum occur?

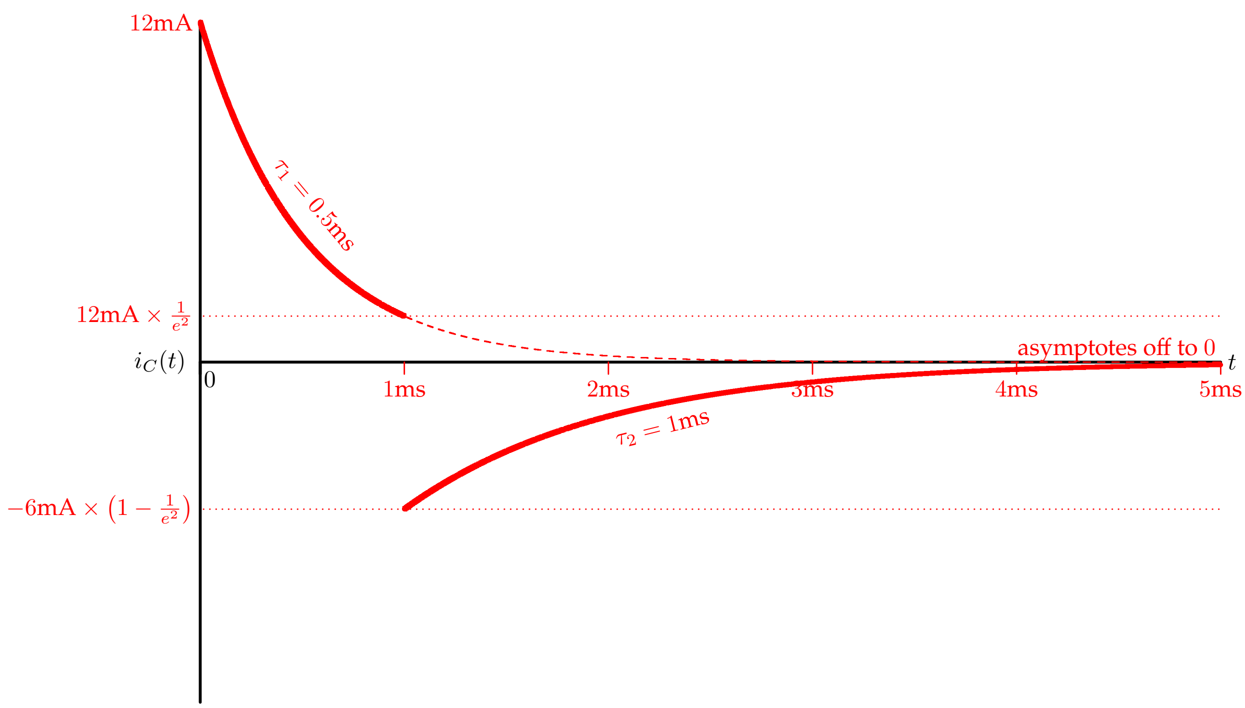
12mA, right away at $t = 0^+$ (just after the switch closes)

What is the minimum (or most negative) value that $i_C(t)$ reaches, and at what time does this minimum occur?

$\frac{-v_{\text{max}}}{1\text{k}\Omega}$ at $t = 1\text{ms}^+$ (just after the switch opens)

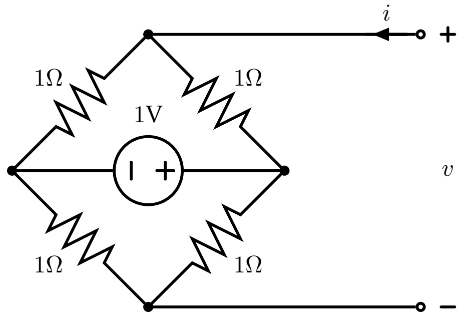
Sketch the values of $v_C(t)$ and $i_C(t)$ as functions of time on the axes below. In your sketches, label all key values/asymptotes (with units); and for any portions of the graph that trace out an exponential curve, indicate the associated time constant(s).



