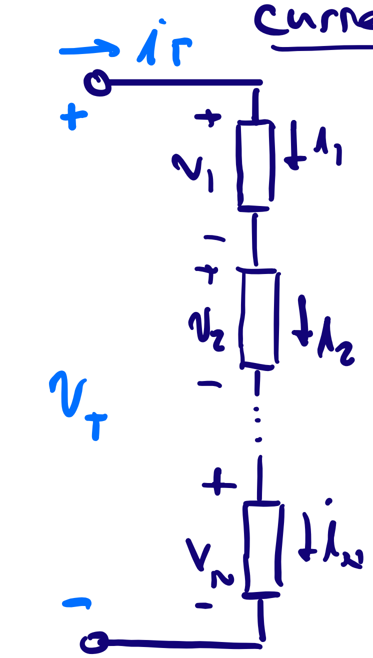
Series: Elements are in series iff they share the same current



$$i_T = i_1 = i_2 = \cdots = i_N$$

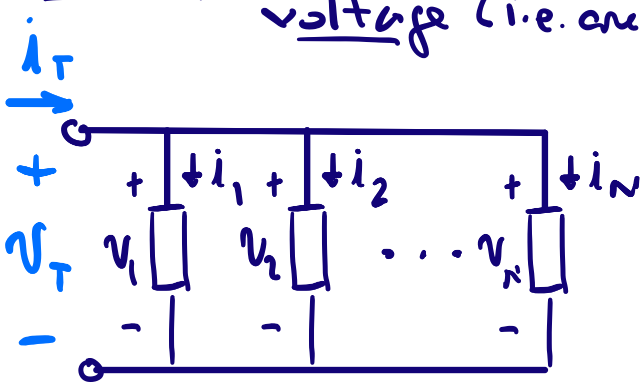
$$v_T = v_1 + v_2 + \cdots + v_N$$

For resistors in series, we get a resistance for the whole "chain" of:

$$\frac{v_T}{i_T} = R_T = \sum_{j=1}^{N} R_j$$

e.g. 2 resistors R_1, R_2 in series form a net resistance $R_T = R_1 + R_2$

Parallel: Elements are in parallel if they share the same voltage (i.e. are connected between the same nodes)



$$v_T = v_1 = v_2 \cdots = v_N$$

$$i_T = i_1 + i_2 + \cdots + i_N$$

For resistors in parallel we get a net resistance for the whole parallel group of:

$$\frac{v_T}{i_T} = R_T = \frac{1}{G_1 + G_2 + \cdots + G_N} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}}$$

e.g. 2 resistors R_1, R_2 in parallel gives a net resistance:

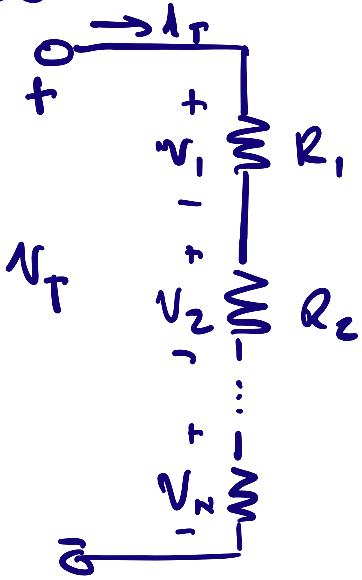
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 R_2} \triangleq R_1 \| R_2$$

- we often combine series resistors into a single resistor to reduce # of nodes and simplify analysis of a network. Once we've done this and found the voltage V_T across the whole resistor "chain", how can we find the voltages across individual resistors?

Voltage dividers:

Suppose we have a set of resistors in series with a total voltage V_T across the series "chain". How can we find the voltages across the individual resistors in terms of the total voltage?

Consider N resistors $R_1, R_2, \ldots R_N$ in series:



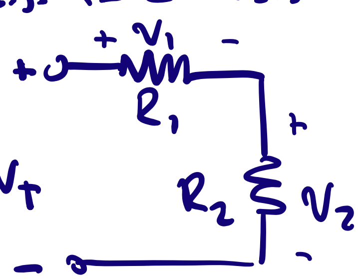
$$v_j = i_T \cdot R_j$$

$$v_T = v_1 + \cdots + v_N$$
$$= i_T \cdot (R_1 + \cdots + R_N)$$

$$\therefore \frac{v_j}{V_T} = \frac{R_j}{R_1 + \cdots + R_N}$$

$\boxed{\text{So resistors in series share the total voltage according to their fraction of the total resistance. Voltage "divides" proportional to the fractional resistance}}$

