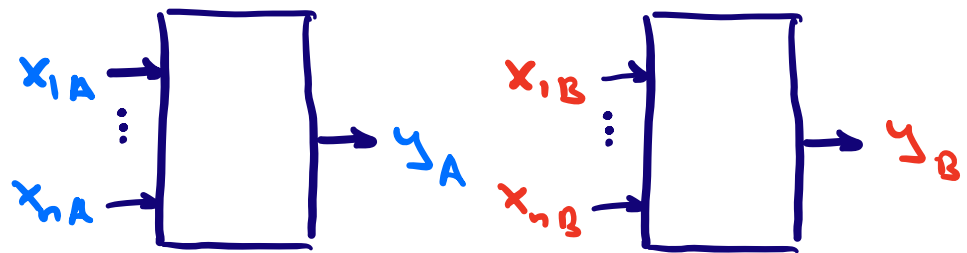
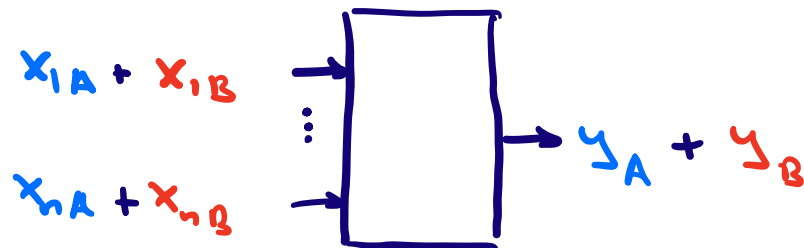


Recall: Linear systems have important properties including Superposition:



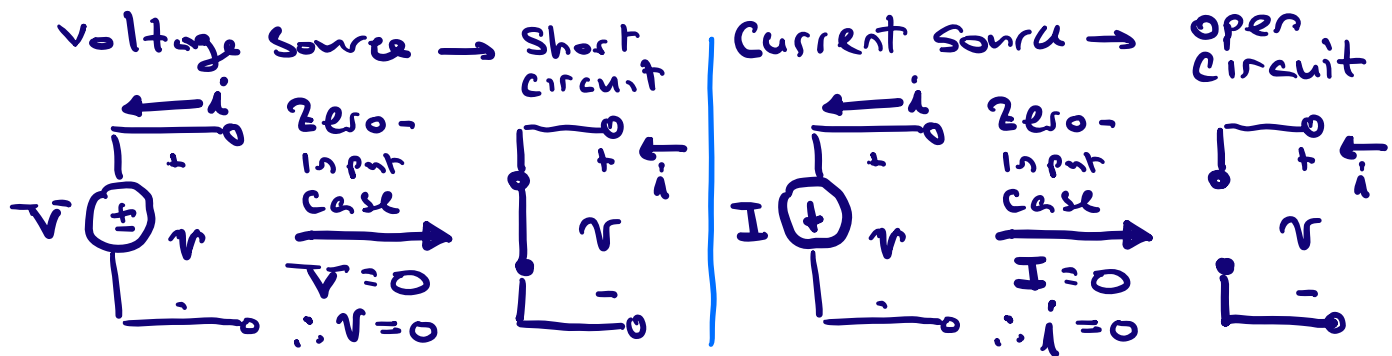
then



A sum of inputs yields a response that is the sum of the responses to the individual inputs

For circuits, our inputs are independent voltage + current sources, and our outputs are the component voltages and currents.

For linear circuits, the total response with all inputs can be found as the sum of the responses to individual inputs with other inputs set to zero (by superposition)

