

6.200 Final Exam

Fall 2025

Name: **Answers**

Kerberos/Athena Username:

5 questions

3 hours

- Please **WAIT** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5" × 11" sheet of handwritten notes (both sides) as a reference. This sheet must be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices other than a multimeter (no computers, calculators, phones, etc.).
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.
- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam. **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

Worksheet (intentionally blank)

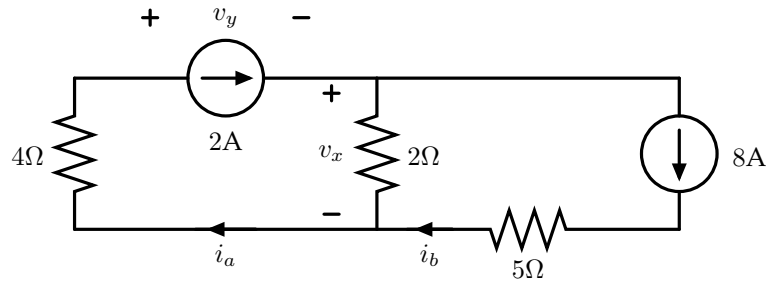
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1 Short Circuits

1.1 Part 1

Solve for v_x , v_y , i_a , and i_b in the circuit below. Provide your answers as exact numbers (with units).

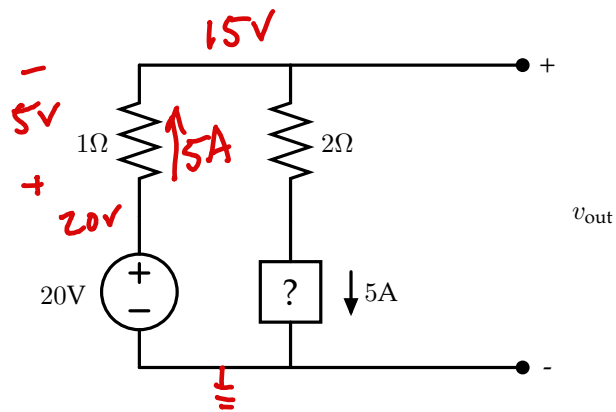


$$v_x = \boxed{-12V} \quad v_y = \boxed{4V} \quad i_a = \boxed{2A} \quad i_b = \boxed{8A}$$

$4V + -16V$ $-12V + 16V$

1.2 Part 2

In the following circuit, the box with a "?" represents an unknown subcircuit consisting of linear components. Although the nature of the subcircuit is unknown, we do know that the current flowing through it is 5A, as indicated below.



Is it possible, from only the information shown above, to solve for v_{out} unambiguously?

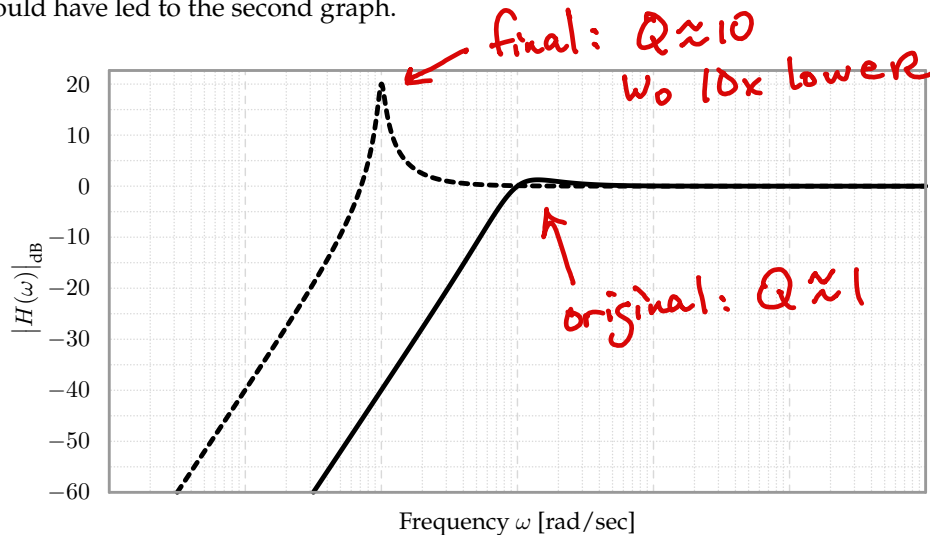
Is this possible? (circle one): **Yes** / No

If it is possible, solve for v_{out} and write your answer below. If it is not possible, briefly explain what additional information you would need, and why that information is necessary.

15V

1.3 Part 3

Consider the following graphs, both of which show the frequency response of a series RLC circuit, measuring the voltage drop across the inductor as the output. The solid line corresponds to the frequency response for some original values of the components, and the dashed curve was made by changing **exactly one component's value**. Compared to the first graph, circle all possible individual changes that could have been made that would have led to the second graph.



R decreased by a factor of $\frac{1}{100}$

R decreased by a factor of $\frac{1}{10}$

R increased by a factor of 10

R increased by a factor of 100

L decreased by a factor of $\frac{1}{100}$

L decreased by a factor of $\frac{1}{10}$

L increased by a factor of 10

L increased by a factor of 100

C decreased by a factor of $\frac{1}{100}$

C decreased by a factor of $\frac{1}{10}$

C increased by a factor of 10

C increased by a factor of 100

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

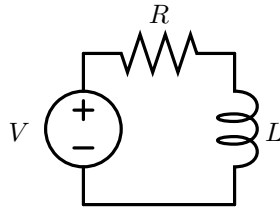
changing R would affect Q
but not ω_0 .

decreasing C so as to increase
 Q would increase ω_0 instead
of decreasing it.

need 100x change in L for 10x in $\sqrt{L}$

1.4 Part 4

In the following circuit, the value of the voltage source is 0 for all $t < 0$ and there is no energy stored in the inductor at time 0. Then, at time $t = 0$, the voltage source's value changes to a constant value V . How much energy is stored in the inductor after two time constants have passed, i.e., at time $t = 2\tau$? Enter your answer as an exact symbolic expression in terms of V , R , and/or L , and simplify your answer to the extent that you can.



