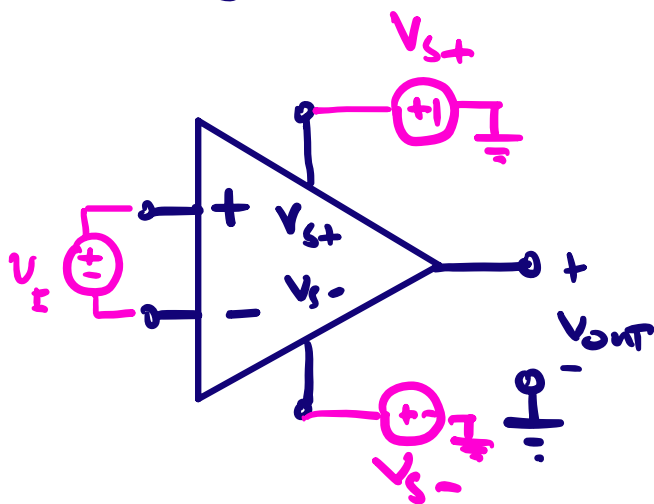


Circuits Op Amp Comparator, Oscillator, dynamics ①

Reminder: The Op Amp

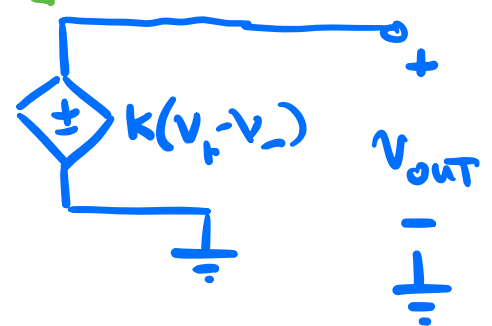
- In its linear range, the op amp ideally looks like a VCVS dependent source with large gain k .
- However, the output voltage can only swing over a range limited by its power supply values
 $V_{s-} < V_{out} < V_{s+}$

The symbol for an op-amp:



V_+
 V_-

★ output voltage will "clip", with limits between (V_{s-}, V_{s+})



Linear range model

- A practical op amp has an "open-loop gain" k that is very large: $10^5 - 10^6$ in practice. Ideally we treat as $k \rightarrow \infty$
- In practice, the output voltage V_{out} is constrained to remain within the range of the power supply, i.e. between (V_{s-}, V_{s+}) or an even narrower range, e.g. $(V_{s-} - \Delta V, V_{s+} + \Delta V)$