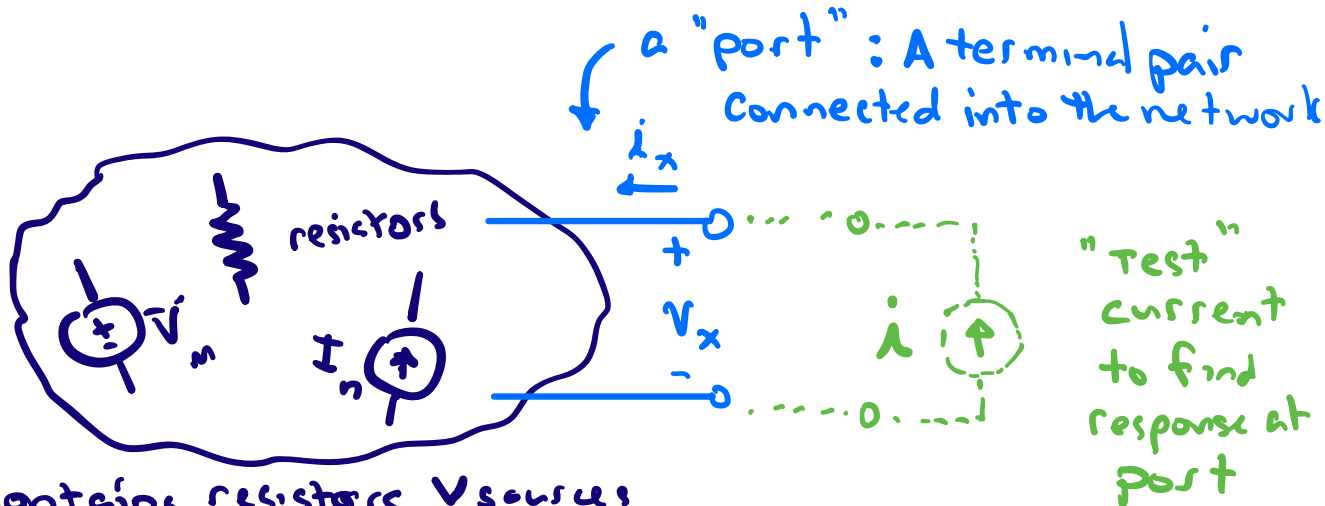


Superposition enables a powerful modeling technique for linear circuits: The Thévenin equivalent circuit

Circuits

Thévenin (2)

Consider an arbitrary "linear resistor plus source" network



Containing resistors, V sources and I sources

By superposition

$$V_x = \left\{ \sum_m \alpha_m \cdot V_m + \sum_n \beta_n I_n \right\} + R_{Th} \cdot i$$

$\uparrow$ units of voltage $\uparrow$ units of resistance $\uparrow$ units of resistance

Response at port with $i = 0$
($\therefore i_x = 0$).

This is the "open circuit" voltage V_{Th}
found when $i_x = 0$.

(The voltage difference between any terminal pair is a difference between node voltages $= k \cdot e = k \cdot G^{-1} S$.)
(V_x response from internal sources only)

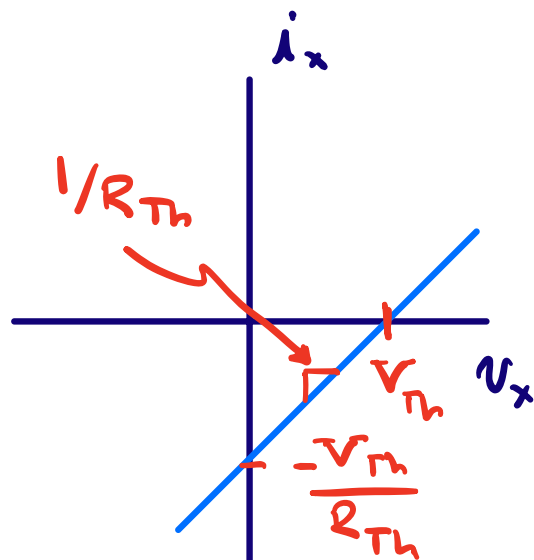
R_{Th} : equivalent resistance looking into terminal pair with all independent sources inside network "killed"

$\therefore$ all $V_{m_i} = 0$ "shorted"
all $I_{n_i} = 0$ "opened"
(V_x from external i only)

By superposition, the response at the terminal pair must have the form:

$$V_x = V_{Th} + R_{Th} \cdot i_x$$

So what is the i - v characteristic of our arbitrary linear resistor + source network?