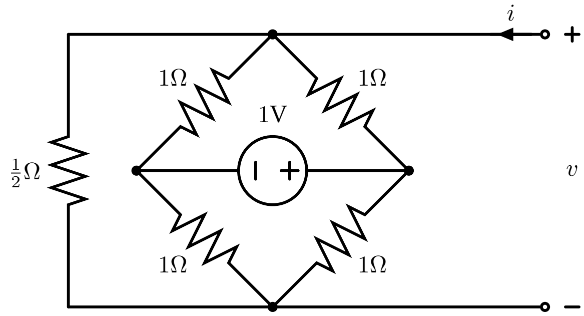


4 Bridging the Gap

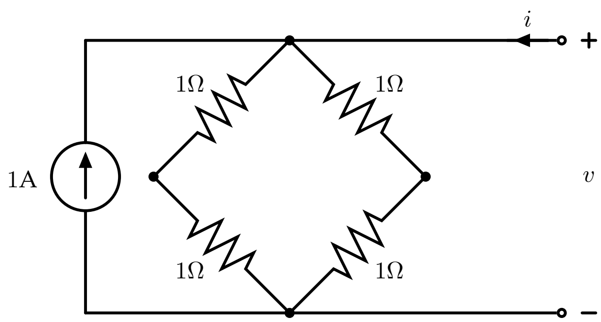
For each of the following circuits, determine which (if any) of plots A-H on the facing page (page 9) show the relationship between the labeled v and i values.