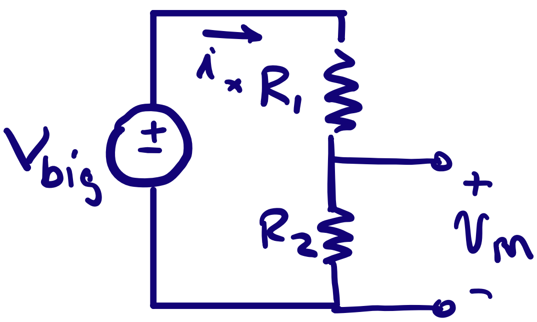
e.g. for 2 resistors:



$$v_2 = \frac{R_2}{R_1 + R_2} V_T \qquad v_1 = \frac{R_1}{R_1 + R_2} \cdot V_T$$

A Series resistor chain is often used as a "voltage divider" to provide a scaled-down version of a large voltage.

Suppose we had a large power supply voltage V_{big} that could be as large as 10,000 V. We might want to monitor that voltage, but we might have monitoring circuitry that can only take a voltage input V_m that is less than 10 V. We could use a voltage divider to make V_m a scaled-down replica of V_{big}:



$$V_m = \frac{R_2}{R_1+R_2} \cdot V_{big}$$

To make $V_m = 10\,V$ @ $V_{big} = 10{,}000\,V$

$$\frac{R_2}{R_1+R_2} = \frac{10}{10{,}000} = 1 \times 10^{-3} = 0.001$$

If $R_1 + R_2 = 10\,M\Omega$, $R_2 = 10\,k\Omega$

$$\therefore\ R_1 = 10\,M\Omega - 10\,k\Omega = 9.99\,M\Omega$$

How much __power__ do we dissipate?

power dissipated in a component x: $P_x = v_x \cdot i_x$

For a resistor $i_x = V_x / R_x$ $\therefore$ $P_x = v_x^2 / R_x$

$\therefore$ Total power in R_1, R_2 is $P_{TOT} = V_{big}^2 / (R_1 + R_2)$

$= (10{,}000)^2 / 10^7 = 10\,W!$

The power in R_2 is $P_2 = \dfrac{V_m^2}{R_2} = \dfrac{(10)^2}{10^4} = 0.01\,W$

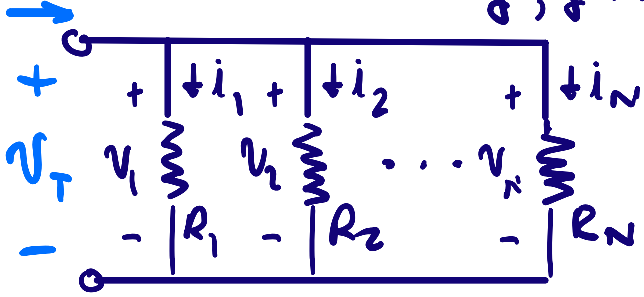
All the rest is dissipated in R_1

In practice we'd likely build R_1 from a series chain of resistors both to get the exact value and to handle the power.

Current Divider: Calculate how current splits among a set of paralleled resistors.

Suppose we have a set of paralleled resistors carrying a total current i_T. Let's find how this current distributes (divides) among the resistors

i_T Resistors R_j, $j = 1..N$ where $G_j = \frac{1}{R_j}$



$$i_j = v_T \cdot G_j$$

$$i_T = i_1 + i_2 + \cdots + i_N$$
$$= v_T \cdot (G_1 + G_2 + \cdots + G_N)$$

The j^{th} branch current over the total current is thus:

$$\frac{i_j}{i_T} = \frac{G_j}{G_1 + \cdots + G_T} \quad \text{or} \quad i_j = \frac{G_j}{G_1 + \cdots + G_N} \cdot i_T$$

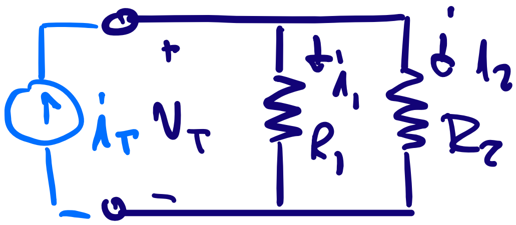
> So resistors in parallel split current according to their fraction of the total conductance. Current "divides" based on fractional conductance.

Expressed in terms of resistances, this gives us:

$$\frac{i_j}{i_T} = \frac{1/R_j}{1/R_1 + \cdots + 1/R_N}$$

so the smaller an individual resistor is, the greater fraction of the current it draws.

