6.200 Midterm

Spring 2025

Name: **Answers**

Kerberos/Athena Username:

5 questions   1 hour, 50 minutes

- Please **WAIT** until we tell you to begin.

- Write your name and kerberos **ONLY** on the front page.

- This exam is closed-book, but you may use one 8.5″ × 11″ sheet of handwritten notes (both sides) as a reference. This sheet must be **handwritten** directly on the page (not printed).

- You may **NOT** use any electronic devices other than a multimeter (no computers, calculators, phones, etc.).

- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.

- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam. **Please do not write on the QR codes.**

- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.

- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

Worksheet (intentionally blank)
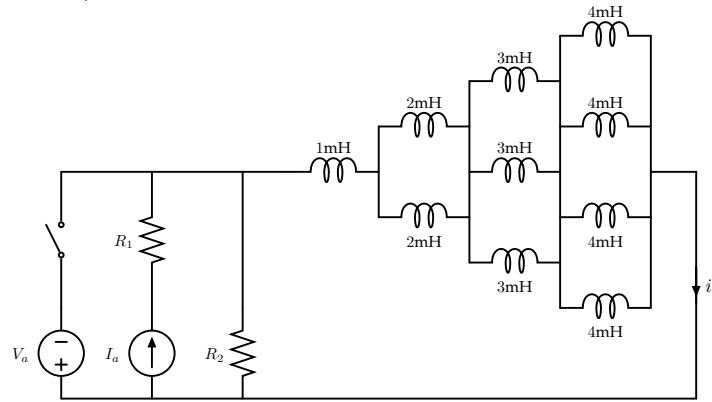
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1   RL Circuit

Consider the following circuit, where V_a and I_a are constant sources, and R_1 and R_2 are constant-valued resistors.

In particular, consider the case where the switch is open for all time $t < 0$, closed for time $0 \le t < 40\mu s$, and open again for all time $t \ge 40\mu s$.


A few important things to notice:

- The tower of inductors collapses to a single equivalent inductance of 4mH.
- When the switch is closed, the voltage source is in parallel with this inductance, so the current source doesn't matter, nor do the resistors.
- When the switch is open, R_1 is irrelevant because the constraint imposed on the circuit by the combination of R_1 and I_a in series is the same as that imposed by just I_a. So in that case, the inductor is effectively connected to a Norton consisting of I_a and R_2.

1.1   Graph 1

Is it possible to adjust the values of V_a, I_a, R_1, and R_2 such that a graph of i_b looks like the following? If so, what values of V_a, I_a, R_1, and R_2 are necessary to make the graph look this way? If not, briefly explain why not.
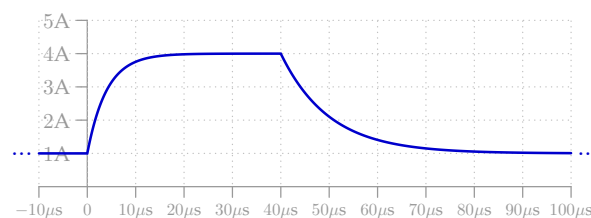
> **Not possible.** With the switch closed ($0 < t < 40\mu s$), we have a constant voltage dropping across our inductor (the effective inductance of the whole combination), which means that the shape of the graph during that time should be a straight line, not an exponential.

1.2 Graph 2

Is it possible to adjust the values of V_a, I_a, R_1, and R_2 such that a graph of i_b looks like the following? If so, what values of V_a, I_a, R_1, and R_2 are necessary to make the graph look this way? If not, briefly explain why not.
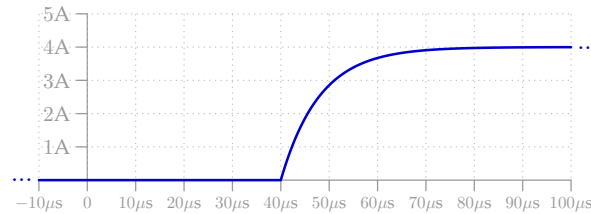
Not possible. The initial (before time 0) and final values must match because they both represent the steady state of the same circuit.