Energy stored in the inductor after two time constants:

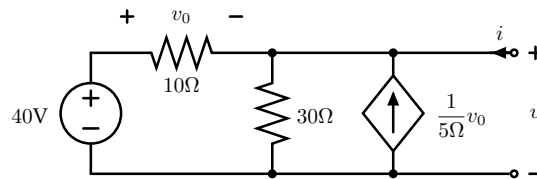
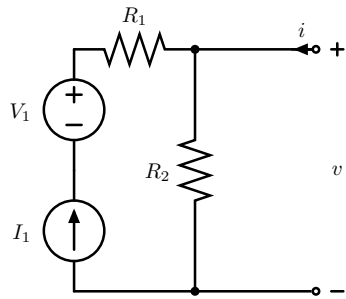
$$\frac{LV^2}{2R^2} \left(1 - \frac{1}{e^2}\right)^2$$

energy is $\frac{1}{2}Li^2$

Current after 2τ is $i = \frac{V}{R} \left(1 - \frac{1}{e^2}\right)$

1.5 Part 5

For the pair of circuits below, find constraints on the parameter values of the unspecified components that would make the two circuits equivalent to one another (in terms of the i - v relation at the indicated port). If any of the unspecified component parameter values are irrelevant, indicate that explicitly in your answer.

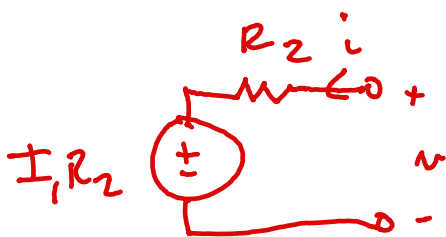


$$I_1 = 12A$$

$$R_2 = 3\Omega$$

V_1 and R_1 don't affect the result

left circuit becomes:



$$v = I_1 R_2 + R_2 i$$

right circuit:

$$\frac{40V - v}{10\Omega} + \frac{40V - v}{5\Omega} + i = \frac{v}{30\Omega}$$

$$120V - 3v + 240V - 6v + 30\Omega \cdot i = v$$

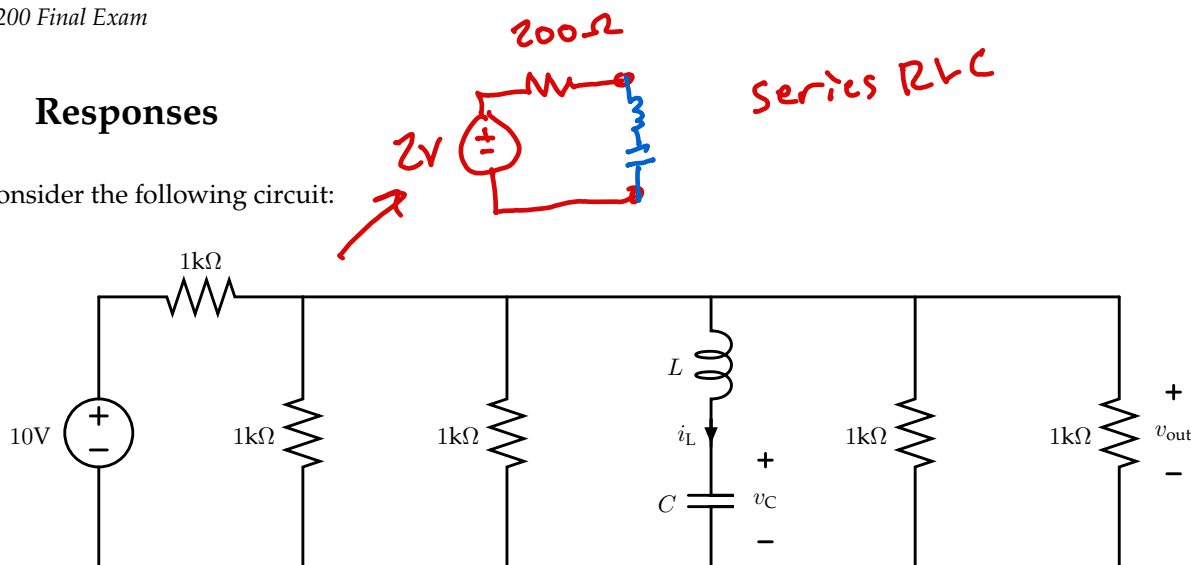
$$360V - 30\Omega \cdot i = 10v$$

$$v = 36V + 3\Omega \cdot i$$

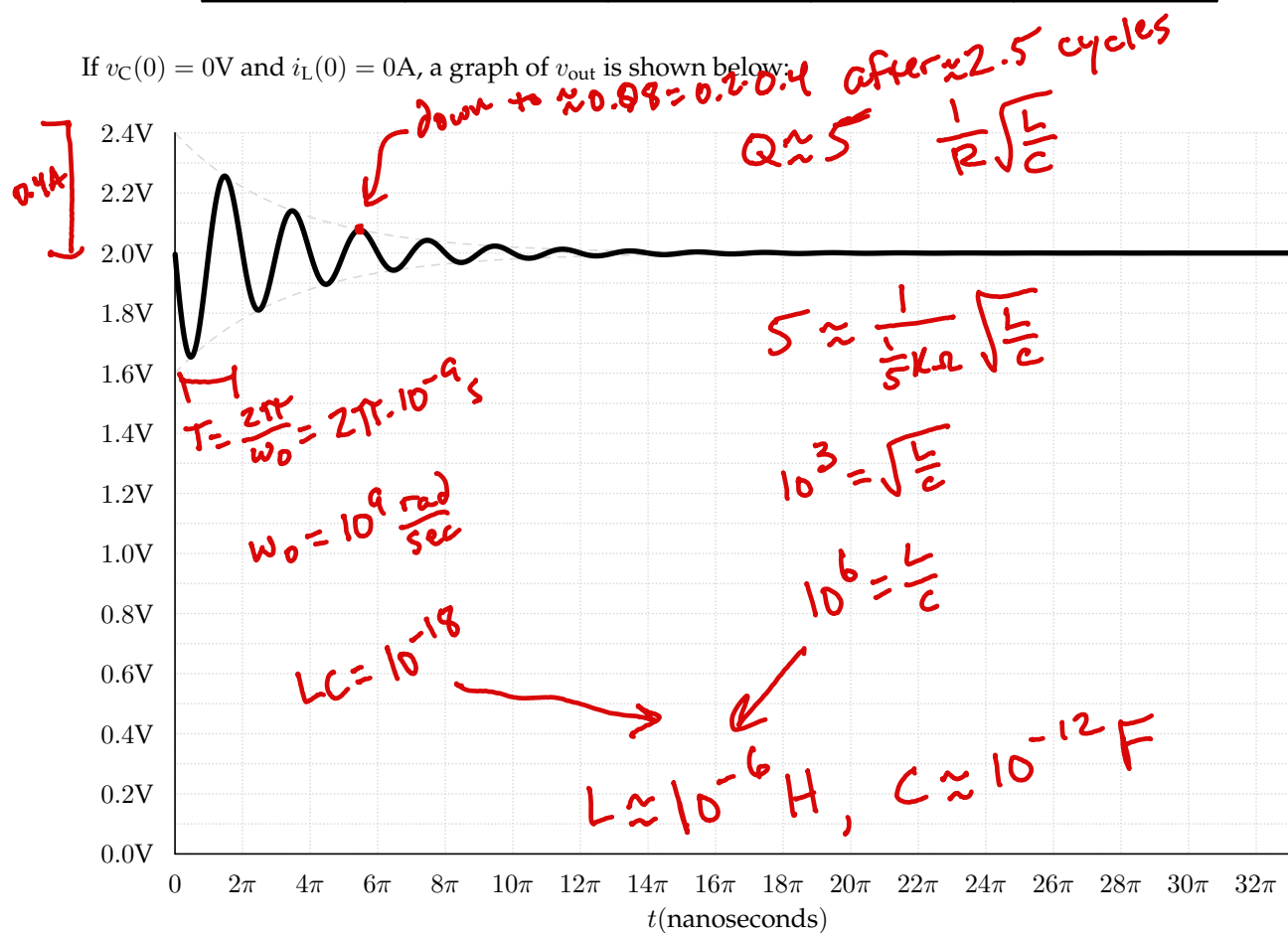
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2 Responses

Consider the following circuit:



If $v_C(0) = 0V$ and $i_L(0) = 0A$, a graph of v_{out} is shown below:



2.1 Part 1

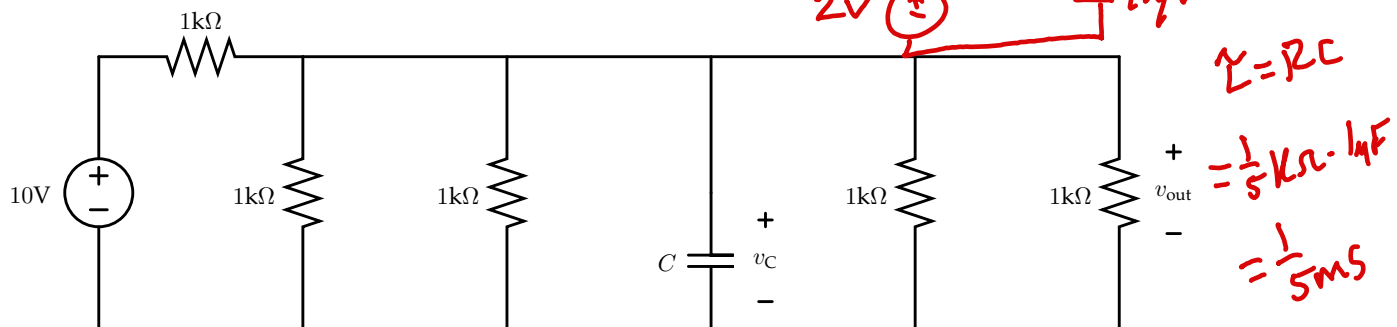
From the information in this graph, estimate the values of L and C . Write a single number in each box below (including units):

