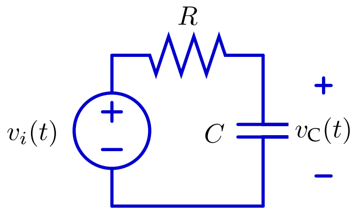

Assume $v_i(t) = A\cos(\omega t)$

Solving for the particular part of $v_C(t)$, which is **any** $v_{C,p}(t)$ that satisfies our diffeq: $v_i = v_{C,p} + RCv'_{C,p}$.

Given the form of our answer, assume:

$$v_{C,p} = B\cos(\omega t + \phi)$$

Using the trig identity that $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$, we can re-express this:

$$v_{C,p} = B\cos(\phi)\cos(\omega t) - B\sin(\phi)\sin(\omega t)$$

And we can also find $v'_{C,p}$:

$$v'_{C,p} = -B\omega\cos(\phi)\sin(\omega t) - B\omega\sin(\phi)\cos(\omega t)$$

Subbing these back into our original diffeq $v_i = v_C + RCv'_C$:

$$A\cos(\omega t) = B\cos(\phi)\cos(\omega t) - B\sin(\phi)\sin(\omega t) - RCB\omega\cos(\phi)\sin(\omega t) - RCB\omega\sin(\phi)\cos(\omega t)$$

To make these match, note that the only non-constant pieces there are $\cos(\omega t)$ and $\sin(\omega t)$. So we can rearrange a little bit:

$$\begin{aligned} A\cos(\omega t) + 0\sin(\omega t) &= B\cos(\phi)\cos(\omega t) - RCB\omega\sin(\phi)\cos(\omega t) - RCB\omega\cos(\phi)\sin(\omega t) - B\sin(\phi)\sin(\omega t) \\ &= (B\cos(\phi) - RCB\omega\sin(\phi))\cos(\omega t) - (RCB\omega\cos(\phi) + B\sin(\phi))\sin(\omega t) \end{aligned}$$

This gives us two relationships that must be satisfied: $A = B\cos(\phi) - RCB\sin(\phi)$, and $0 = B\sin(\phi) + RCB\omega\cos(\phi)$.

We can use these to solve for ϕ and B (note that the result for B depends on the result for ϕ):

$$0 = \sin(\phi) + RC\omega\cos(\phi) \qquad\qquad A = B\cos(\phi) - RCB\omega\sin(\phi)$$

$$\sin(\phi) = -RC\omega\cos(\phi) \qquad\qquad A = \frac{B}{\sqrt{1 + (\omega RC)^2}} + \frac{B(RC\omega)^2}{\sqrt{1 + (\omega RC)^2}}$$

$$\left(\frac{\sin(\phi)}{\cos(\phi)}\right) = -RC\omega \qquad\qquad A = \frac{B(1 + (\omega RC)^2)}{\sqrt{1 + (\omega RC)^2}}$$

$$\phi = \tan^{-1}(-RC\omega) \qquad\qquad B = \frac{A}{\sqrt{1 + (\omega RC)^2}}$$

Therefore, $v_{C,p} = \dfrac{A}{\sqrt{1 + (\omega RC)^2}}\cos\left(\omega t + \tan^{-1}(-RC\omega)\right)$