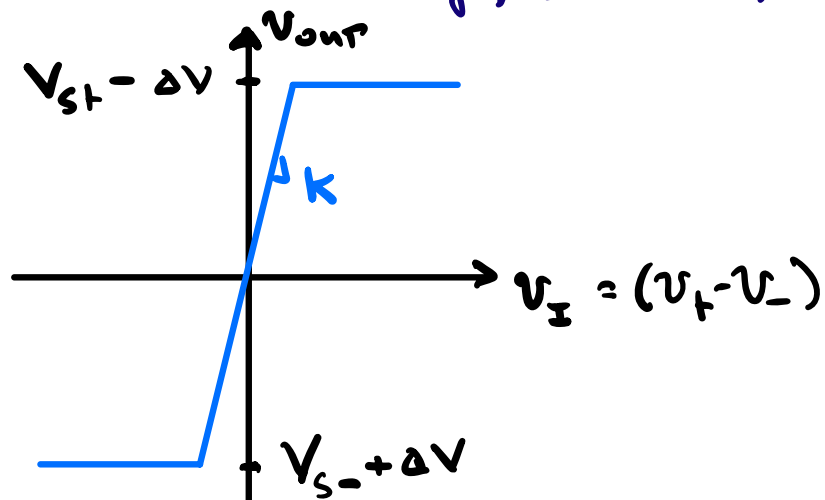
e.g., if

$$V_{s+} = +15V$$

$$V_{s-} = -15V$$

$$\Delta V = 3V$$

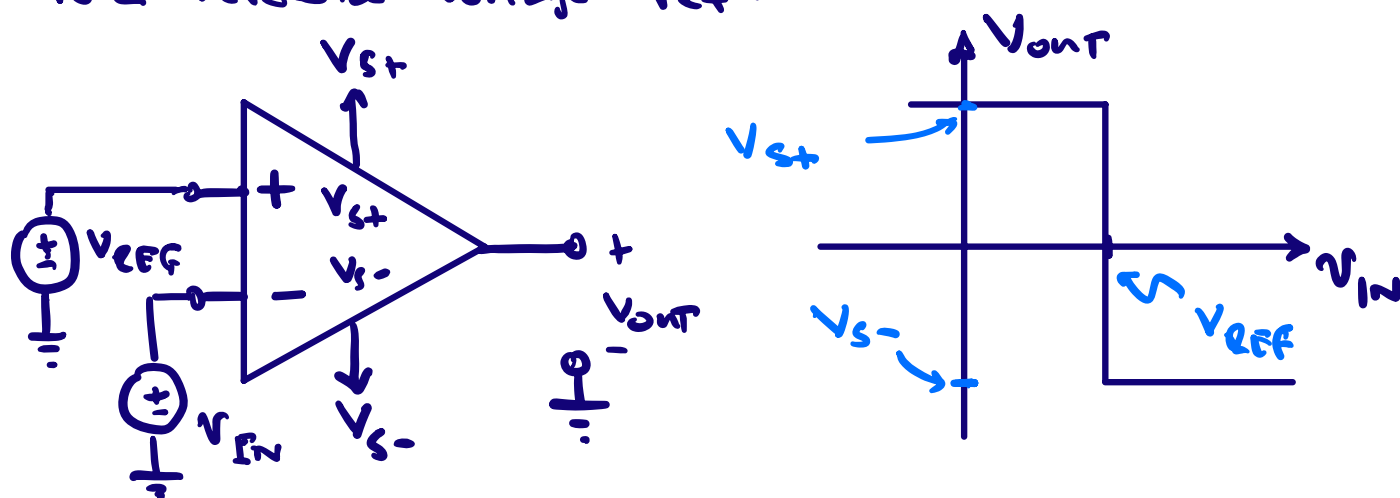
V_{out} in $(-12, 12)V$ range



Circuits Op Amp Comparator, Oscillator, dynamics ②

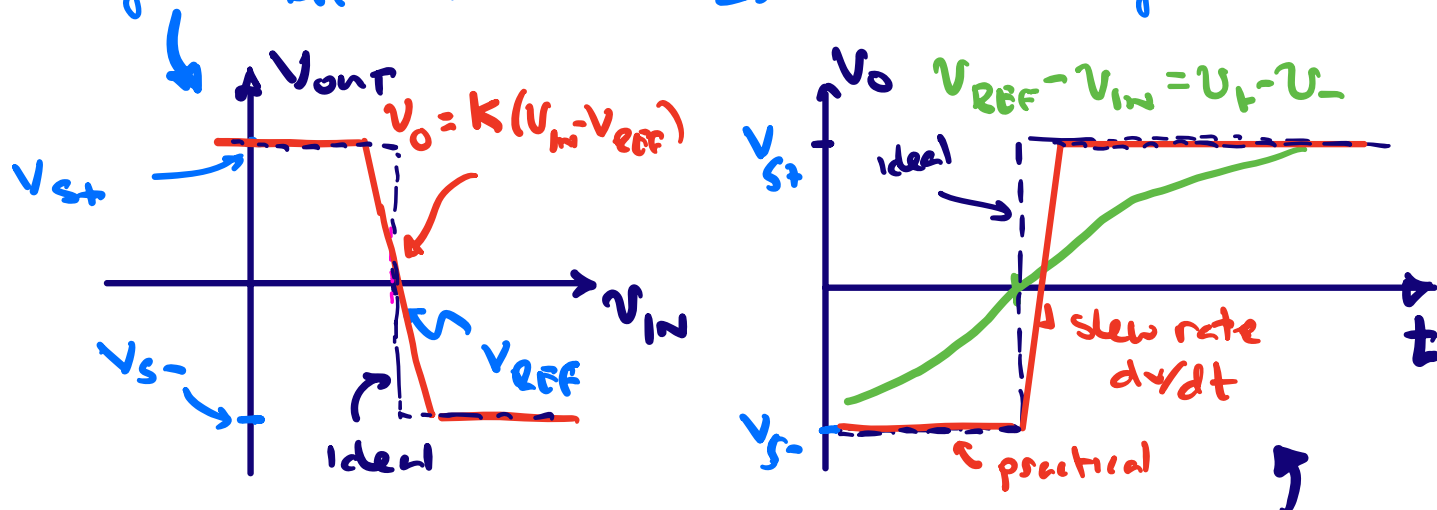
One way we can use an op-amp is as a comparator. (there are special versions designed specifically for use as comparators, though we can do this function with most op amps).