$$L = \boxed{1 \mu H}$$

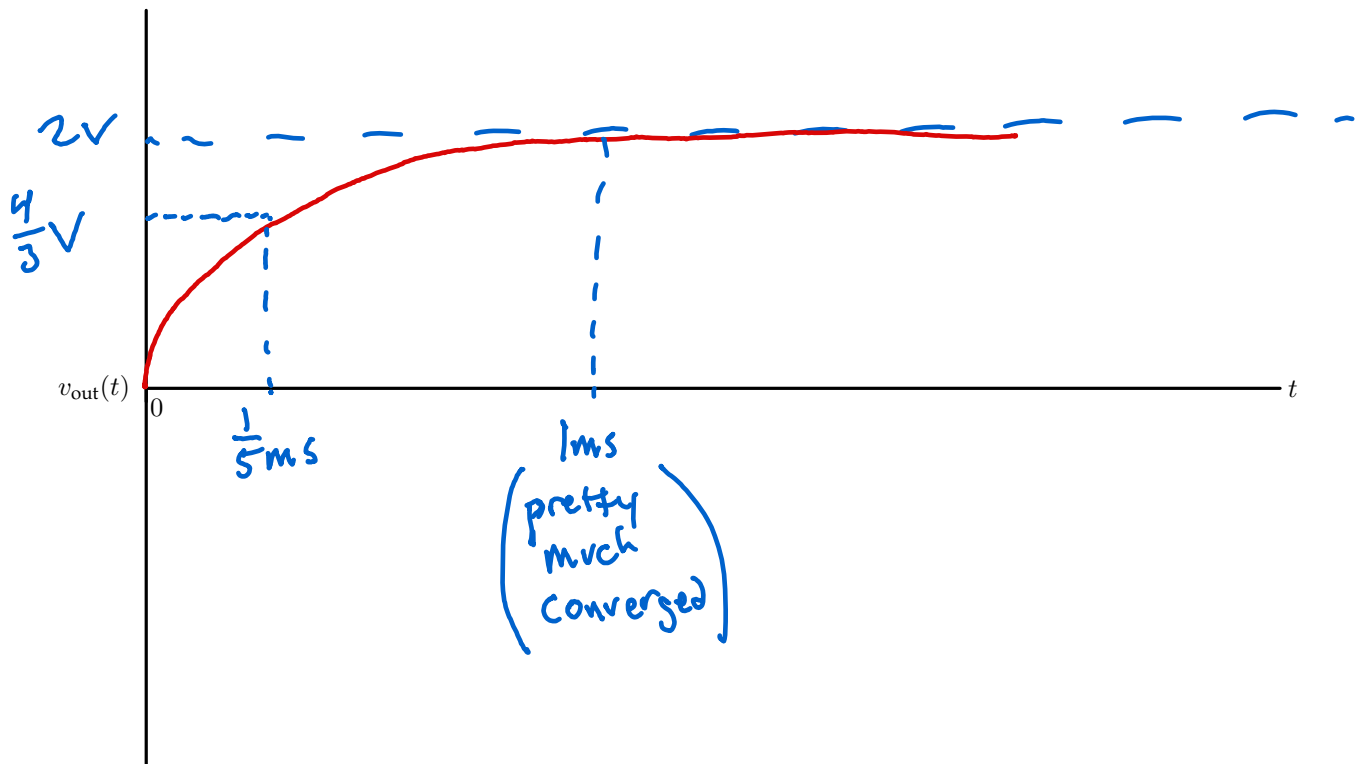
$$C = \boxed{1 pF}$$

2.2 Part 2

Now consider a variant of the circuit from the previous part, with the inductor replaced with a short (but where all other components remain the same as in the original circuit):



Assuming that $v_C(0) = 0\text{V}$, and now using $C = 1\mu\text{F}$ (regardless of the value you found in the previous section), sketch v_{out} as a function of time on the axes below. Label all key values/asymptotes (with units).

