6.200 Midterm

Fall 2026

Name: **Answers**

Kerberos/Athena Username: _____@mit.edu

5 questions 1 hour, 50 minutes

- Please **WAIT** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5" × 11" sheet of handwritten notes (both sides) as a reference. This sheet must be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices (no computers, calculators, phones, etc.) except for a multimeter.
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.
- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam. **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until overall exam grades have been released.

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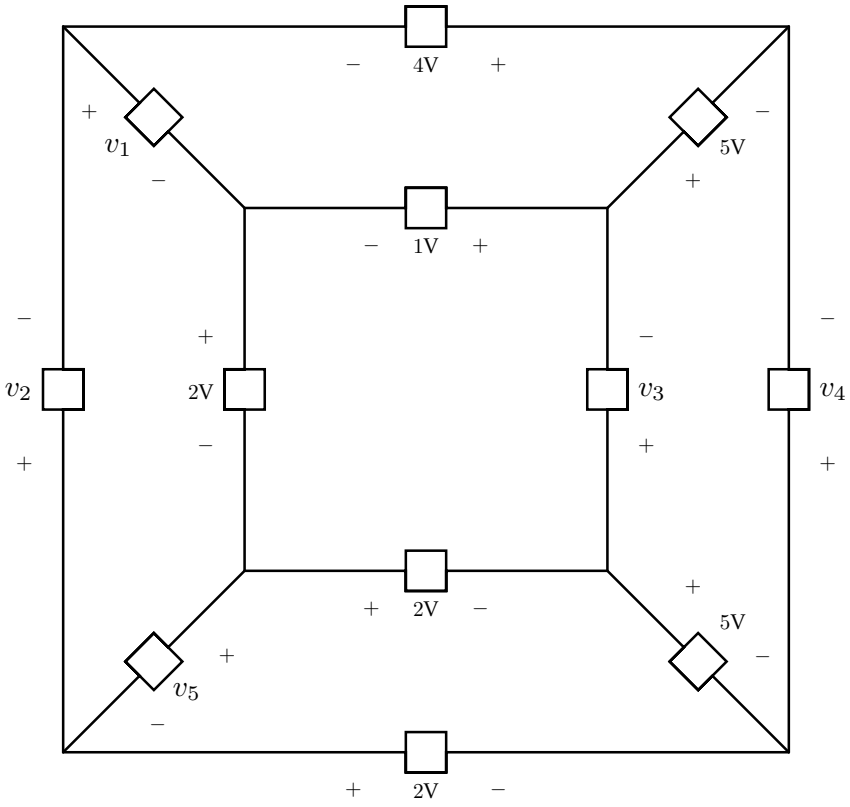
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1 Short Circuits

1.1 Part 1

Solve for the unknown voltages v_1, v_2, v_3, v_4, and v_5 in the circuit below, where each box represents an arbitrary component:



KVL Equations (can be solved one at a time, no need for much algebra):

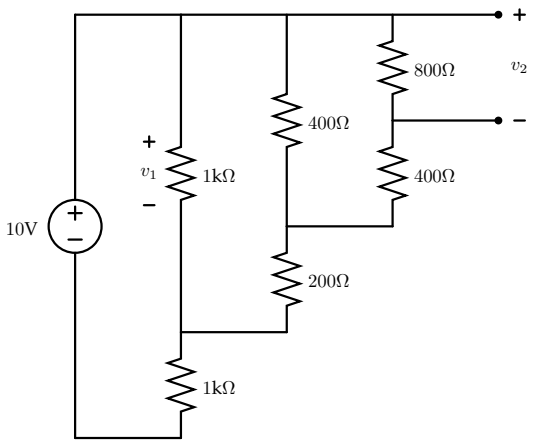
$$v_1 + 4\text{V} + 5\text{V} = 1\text{V}$$
$$v_1 + v_2 + v_5 + 2\text{V} = 0$$
$$v_3 + 2\text{V} + 2\text{V} + 1\text{V} = 0$$
$$v_4 + 5\text{V} = v_3 + 5\text{V}$$
$$v_5 + 2\text{V} = 2\text{V} + 5\text{V}$$