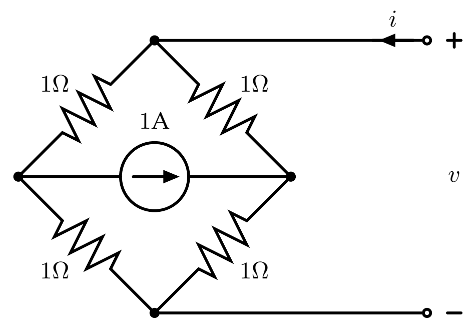
Plot (**A-H** or **None**): C



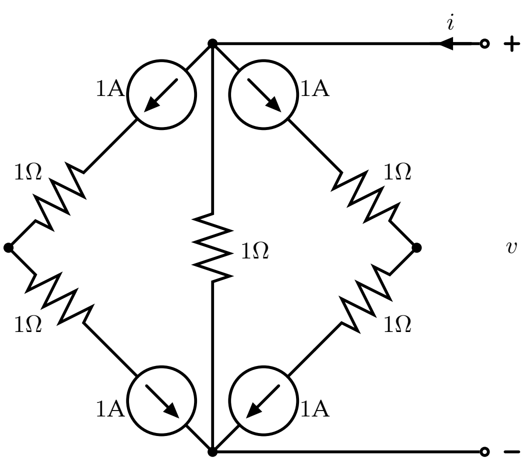
Plot (**A-H** or **None**): B



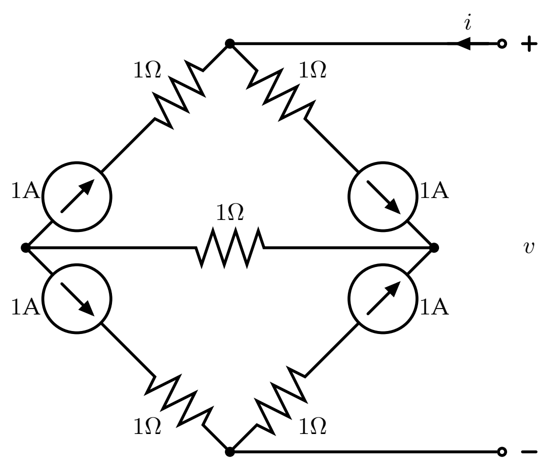
Plot (**A-H** or **None**): D



Plot (**A-H** or **None**): C

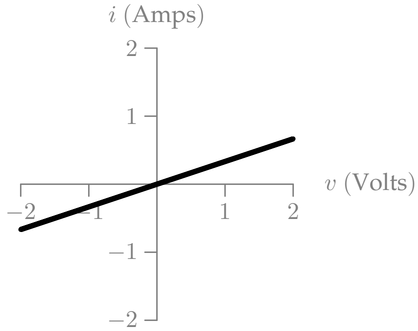


Plot (**A-H** or **None**): E

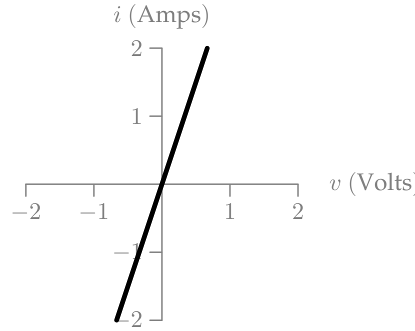


Plot (**A-H** or **None**): F

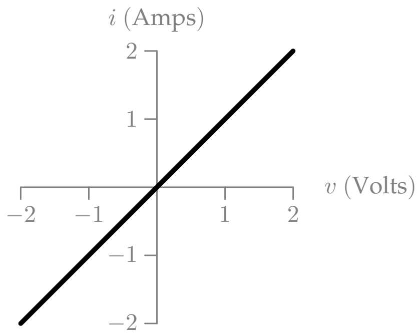
Plot A



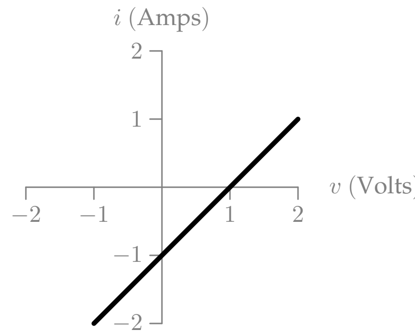
Plot B



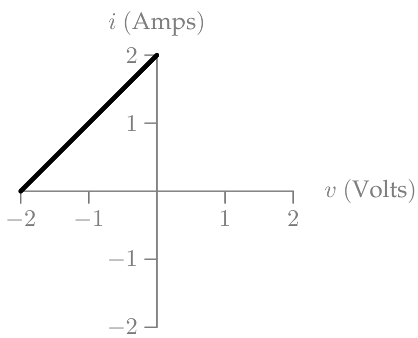
Plot C



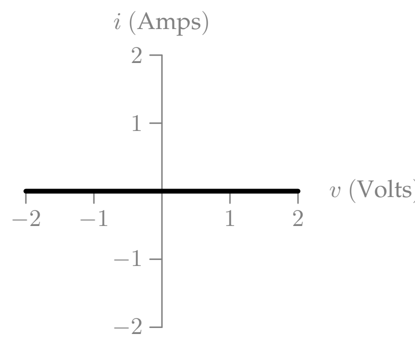
Plot D



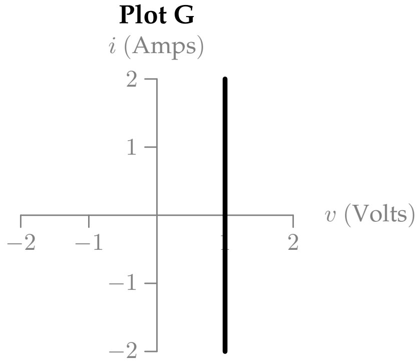
Plot E



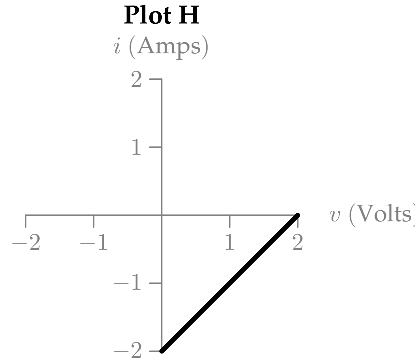
Plot F



Plot G

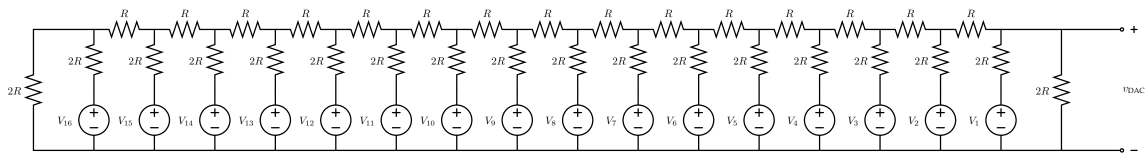


Plot H

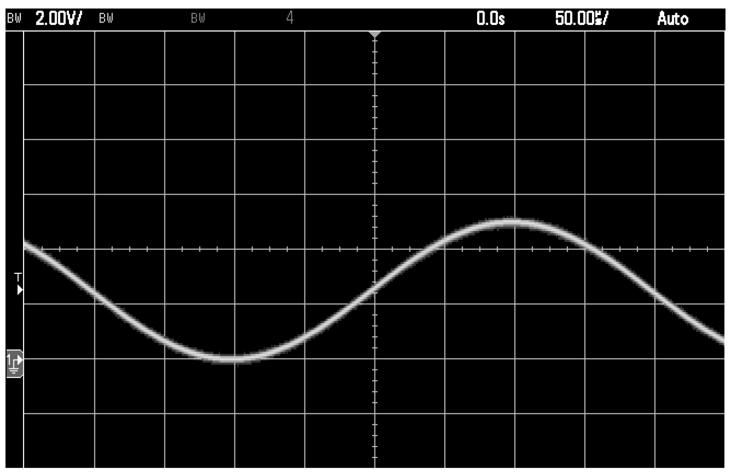


5 DAC To Reality; Ope, There Goes Gravity...

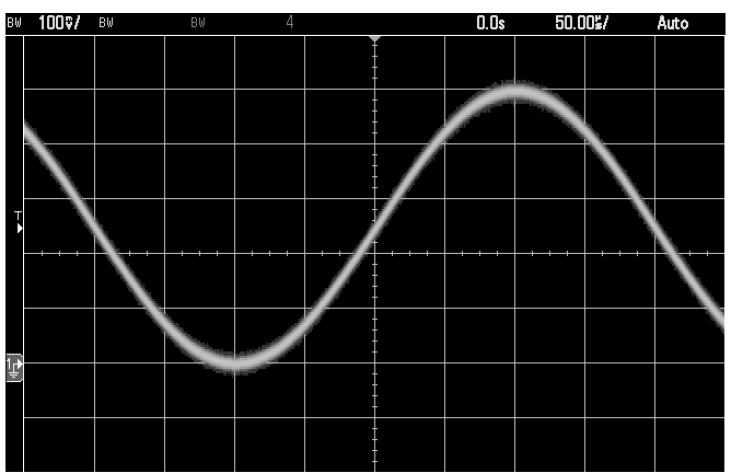
To help prepare for your dorm's big party this weekend, you've decided to build a 16-bit version of our familiar DAC circuit to provide the music for the party (6-bit audio quality just won't cut it!). And as before, a program on the Teensy will turn the various sources on and off to produce different voltages at v_{DAC}. Here is your circuit schematic:



You finish building your circuit, but unfortunately you've worked in such a rush that you can't remember what value you used for R in the circuit (and the lights in the lab are turned down low, so you can't see the resistor color codes)! Instead, you decide to take some measurements using the oscilloscope to backsolve for R. You hook up a scope directly to measure v_{DAC} and see the following:



Then, with the scope probes still connected, you hook up a speaker (modeled as a 4Ω resistor) to the port labeled v_{DAC} and see the following instead (note that the scales on the scope have been set differently between the two measurements):



On the facing page (page 11), answer the questions about this circuit.

What is the frequency of the sine wave measured at v_{DAC}, in units of Hz?

2.5kHz

What is the peak-to-peak amplitude of v_{DAC} without the speaker connected? Include units.

5V

What is the peak-to-peak amplitude of v_{DAC} *with* the speaker connected? Include units.

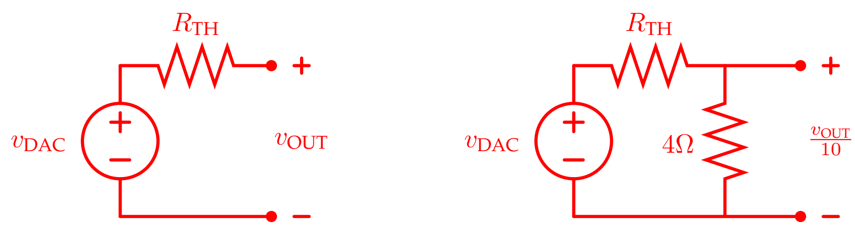
500mV

Based on these results, approximately what value of R was used in the original circuit? Include units.

54Ω

Briefly explain your method for calculating R:

Connecting the speaker caused the output voltage to drop by about a factor of 10. Importantly, we can model the entire DAC as a Thévenin, so our circuit (with and without the speaker) looks like:



Using the voltage divider equation, this means that R_{TH} must be about 9 times as big as our 4Ω speaker, so $R_{\text{TH}} \approx 36\Omega$. And, solving, we find that $R_{\text{TH}} = \dfrac{2R}{3}$, so $R = \dfrac{3}{2}R_{\text{TH}} \approx 54\Omega$.