1.3 Graph 3

On the axes below, sketch a graph of i_b as a function of time when $V_a = 200\text{mV}$, $I_a = 5\text{mA}$, $R_1 = 250\Omega$, and $R_2 = 1\text{k}\Omega$. Clearly label all key values, slopes, asymptotes, and time constants in your graph, including units.
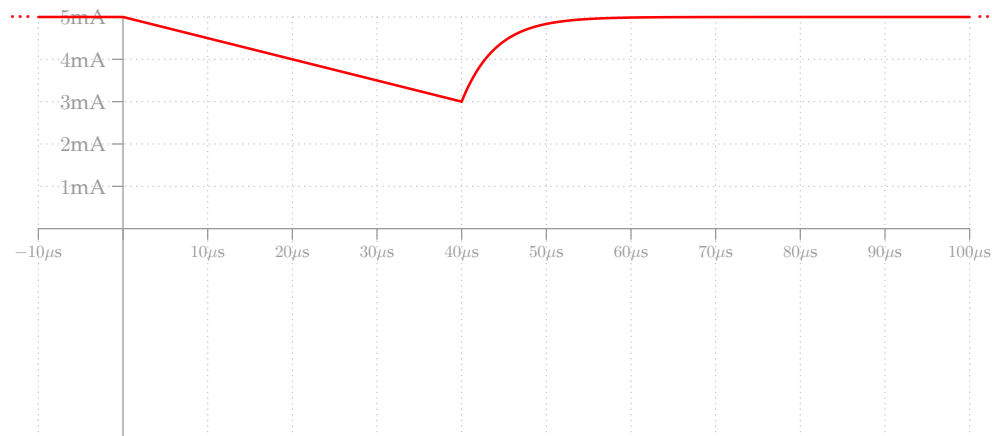

- Before $t = 0$ (and in the long term), $i_b = 5\text{mA}$.
- From $0 < t \leq 40\mu\text{s}$, we have a constant drop of -200mV across the inductor, so $\frac{di_b}{dt} = \frac{-200\text{mV}}{4\text{mH}} = -50\text{A/s}$. Thus, starting from 5mA at time $t = 0$, the current decreases linearly, ultimately reaching $5\text{mA} - (50\text{A/s}) \times 40 \times 10^{-6}\text{s} = 3\text{mA}$ at $t = 40\mu\text{s}$.
- Then, for $t > 40\mu\text{s}$, we see an exponential rise with a time constant of $\tau = 4\text{mH}/1\text{k}\Omega = 4\mu\text{s}$, which is pretty much converged to its final value by $t = 60\mu\text{s}$.

2 Dependent Sources and Op-Amps

2.1 Dependent Source

Using the node method, set up (but do not solve) a system of two linearly-independent KCL equations involving v_1 and v_{out} (as well as other constants in the circuit) that, when solved, would correctly produce a solution for v_{out} in the circuit below. As a self-check, the solution is $v_{\text{out}} = 13.5$ Volts.
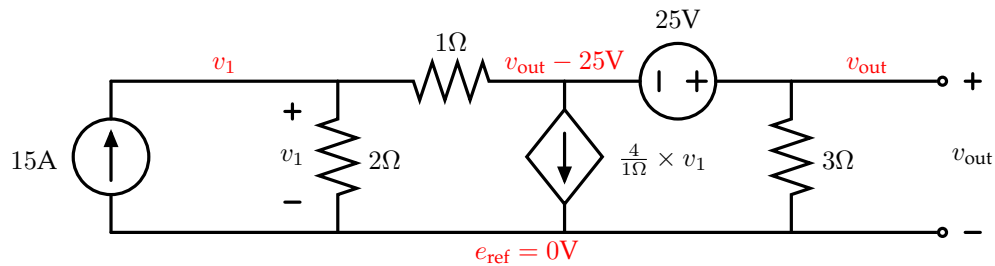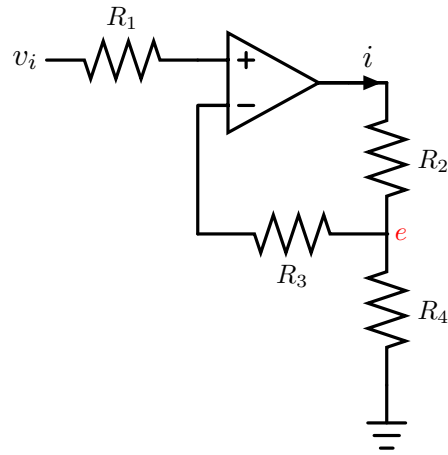

Equation 1:

$$15\text{A} = \frac{v_1 - (v_{\text{out}} - 25\text{V})}{1\Omega} + \frac{v_1}{2\Omega}$$