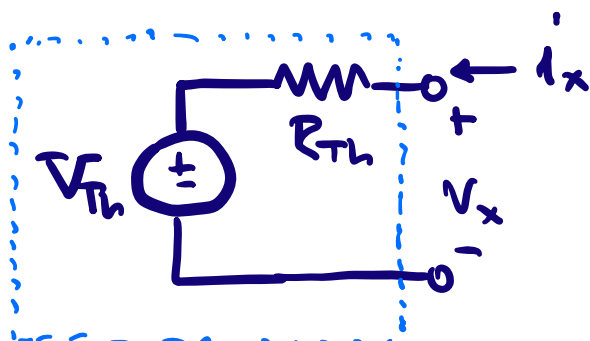
Open-circuit voltage ($i_x = 0$)

$$v_x = \sum_m \alpha_m V_m + \sum_n \beta_n I_n = V_{TH}$$

Short-circuit current ($v_x = 0$)

$$i_x = -\frac{V_{TH}}{R_{TH}}$$

The terminal characteristics i_x - v_x of our arbitrary network are the same as:



where

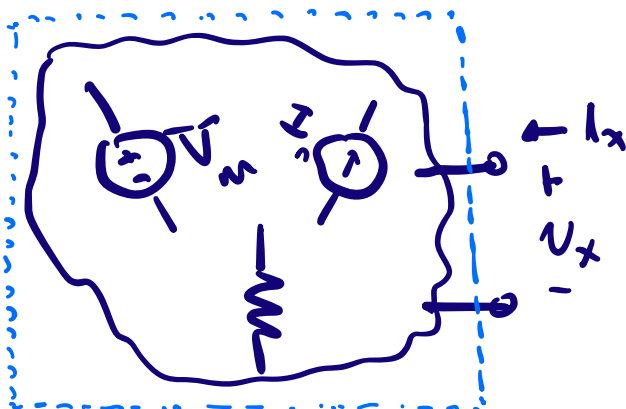
1. V_{TH} is the voltage at v_x with nothing connected externally ($i_x = 0$)

2. R_{TH} is the resistance looking into the i_x - v_x "port" with all internal independent sources "killed" (set to zero), i.e.

$I_{n'k} = 0$ "opened"

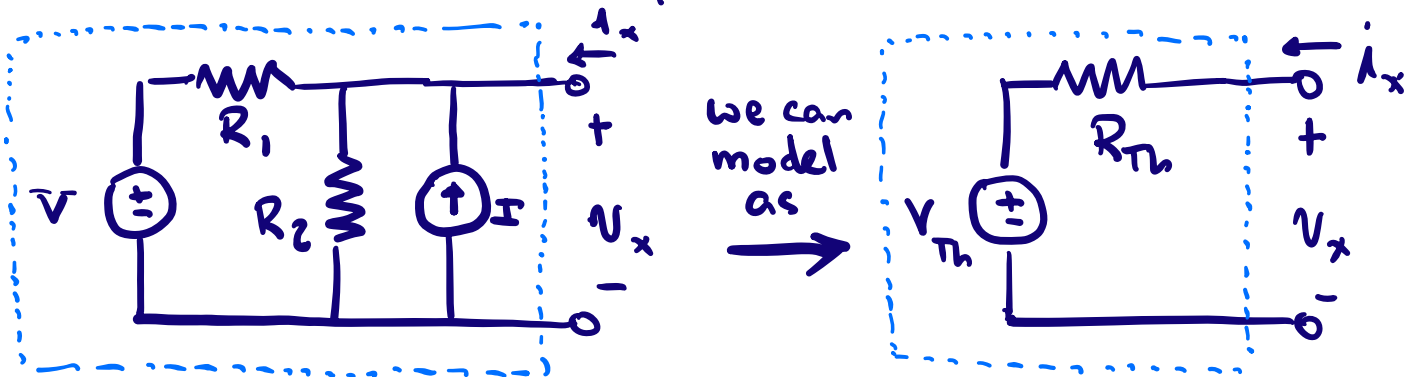
$V_{m'k} = 0$ "shorted"

↑ same behavior as



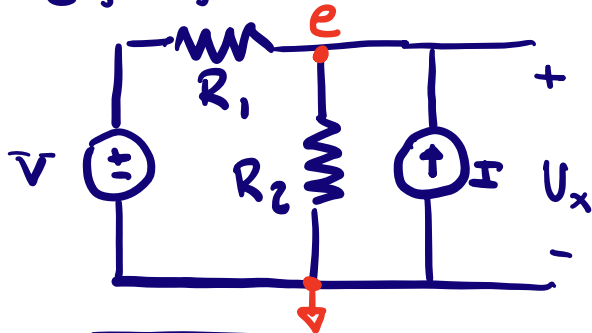
This V_{TH} , R_{TH} model for a circuit is called a "Thévenin equivalent circuit"