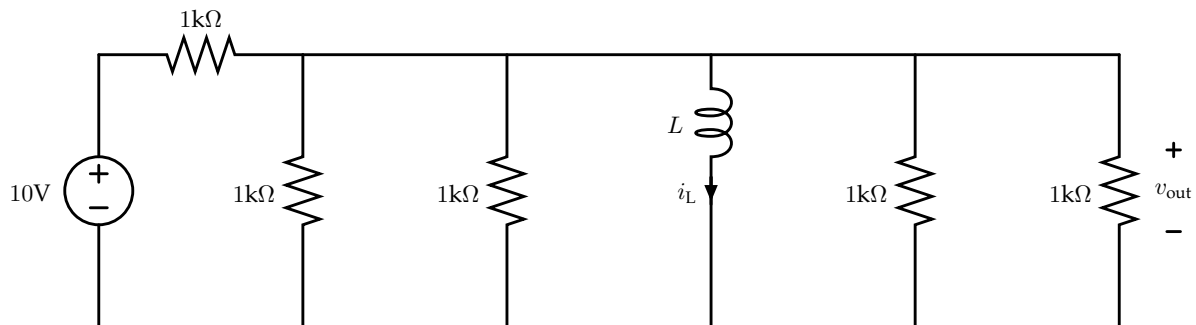


2.3 Part 3

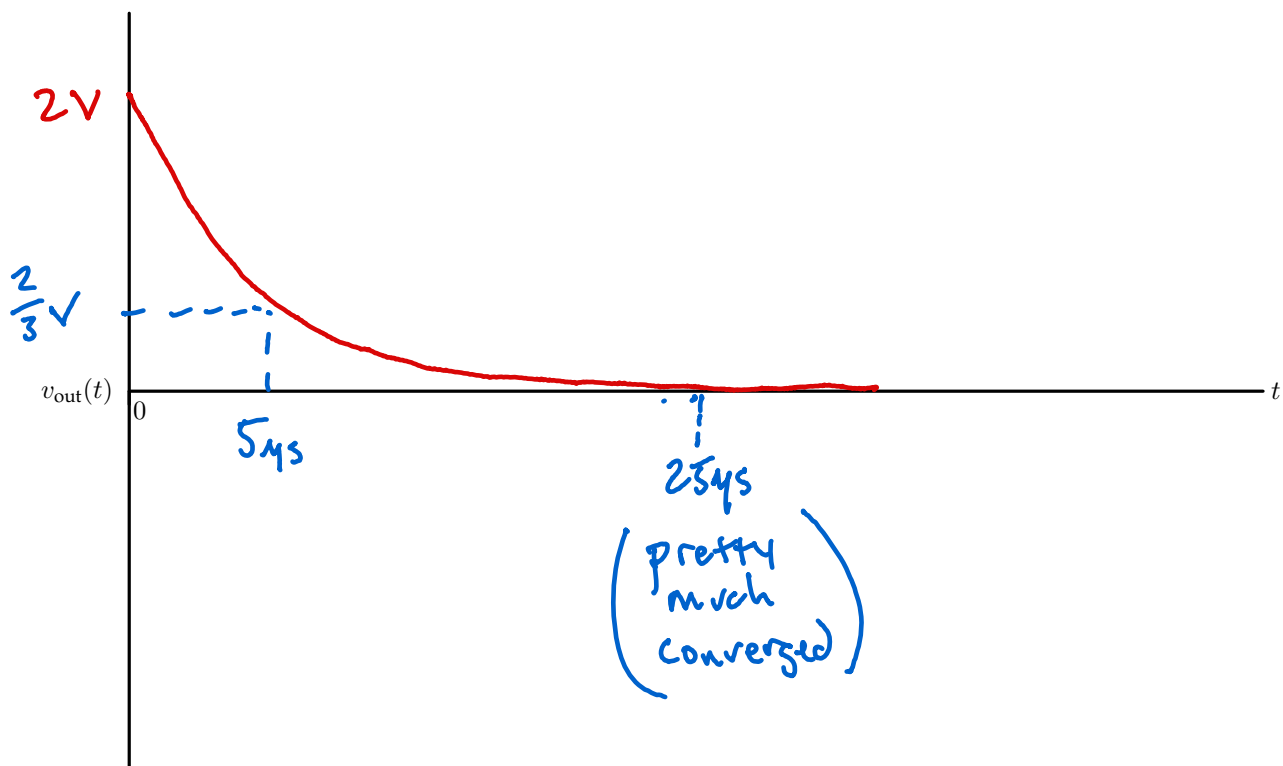
Handwritten notes above the circuit diagram:

200Ω
 $2V$ (with a circled plus-minus sign)
 $1mH$
 $\tau = \frac{L}{R} = \frac{1mH}{\frac{1}{5}k\Omega} = 5\mu s$

Finally, consider another a variant of the original circuit, this time with the *capacitor* replaced with a short (but where all other components remain the same as in the original circuit):



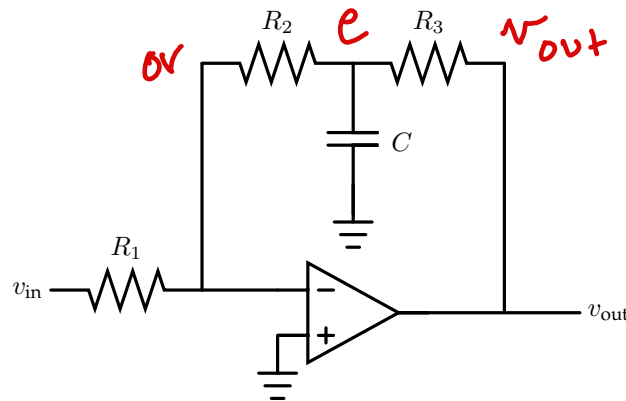
Assuming that $i_L(0) = 0A$, and now using $L = 1mH$ (regardless of the value you found in the first part of this problem), sketch v_{out} as a function of time on the axes below. Label all key values/asymptotes (with units).



3 Filters with Op-Amps

nothing in series/parallel nodes!

Consider the following circuit:



Find the transfer function that relates v_{out} to v_{in} . Make the ideal op-amp assumption, and ignore limitations imposed on the op-amp's output by its power supply.

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{-(R_2 + R_3 + R_2 R_3 s C)}{R_1}$$