Equation 2:

$$\frac{v_1 - (v_{\text{out}} - 25\text{V})}{1\Omega} = \frac{4v_1}{1\Omega} + \frac{v_{\text{out}}}{3\Omega}$$

2.2   Op-Amp

In the circuit below, solve for the indicated current i. Enter your answer in terms of v_i, R_1, R_2, R_3, and/or R_4. Make the ideal op-amp assumption, and ignore any limitations on the op-amp's output from its power supply.

$$i = \frac{v_i}{R_4}$$

- By the ideal op-amp assumption, $v_- = v_+ = v_i$.

- No current into or out of the input terminals of the op-amp means that the voltage at the node labeled e above is also equal to v_i, which means that the voltage drop across R_4 is v_i.

- Thus, the current flowing through R_4 is v_i/R_4 (flowing from top to bottom), and this is precisely equal to i because no current flows through R_3.

3 Superposition

Use superposition to solve the following circuit, and show your work on the facing page (if you need additional space, please clearly reference the page number(s) where your additional work can be found). Then enter the values of the specified quantities in the boxes below (v_x, i_y, and the power supplied by each source in the circuit).
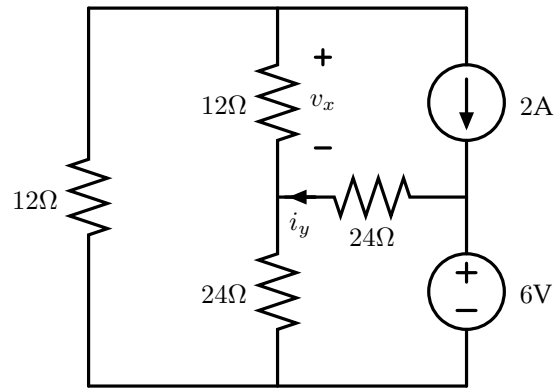
$v_x =$ | −9V

$i_y =$ | 500mA

Power supplied by 6V voltage source: | −9W
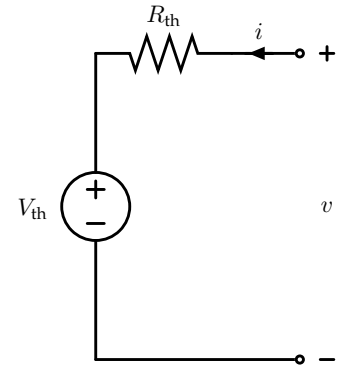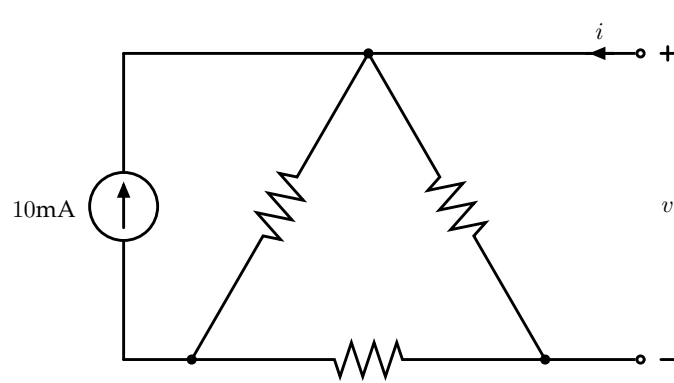
Power supplied by 2A current source: | 42W

Work for Superposition Problem:

4 Return of the Triangle Circuits

4.1 Part 1

Assume that each of the resistors in the left circuit has a resistance of $10\text{k}\Omega$. Determine the voltage V_{th} and the resistance R_{th} such that these two circuits are equivalent when viewed from the port labeled with v and i. Include units in your answers.

$$V_{\text{th}} = \boxed{\frac{100}{3}\text{V}} \qquad R_{\text{th}} = \boxed{\frac{20}{3}\text{k}\Omega}$$