Suppose we want to compare some "input" voltage V_{in} to a "reference" voltage V_{ref} :



Our comparator can tell us whether or not our input signal is greater than or less than the reference voltage.

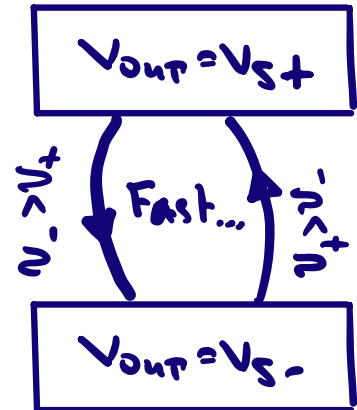
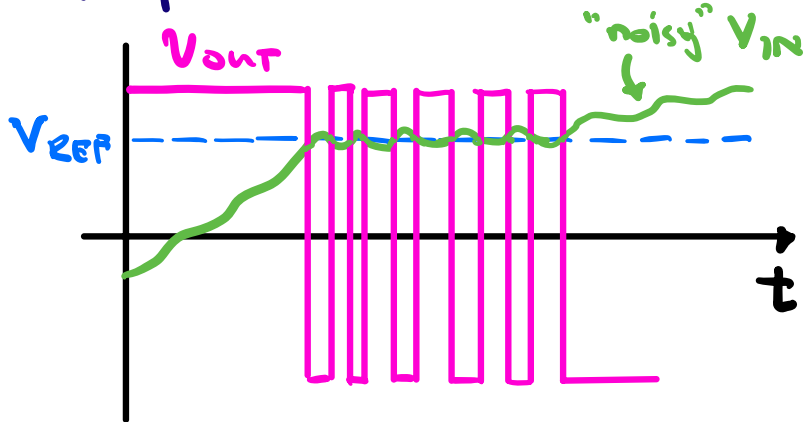
In reality the output transitions over a small but nonzero range of $V_{REF} - V_{IN}$ (but k is big so transition range is small)



In reality, the speed with which the output can switch is finite, with a limiting "slew rate" in output voltage that depends on the difference between V_+ , V_- up to a practical maximum that depends on the device,

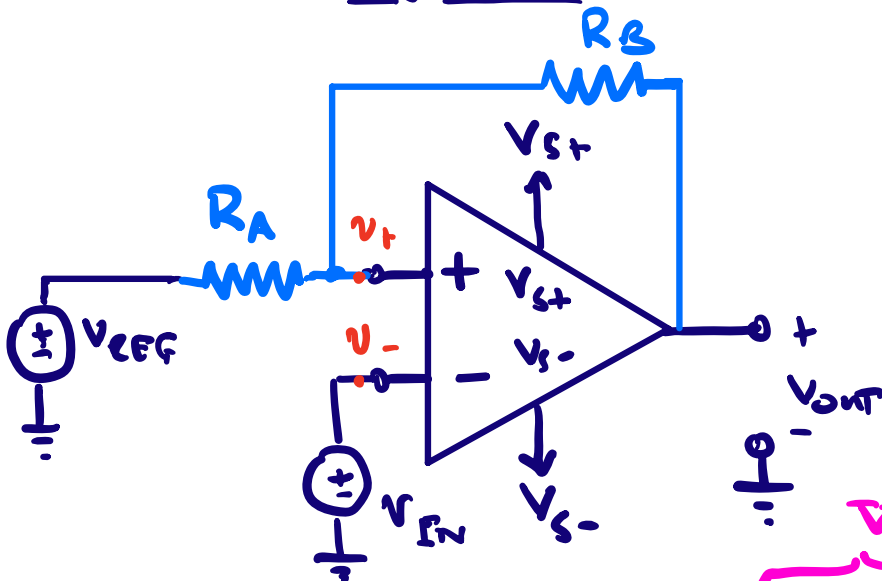
Circuits Op Amp Comparator, Oscillator, dynamics ③

A problem with comparator circuits is that the output can "chatter" back and forth for input voltages near the reference voltage (such that V_+ is very close to V_-), especially in the presence of noise.