$$\frac{-e}{R_2} = \frac{e}{1/sC} + \frac{e - v_{out}}{R_3}$$

$$\frac{v_{in}}{R_1} = \frac{-e}{R_2} \Rightarrow e = -\frac{R_2}{R_1} v_{in}$$

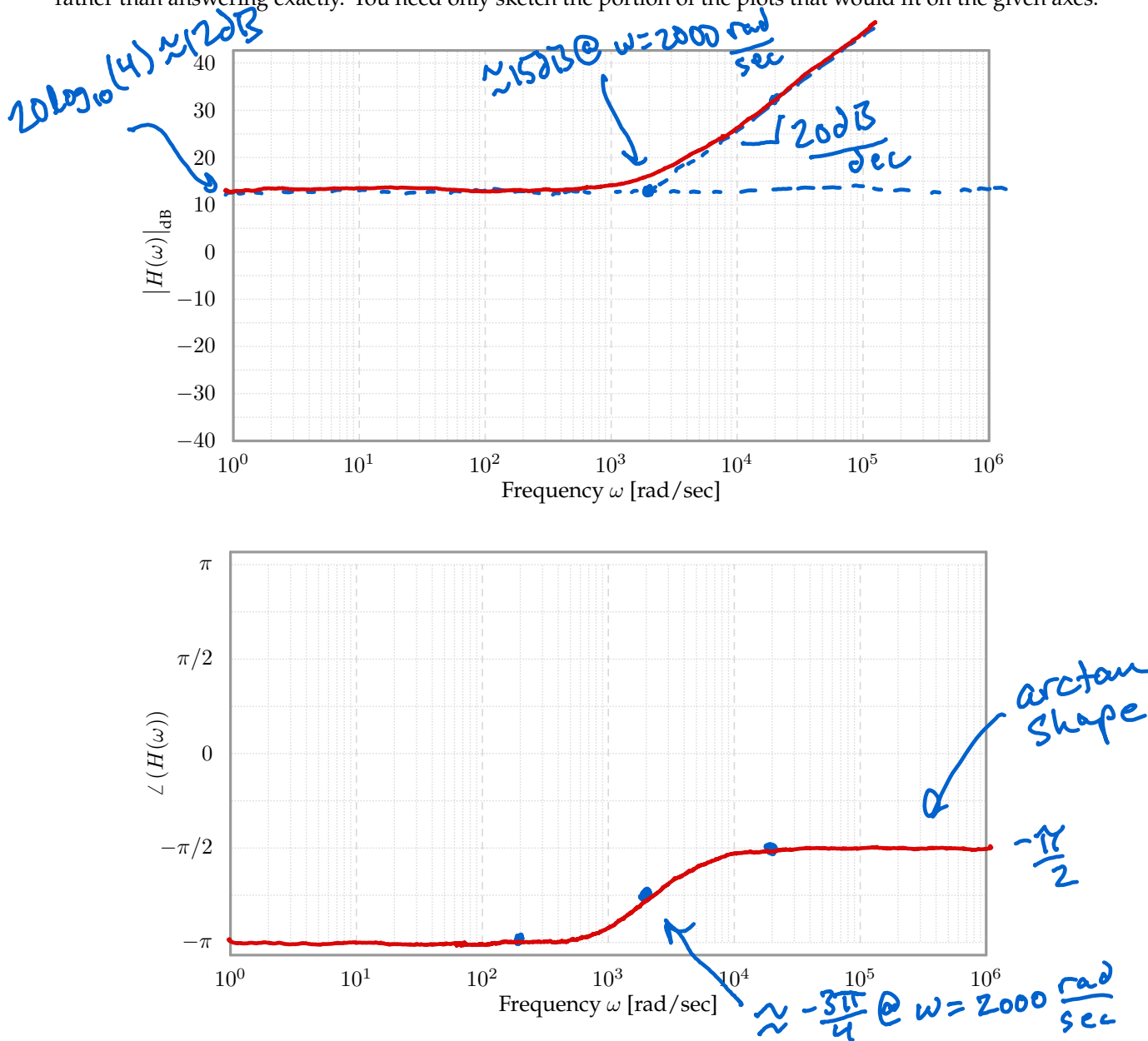
$$\frac{R_2}{R_1} v_{in} = \frac{-R_2 s C}{R_3} v_{in} + \frac{-R_2}{R_1} v_{in} - \frac{v_{out}}{R_3}$$

$$\frac{1}{R_1} v_{in} = -\frac{R_2}{R_1} s C v_{in} + \frac{-R_2}{R_1 R_3} v_{in} - \frac{1}{R_3} v_{out}$$

$$R_3 v_{in} = -R_2 R_3 s C v_{in} - R_2 v_{in} - R_1 v_{out}$$

$$-v_{in}(R_2 R_3 s C + R_3 + R_2) = R_1 v_{out}$$

On the axes below, sketch the Bode plot for this circuit when $R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 2\text{k}\Omega$, and $C = 0.5\mu\text{F}$. Please label all crossover frequencies (as well as the value of each graph at all of the crossover frequencies), and clearly label and provide expressions for all asymptotes. It is OK to make reasonable approximations rather than answering exactly. You need only sketch the portion of the plots that would fit on the given axes.



single zero @ $\frac{-R_2 + R_3}{R_2 R_3 C} = s = \frac{-4\text{k}\Omega}{4\text{M}\Omega^2 \cdot \frac{1}{2}\mu\text{F}} = -2000$

(crossover freq @ 2000 rad/sec)

or, using freq response:

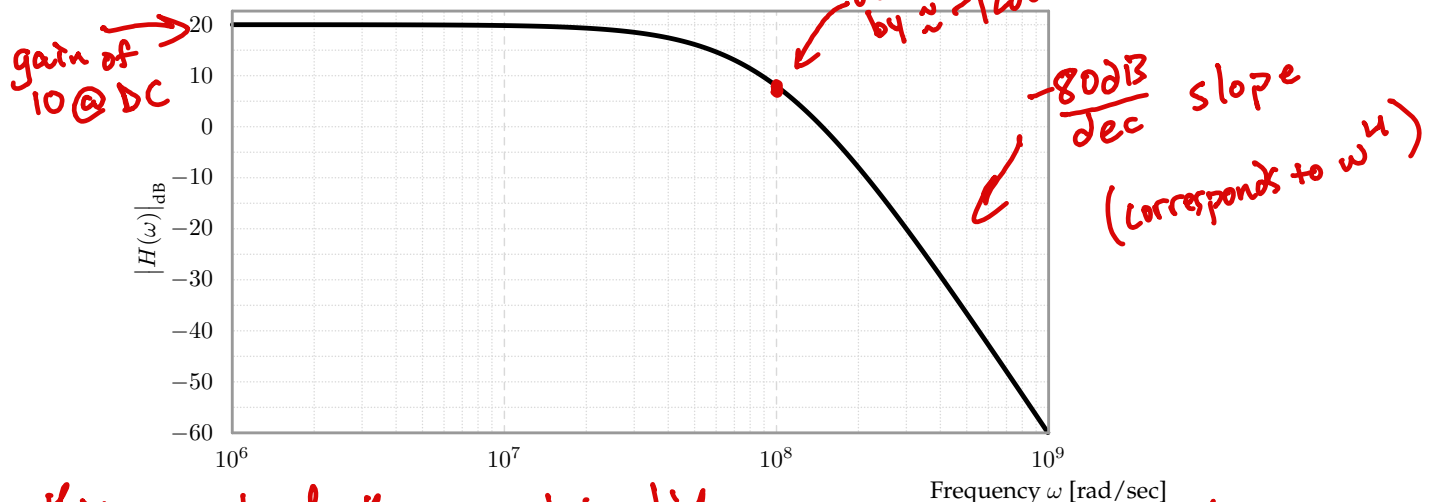
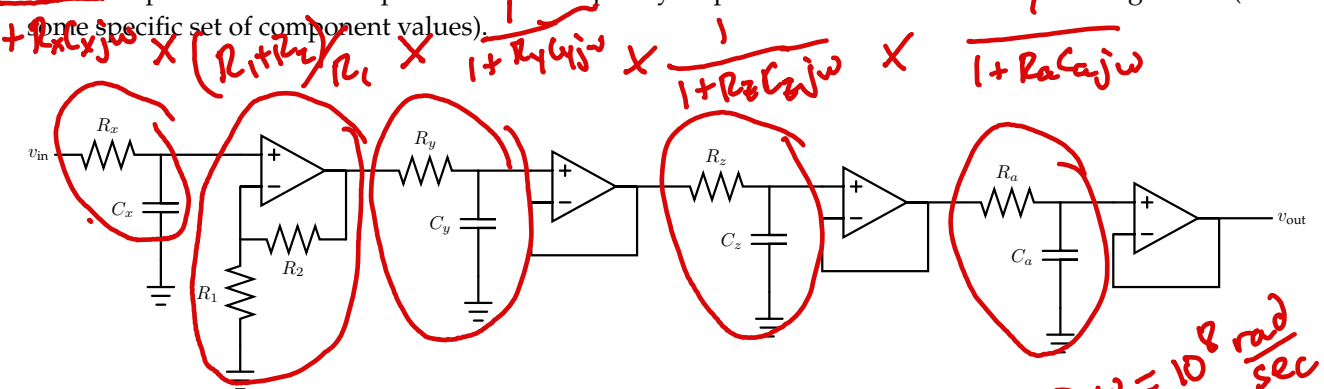
$$H(\omega) = \frac{-(4000 + 2j\omega)}{1000}$$

big $\omega \rightarrow -\frac{2}{1000}j\omega \rightarrow \text{mag } \frac{2}{1000}\omega$
 $\rightarrow \text{phase } -\frac{\pi}{2}$

small $\omega \rightarrow -4 \rightarrow \text{mag } 4$
 $\rightarrow \text{phase } \pi$ | crossover @ $\frac{2}{1000}\omega = 4$

4 Chaining Filters

The Bode plot below shows a portion of the frequency response associated with the following circuit (for some specific set of component values).



this graph looks a lot like our usual LPF, but with a -80 dB/dec slope instead. if we assume the cutoffs for each RC are the same, we expect to be 120 dB down at that cutoff

4.1 Part 1

Below, please enter values (including units) for each of the components in order to make the circuit's frequency response $H(\omega)$ match what is shown above. Note that there are multiple correct answers; you only need to provide one. (4 x 30)

You may make the ideal op-amp assumption and ignore power supply limitations.

$R_x = R_y = R_z = R_a = 1\ \Omega$
 $C_x = C_y = C_z = C_a = 10\text{ nF}$

$R_2 = 9\text{ k}\Omega$
 $R_1 = 1\text{ k}\Omega$

$R_i C_i = 10^{-8}$
 any $R_2 = 9R_1$

4.2 Part 2

Now, consider the case where $v_{\text{in}}(t)$ is given by the following function for all time:

$$v_{\text{in}}(t) = 2V + 2V \times \cos(10^8 t) + 2V \times \cos(10^{10} t)$$

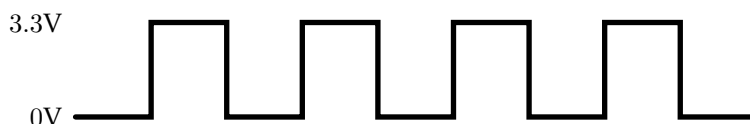
Given this input, find an approximate expression for $v_{\text{out}}(t)$ after the system has settled to its steady-state behavior. Enter your expression below:

$$\begin{aligned}
 & 2V \cdot 10V + 2V \cdot 10V \cdot \underbrace{\left(\frac{1}{\sqrt{2}}\right)^4}_{\substack{\frac{1}{\sqrt{2}} \text{ from} \\ \text{each RC}}} \cos\left(10^8 t - \underbrace{\frac{\pi}{4} \cdot 4}_{\substack{-\frac{\pi}{4} \text{ from} \\ \text{each RC}}}\right) + 2V \cdot 10V \cdot \underbrace{(10^{-4})^2}_{\substack{-80\text{dB/dec}, \\ 2 \text{ decades} \\ \text{from cutoff}}} \cos\left(10^{10} t - \underbrace{\frac{\pi}{2} \cdot 4}_{\substack{-\frac{\pi}{2} \text{ from} \\ \text{each RC}}}\right) \\
 & = 20V - 5V \cos(10^8 t) + 0.24V \cdot \cos(10^{10} t)
 \end{aligned}$$

