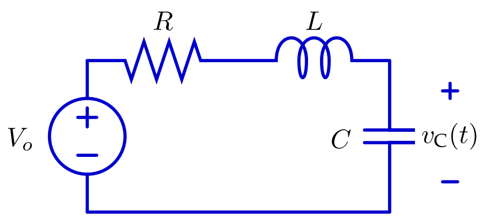

$$v_R = iR$$

$$v_L = Li'$$

$$i = Cv_C'$$

We can set up our differential equation as before, using KVL:

$$v_L + v_R + v_C = V_o$$

For now, let's focus on the homogeneous part of the solution here, which is what we get by setting the left-hand side of the equation equal to 0:

$$v_L + v_R + v_C = 0$$

We can then simplify using the constitutive equations of the various components to get an equation involving only v_C:

$$Li' + iR + v_C = 0$$

$$LCv_C'' + RCv_C' + v_C = 0$$

$$v_C'' + \frac{R}{L}v_C' + \frac{1}{LC}v_C = 0$$

We'll also define $\omega_o = \frac{1}{\sqrt{LC}}$ and $\alpha = \frac{R}{2L}$ so that our equation becomes:

$$v_C'' + 2\alpha v_C' + \omega_o^2 v_C = 0$$

Then we can proceed as we have done in the past, by assuming a form of v_C; here we assume $v_C = Ae^{st}$ and plug in:

$$s^2Ae^{st} + 2\alpha sAe^{st} + \omega_o^2Ae^{st} = 0$$

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

We can then solve for s with the quadratic formula:

$$s = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4\omega_o^2}}{2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

There are two answers here, so our actual answer looks more like $v_C = A_1e^{s_1t} + A_2e^{s_2t}$.

Depending on the specific values of R, L, and C (which affect α and ω_o), we can get dramatically different behaviors out of the circuit.