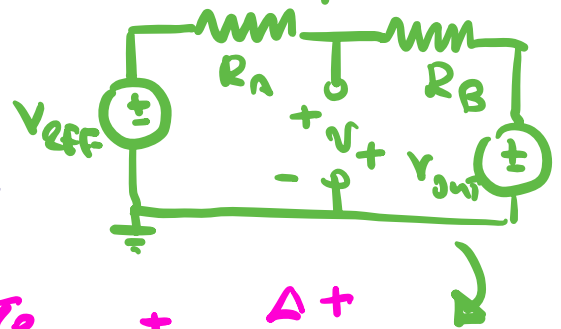
Such chatter can cause problems for circuits that try to use V_{out} .

We can fix the chattering problem with "positive feedback" that causes hysteresis in the switching.



See demo

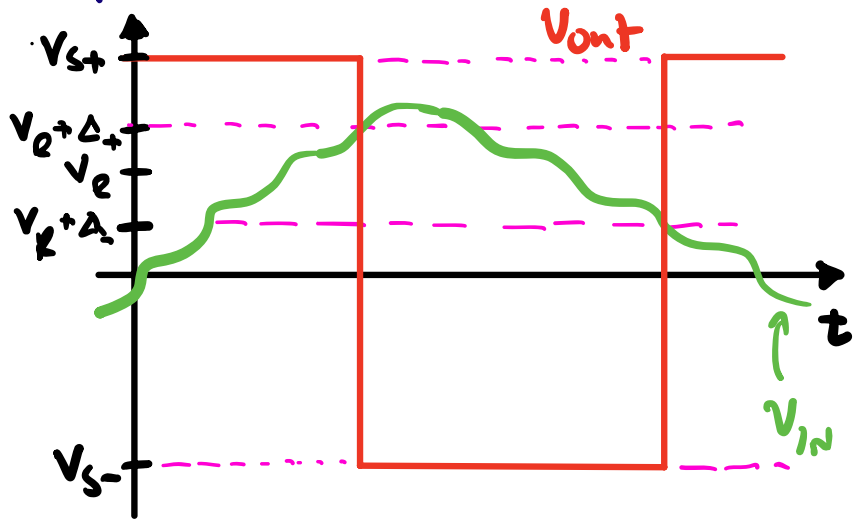
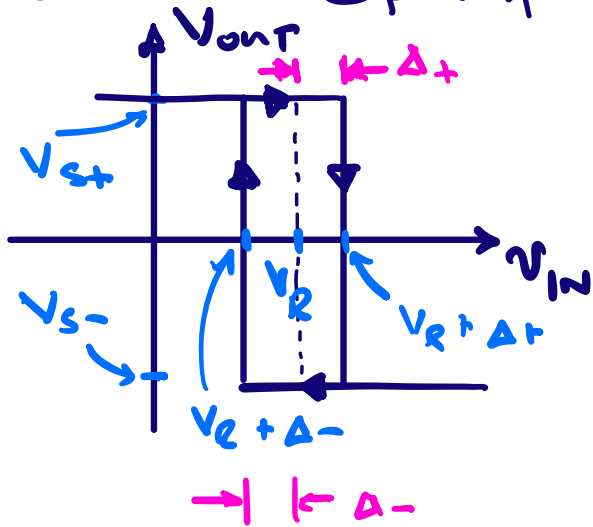
Superposition to find V_+ :



$$\text{If } V_{out} = V_{S+} \iff V_{IN} < \underbrace{V_{REF} \cdot \frac{R_B}{R_A + R_B}}_{V_2} + \underbrace{V_{S+} \cdot \frac{R_A}{R_A + R_B}}_{\Delta+}$$