$v_1 =$ -8V $v_2 =$ 1V $v_3 =$ -5V

$v_4 =$ -5V $v_5 =$ 5V

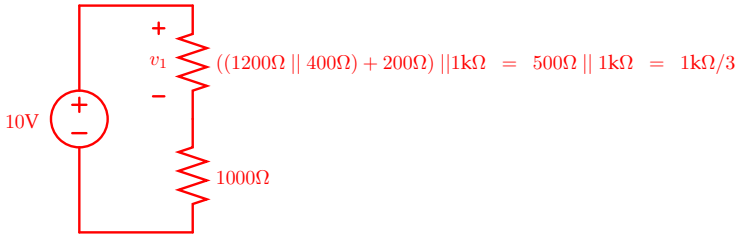
1.2 Part 2

Solve for the voltages v_1 and v_2 in the circuit below, using any method you prefer:

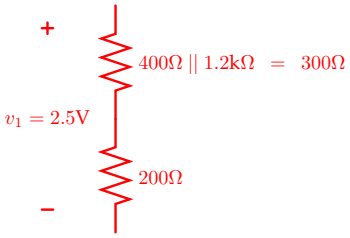


$v_1 =$ 2.5V $v_2 =$ 1V

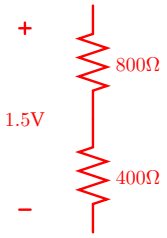
Squish everything down to a single voltage divider to find v_1:



The top resistor accounts for $1/4$ of the total resistance of the divider, so $1/4$ of the voltage drops there ($v_1 = 2.5\text{V}$). That same 2.5V drops across everything to the right of that point. Unsquishing one layer, we have:



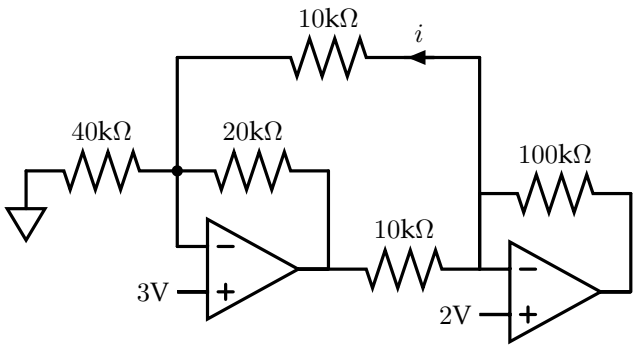
The drop across the top resistor is $\frac{3}{5} \times 2.5\text{V} = 1.5\text{V}$. This same 1.5V drops across the series combination of 800Ω and 400Ω:



v_2 is the drop across the 800Ω resistor, so $v_2 = \frac{2}{3} \times 1.5\text{V} = 1\text{V}$.

1.3 Part 3

Solve for the current i in the circuit below. Make the ideal op-amp assumption and ignore limitations on the op-amps' outputs imposed by their power supplies. Assume that all indicated voltages are measured with respect to the same reference node.



$i =$ -0.1mA

Making the ideal op-amp assumption, we can know the voltages at the $-$ terminals of each op-amp: the left op-amp's $-$ terminal is at 3V relative to our reference, and the right op-amp's $-$ terminal is at 2V relative to our reference.

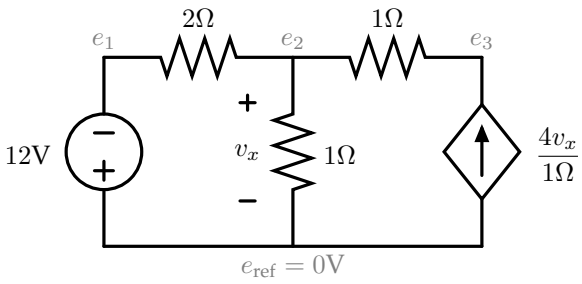
Those two nodes are the same nodes that are connected to the $10\text{k}\Omega$ resistor we're interested in, so we can simply use Ohm's Law:

$$i = \frac{2\text{V} - 3\text{V}}{10\text{k}\Omega} = -0.1\text{mA}$$

2 Dependent Sources

2.1 Solving Circuits

Solve for each of the indicated node potentials in the circuit below.