5 Blink Blink

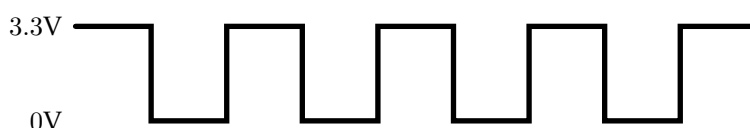
In the folder at your table (which should be labeled with the same number that is printed on the bottom of this exam near the QR codes), you will find a breadboard containing the circuit we'll use for this problem, as well as some parts you can use to add to the circuit.

Currently on the board, you should see a Teensy, powered by a 6-Volt battery pack (yours may not be exactly 6 Volts, but it should be close). Pin 14 on the Teensy is producing a signal that slowly alternates between 3.3 Volts and 0 Volts at a fixed rate:



A little series combination of a $2k\Omega$ resistor and an LED is also hooked up to pin 14, so you should be able to see it blinking on and off when you flip the power switch on the battery case to “on.”

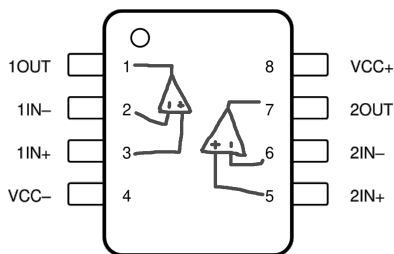
Your goal for this problem will be to keep this signal intact, but also to create a separate signal that also alternates between 3.3 Volts and 0 Volts (measured relative to the same ground), but out of time with our original signal (so then when one is on, the other is off, and *vice versa*):



You should use the components in the bag, as well as the Teensy, to help construct this output. Also, although you are welcome to add to the circuit, please do not remove any components that were in the board when you received it.

The small baggie in your envelope contains several components that you can use to accomplish this goal. Note that you do not need to use all of the components in the baggie. In addition to jumper wires, your baggie should contain:

- A small number of $2k\Omega$ resistors.
- An MCP6002 op-amp package. This is the same one that we've used in lab a few times; as a refresher, each package contains 2 op-amps, and its pinout looks like the following:



- One additional LED, which you should connect up to your output when you're done (in series with a $2k\Omega$ resistor like the first one). Note that the LEDs have polarity: when connecting your LEDs, the longer leg should be connected to a higher potential than the shorter leg.
- There should also be a multimeter nearby, which you can feel free to use to help with testing and debugging.

Our chosen ground/reference node is available on the Teensy pin labeled G. The Teensy also provides a couple of other signals, each of which can be modeled as a voltage source under most conditions:

- A constant 3.3-Volt signal is available on the pin labeled 3V and on pin 11.
- The alternating signal discussed earlier is available on pin 14.

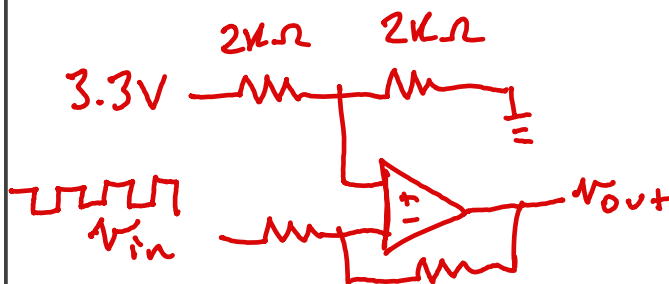
5.1 Planning

In the box below, please draw a schematic for the circuit you plan to implement, clearly labeling both the input and output signals. You do not need to explicitly draw the load ($2k\Omega$ resistor and LED in series), but it's fine to include those if you want.

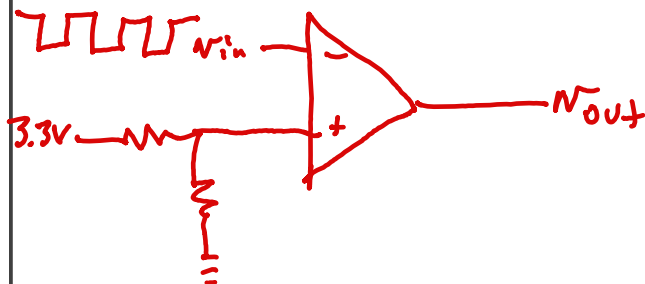
This problem will be graded primarily on your physical implementation of the circuit, but we will consider your schematic when assigning partial credit.

we have at least a couple of options here.

3.3V - v_{in} with a subtractor:



Comparator (iff we power the op-amp w/ 3.3V and 0V)



Planning is all well and good, but don't forget to build your circuit as well (instructions on the following page!)

Note that some options (like inverting the input around 0V) are not viable because of output limitations of the op-amp.

5.2 Building

Now, go ahead and build your circuit. If everything is working, you should have two LEDs blinking on and off: when one is on, the other is off, and *vice versa*. They do not need to be the same brightness, but measuring the drop across these resistor-and-LED combinations (measured relative to our chosen ground) should yield the signals described in the first section of this problem.

When you are done, please:

- Double-check that the number on your envelope matches the number printed at the bottom of this exam near the QR codes.
- Make sure that your battery pack is turned **OFF**.
- Carefully slide your completed circuit back into its envelope and **leave it at your table**; we'll pick it up.

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