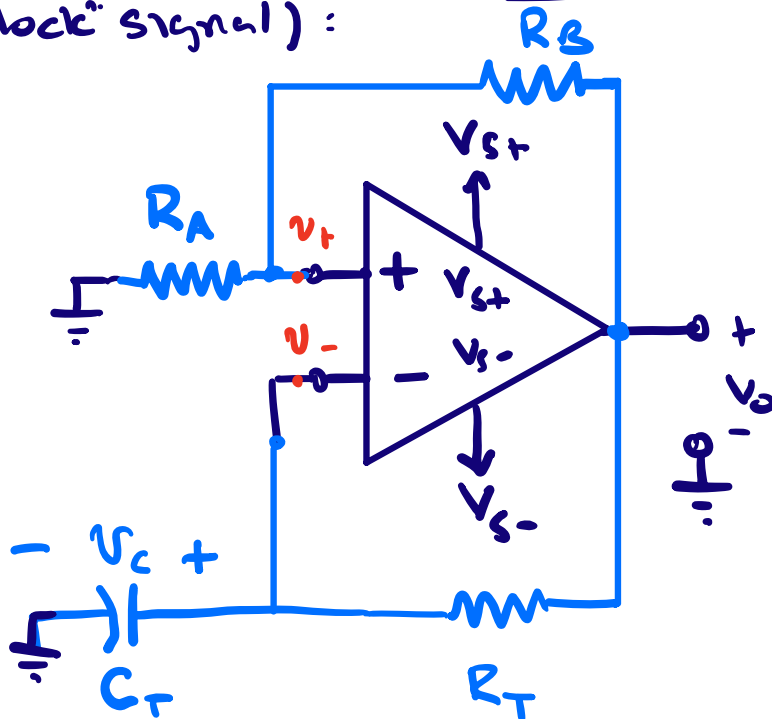
$$\text{If } V_{out} = V_{S-} \iff V_{IN} > \underbrace{V_{REF} \cdot \frac{R_B}{R_A + R_B}}_{V_2} + \underbrace{V_{S-} \cdot \frac{R_A}{R_A + R_B}}_{\Delta-}$$

Circuits Op Amp Comparator, Oscillator, dynamics (4)



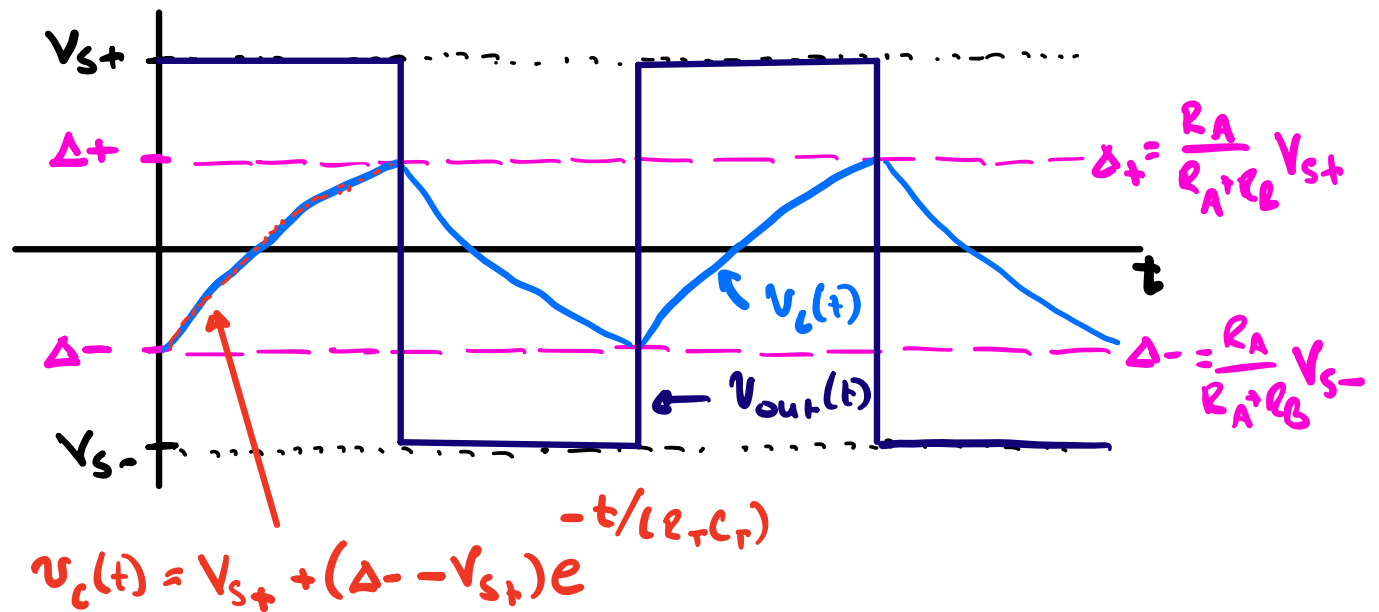
The transition threshold is different for positive - and negative-going transitions, so we do not get chattering at the boundary! (Positive feedback provides hysteresis.)