Consider current division of a total current i_T among 2 resistors R_1, R_2 in parallel



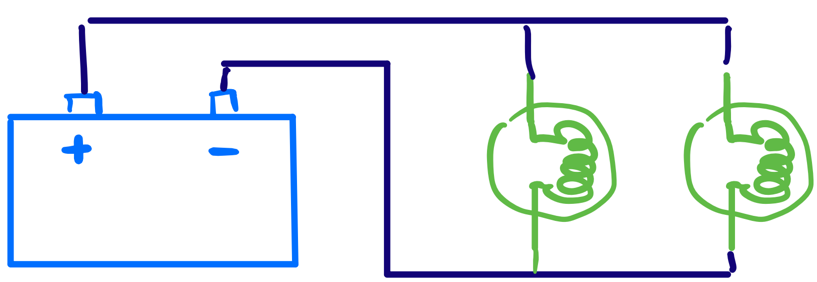
$$i_1 = \frac{1/R_1}{1/R_1 + 1/R_2}\, i_T = \frac{R_2}{R_1 + R_2}\, i_T$$

$$i_2 = \frac{1/R_2}{1/R_1 + 1/R_2}\, i_T = \frac{R_1}{R_1 + R_2}\, i_T$$

So expressed in terms of resistances, the current divider relationship for parallel resistors is "opposite" to that of the voltage divider relationship for series resistors.

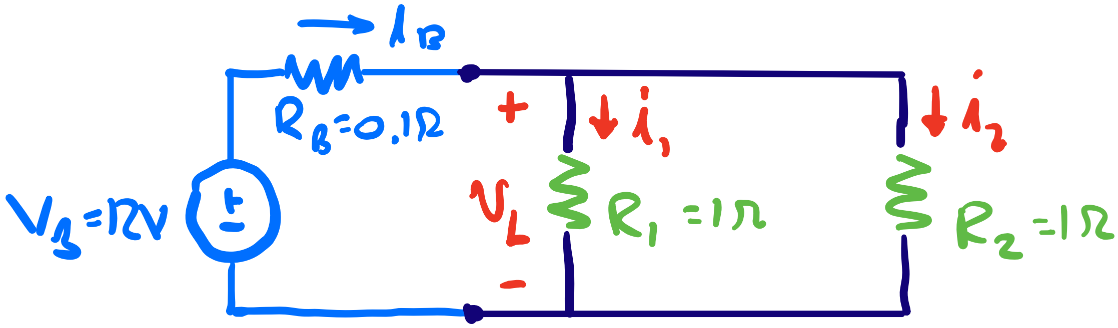
Series/Parallel resistor combinations and dividers appear so frequently and are so useful that they are worth memorizing.

Example: Consider a battery feeding two lights:



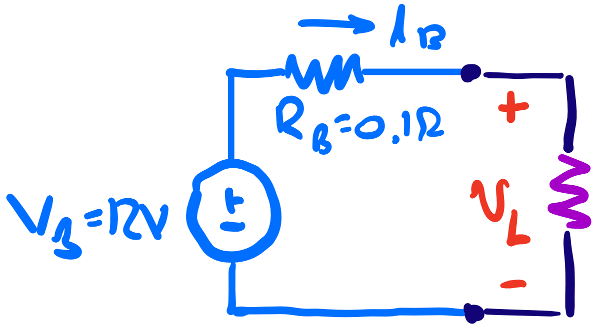
model the battery as having an "open circuit" voltage V_B in series with an "internal resistance" R_b: $V_B = 12V$, $R_b = 0.1\Omega$

model the lights as resistors $R_1, R_2 = 1\Omega$



- Suppose we want to know the voltage at the lights:

We can quickly simplify the circuit (by parallel resistor combination) and then use voltage division to find V_L:



$$R_{eq} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{2}\,\Omega$$

By voltage division from V_B: $V_L = \frac{R_{eq}}{R_{eq}+R_B} \cdot V_B$

$$= \frac{0.5}{0.6} \cdot 12 = 10V$$

- What is the battery current, i_B?

The total series resistance is $R_T = R_B + R_{eq} = 0.6\,\Omega$

$$\therefore i_B = \frac{V_B}{R_T} = \frac{12V}{0.6\Omega} = 20A$$

- How does i_B divide between the two lights?

Current division!

$$i_1 = \frac{R_2}{R_1+R_2} \cdot i_B = \left(\tfrac{1}{2}\right)(20) = 10A$$

$$i_2 = \frac{R_1}{R_1+R_2} \cdot i_B = 10A \quad \text{(current splits equally)}$$

Note: This is expected by symmetry (equal resistances). If R_1 were smaller than R_2, we'd expect R_1 to get a <u>greater</u> fraction of the total current.