4.2 Part 2

Assume that each of the resistors in the left circuit has a resistance of $10\text{k}\Omega$. Determine the voltage V_{th} and the resistance R_{th} such that these two circuits are equivalent when viewed from the port labeled with v and i. Include units in your answers.
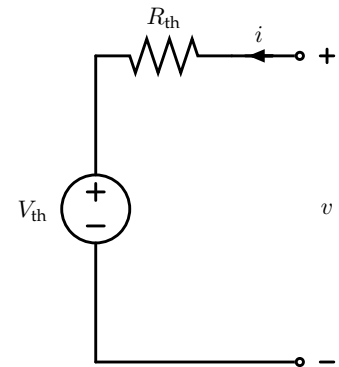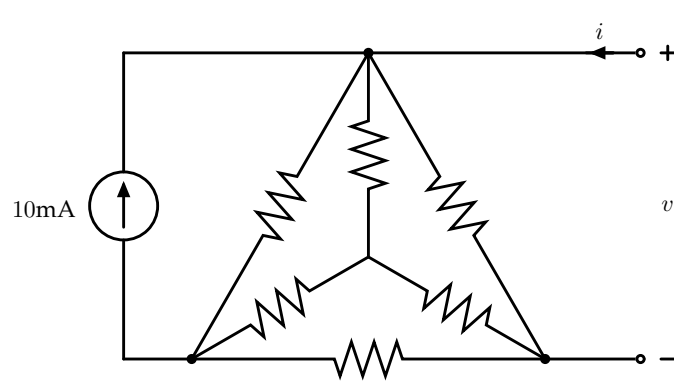

$V_{\text{th}} =$ | 25V

$R_{\text{th}} =$ | 5kΩ

5 Circuit Design

Your goal for this problem is to design and build a circuit consisting of a 9-Volt battery, a button, and no more than eight $1\text{k}\Omega$ resistors that has the following properties:

- The total power provided by the battery is 81mW when the button is open (not pressed), and 108mW when the button is closed (pressed).

- Some branch voltage in the circuit (call it v_a) is 4.5V when the button is open (not pressed), and 6V when the button is closed (pressed).

- The current i_b through the button is 0 when the button is open (not pressed), and 6mA when the button is closed (pressed).

5.1 Schematic

In the box on the facing page, draw a schematic for a circuit that meets these specifications. Model the battery as a 9-Volt voltage source. Clearly label v_a and i_b in your schematic.

5.2 Physical Circuit

In the folder at your table (which should be labeled with the same number that is printed on the bottom of this exam near the QR codes), you will find a breadboard, a 9-Volt battery, eight $1\text{k}\Omega$ resistors, a button, and a small number of jumper wires. Use these components to physically construct a circuit matching the description from above. Connecting the battery to the connector on the breadboard will connect it between the rec and blue power rails on that side of the breadboard.
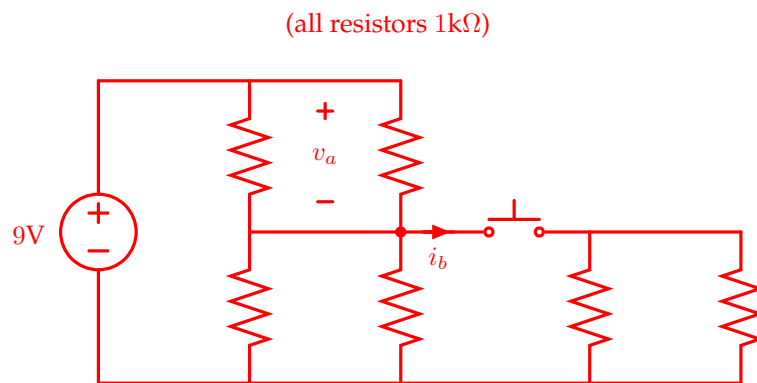
Additionally, your kit contains 2 longer wires labeled v_a+ and v_a-, respectively; plug one end of each wire into the breadboard where we would need to connect a multimeter to measure v_a. Your final circuit should be such that attaching a multimeter to those points would result in measuring v_a as described above.

Note as you are building and debugging your circuit that the resistors you've been given are not exactly $1\text{k}\Omega$, nor is the battery perfectly modeled as a 9-Volt voltage source, which means that the values you measure will not be exactly the values you designed for. This is expected, and you should not adjust your circuit to make the values closer to what's described above; rather, you should build your circuit such that if those components matched their models exactly, your circuit would behave exactly as described above.

When you are done, please:

- Double-check that the number on your envelope matches the number printed at the bottom of this exam near the QR codes.

- Disconnect your battery and re-cover its terminals. Leave your battery on your table outside of the envelope.

- Carefully slide your completed circuit back into its envelope and leave it at your table.

Schematic:

(all resistors 1kΩ)

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