We can also use hysteresis (with both +ve and -ve feedback) to build an oscillator. (e.g. to generate a "clock" signal):



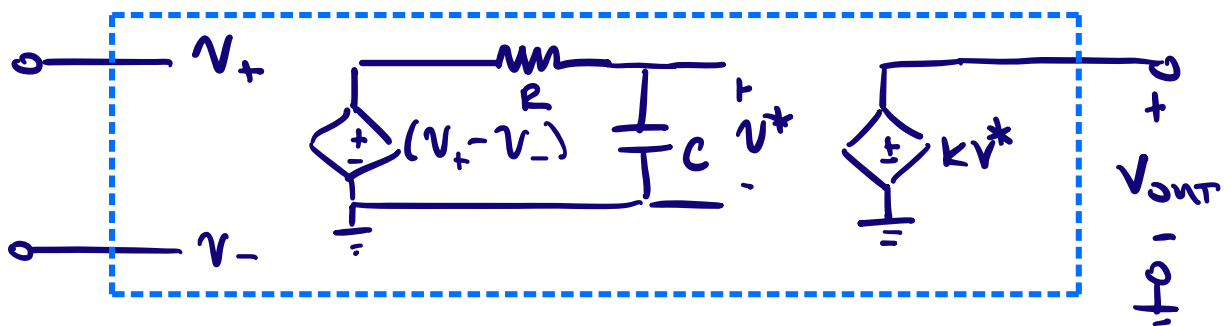
Consider V_O, V_C as a function of time

Circuits Op Amp Comparator, Oscillator, dynamics ⑤



The oscillation period is related to the output saturation voltages (here V_{s+}, V_{s-}), hysteresis band size $\Delta+ - \Delta-$ (determined by R_A, R_B) and the time constant $R_T C_T$

In practice, op amps have internal dynamics (yielding response speed limits). A slightly more sophisticated model for a real op amp might look like



e.g., the LM741 op amp has $K \approx 200,000$; $\tau = RC \approx 0.02$ Sec (The open-loop bandwidth is low, but closed-loop bandwidth can be high)

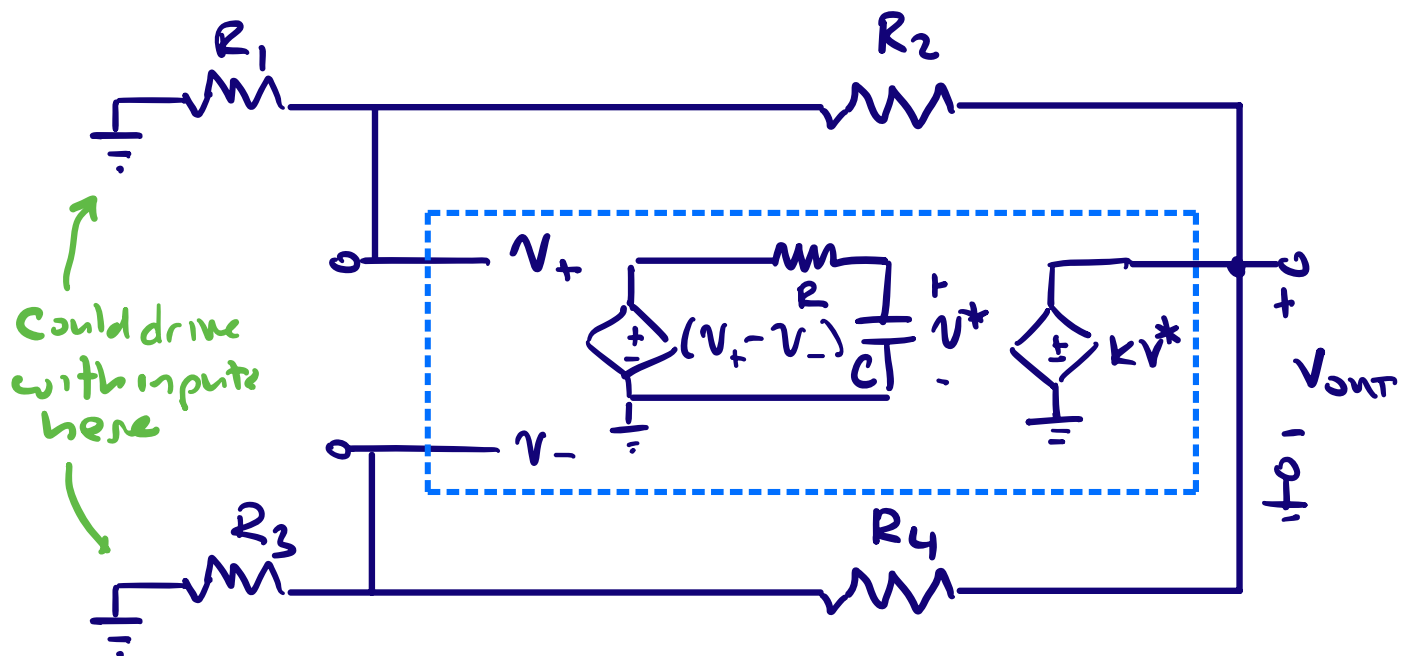
Considering internal dynamics, we can speak more quantitatively about how op amps respond differently to positive and negative feedback