Example: Suppose we will make connections to the following circuit and would like a simpler model for its behavior:



- What is the Thévenin voltage V_{Th} ? It is the open-circuit voltage V_x with $i_x = 0$.

e.g. by node method



$$(V - e) \frac{1}{R_1} - e \frac{1}{R_2} + I = 0$$

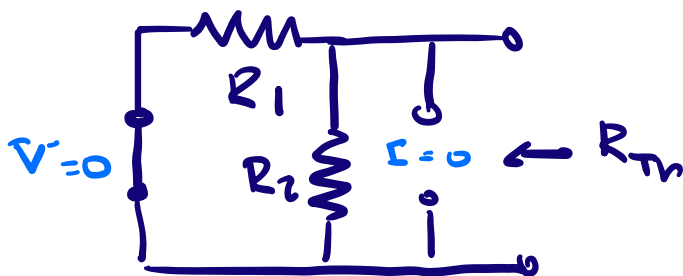
$$e \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{1}{R_1} V + I$$

$$e [R_1 + R_2] = R_2 V + R_1 R_2 I$$

$$e = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

$$\therefore \boxed{V_{Th} = \frac{R_1 R_2}{R_1 + R_2} I + \frac{R_2}{R_1 + R_2} V} \quad (\text{Thévenin equiv. voltage})$$

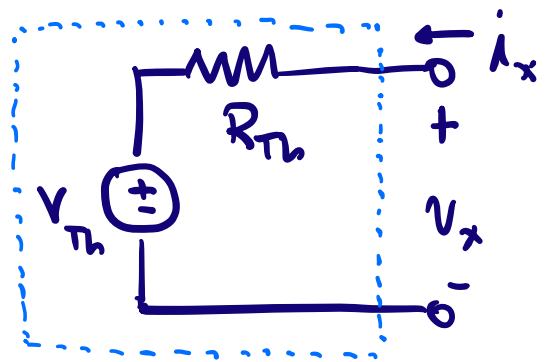
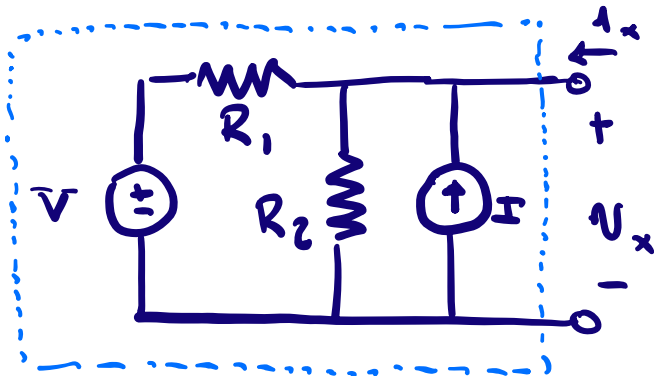
- The Thévenin equivalent resistance R_{Th} of our model is the input resistance with sources "killed"



$$\boxed{R_{Th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}}$$

(Thévenin equiv. current)

So we can model as

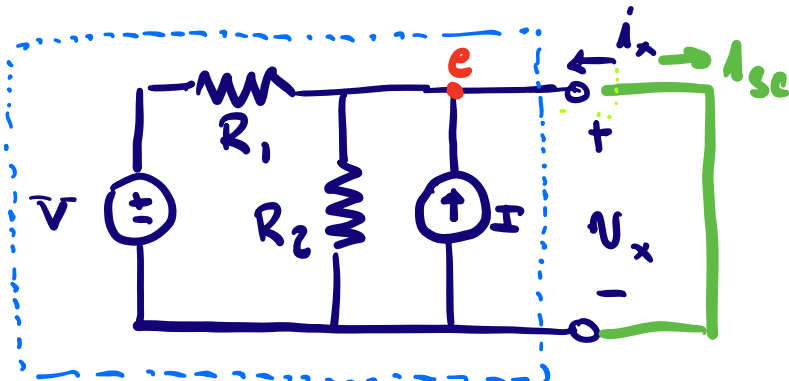


where $R_{Th} = \frac{R_1 R_2}{R_1 + R_2}$

$$V_{Th} = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

Both networks have the same $V_x \cdot i_x$ terminal relationship: electrically they do the exact same thing at the terminals.