Circuits Op Amp Comparator, Oscillator, dynamics ⑥

Consider how such a system responds to positive and/or negative feedback (i.e. to positive + negative inputs)

We will see that positive feedback tends to drive the system unstable, while negative feedback makes it more stable.

If we have feedback to both + and - inputs, whether the system is stable (and will attain equilibrium) depends on whether positive or negative feedback predominates



$$V_{OUT} = KV^* \quad \therefore \quad V^* = \frac{V_{OUT}}{K}$$

positive feedback $V_+ = \frac{R_1}{R_1 + R_2} \cdot V_{OUT} = \gamma^+ \cdot V_{OUT}$

negative feedback $V_- = \frac{R_3}{R_3 + R_4} \cdot V_{OUT} = \gamma^- \cdot V_{OUT}$

Circuits Op Amp Comparator, Oscillator, dynamics (7)

Consider the dynamics of this system:

$$C \frac{dV^*}{dt} = \frac{1}{R} [V_+ - V_- - V^*]$$

$$RC \frac{dV^*}{dt} + V^* + V_- - V_+ = 0$$

$$\frac{1}{k} RC \frac{dV_{out}}{dt} + \frac{1}{k} \cdot V_{out} + \gamma_- V_{out} - \gamma_+ V_{out} = 0$$

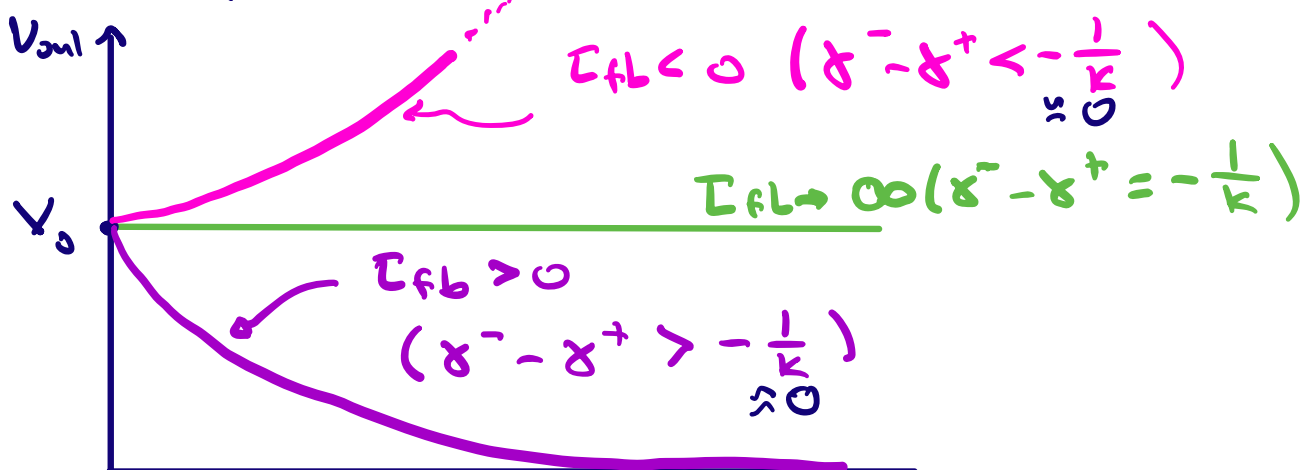
$$\text{or } \frac{dV_{out}}{dt} + \underbrace{\frac{1}{RC} [1 + k(\gamma_- - \gamma_+)]}_{1/\tau_{fb}} V_{out} = 0$$