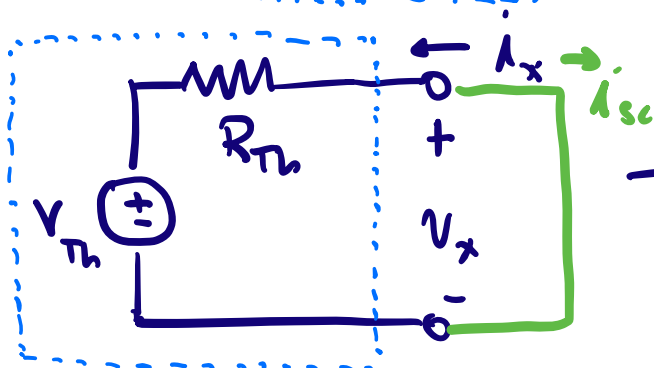
Let's check this: Suppose we impose $V_x = 0$ (e.g. by putting a short circuit at the terminals) and look at the resulting current: ($i_{sc} = -i_x$)



KCL @ node e, when $e = V_x = 0$

$$\frac{V}{R_1} - \frac{0}{R_2} + I - i_{sc} = 0$$

$$i_{sc} = \frac{V}{R_1} + I$$



$$i_{sc} = \frac{V_{Th}}{R_{Th}}$$

$$= \frac{\frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I}{\left(\frac{R_1 R_2}{R_1 + R_2} \right)} = \frac{V}{R_1} + I \quad \checkmark$$

We get the same val. of i_{sc} !

Circuits

Thévenin

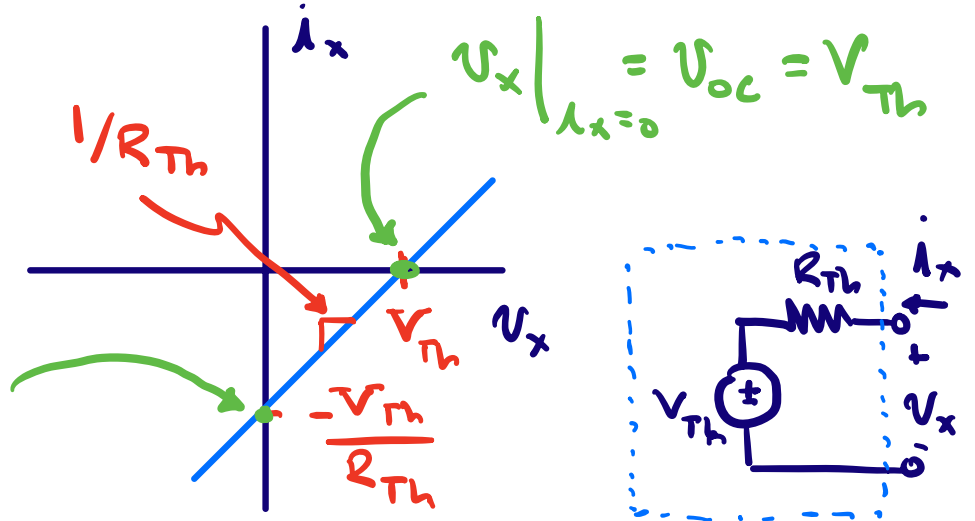
⑥

Another check: Lets set $i_x = 0$ (open circuit):

@ $i_x = 0$, open circuit $V_x = V_{oc} = V_{Th}$ in both circuits

The general characteristic is:

$$i_x \big|_{V_x=0} = -i_{sc}$$



$$R_{Th} = \frac{\Delta V}{\Delta i} = \frac{V_x \big|_{i_x=0}}{-i_x \big|_{V_x=0}} = \frac{V_{oc}}{i_{sc}} \quad \begin{matrix} \leftarrow \text{open ckt voltage} \\ \leftarrow \text{short ckt current} \end{matrix}$$

So in principle we could put an open circuit at the terminals and measure the voltage ($V_{oc} = V_{Th}$), put a short circuit & measure the current (i_{sc}), and then get R_{Th} as their ratio, finding the full Thévenin model.

⇒ Be careful, however! many circuits may be damaged if one does this in practice!

Why is Thévenin so powerful? When working with linear circuits having many (e.g. 100's) of components, we are able to focus on a small portion of the circuit and replace everything else with a simple equivalent circuit! This helps us break down a large circuit into smaller blocks that are each tractable, and focus only on one part of a circuit at a time.