τ_{fb} is system time constant with feedback $\rightarrow 1/\tau_{fb}$

This is a first-order linear constant coefficient differential eqn. with a time constant

$$\tau_{fb} = \frac{RC}{1 + k(\gamma_- - \gamma_+)}$$

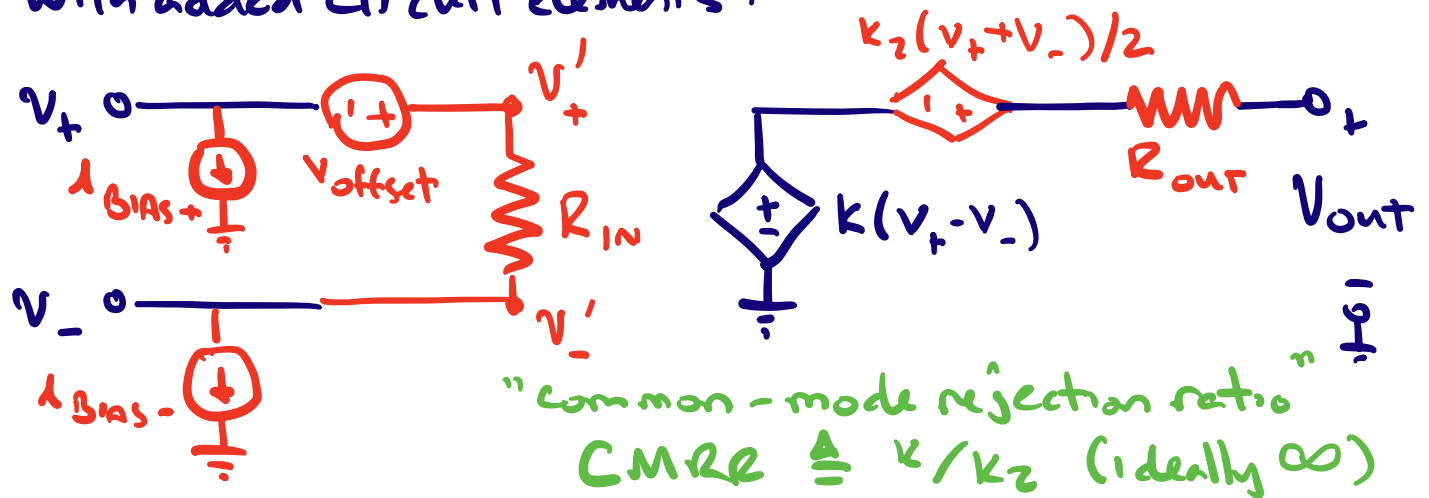
Natural response has form $V_{out} = V_0 e^{-t/\tau_{fb}}$



- So if feedback is net negative ($\gamma_- > \gamma_+$) we are stable but if feedback is net positive ($\gamma_- < \gamma_+$) we will become unstable

Circuits Op Amp Comparator, Oscillator, dynamics ⑧

There are also other nonidealities that can be captured with added circuit elements:



Typical Specs	Ideal	LM741	LF356
k	∞	$>10^5$	$>10^5$
CMRR	∞	>3200	$>10^4$
R_{OUT}	①	$<75\Omega$	0.1-40 Ω (frequency dependent)
R_{IN}	∞	$>2M\Omega$	$>10^{12}\Omega$
V_{offset}	①	$<10mV$	$<10mV$
$I_{bias} +/-$	①	$<500nA$	$<100pA$

Achievable bandwidth also varies among op amps.