$e_1 = -12\text{V}$	$e_2 = \frac{12}{5}\text{V}$	$e_3 = 12\text{V}$

From the voltage source's constitutive equation, we have $e_1 = -12\text{V}$.

We can then write KCL at node e_2 (also noting that $e_2 = v_x$):

$$\frac{-12\text{V} - v_x}{2\Omega} + \frac{4v_x}{1\Omega} = \frac{v_x}{1\Omega}$$

$$-12\text{V} + 7v_x = 2v_x$$

$$-12\text{V} = -5v_x$$

$$\frac{12\text{V}}{5} = v_x$$

Then KCL at e_3 says:

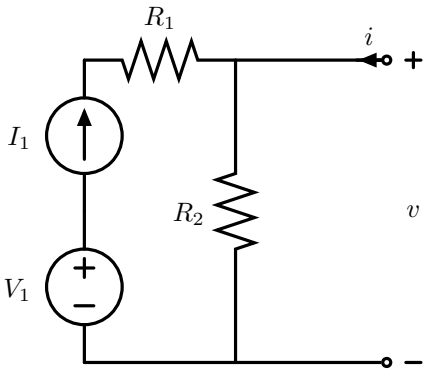
$$e_3 = v_x + \left(\frac{4v_x}{1\Omega}\right) 1\Omega$$

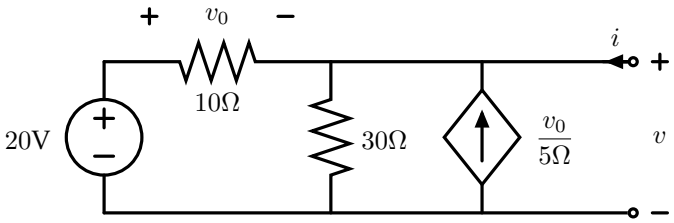
$$e_3 = 5v_x$$

Plugging in, we find $e_3 = 12\text{V}$

2.2 Equivalent Circuits

Find values of V_1, I_1, R_1, and R_2 that make the following two circuits equivalent (in terms of the i/v relationship at the indicated port). If any value works for a given variable, write ANY in the box. If multiple specific values work, you only need to provide one.





$V_1 =$ ANY

$I_1 =$ 6A

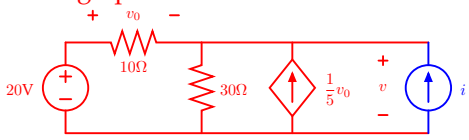
$R_1 =$ ANY

$R_2 =$ 3Ω

In order for these to be equivalent, it suffices to show that they have the same Thévenin equivalent.

Let's start by finding a Thévenin equivalent for the left. Using your favorite method of choice, we should find that $R_{th} = R_2$ and $V_{th} = I_1 R_2$ (note that this is enough to know that V_1 and R_1 are irrelevant).

For finding the Thévenin equivalent of the circuit on the right, we can hold i constant and solve for v in terms of i (from which we can pick out the Thévenin voltage and Thévenin resistance). One way of holding i constant is to pretend that we're hooking up a current source between the terminals:



Then we solve. KCL at the top node (and subbing $20\text{V} - v = v_0$):

$$\frac{20\text{V} - v}{10\Omega} + \frac{1}{5}(20\text{V} - v) + i = \frac{v}{30\Omega}$$

$$2\text{A} - \frac{v}{10\Omega} + 4\text{A} - \frac{v}{5\Omega} + i = \frac{v}{30\Omega}$$

$$6\text{A} + i = \frac{10v}{30\Omega}$$

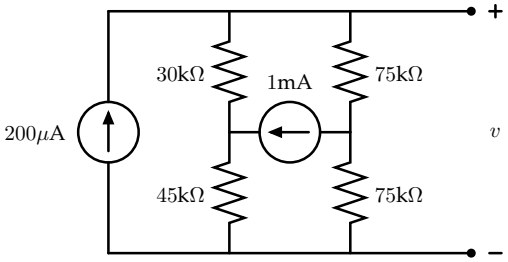
$$18\text{V} + 3\Omega \times i = v$$

From this, we can see that $V_{th} = 18\text{V}$ and $R_{th} = 3\Omega$. Combining with the results from the left-hand circuit, we arrive at our final result, shown in the boxes above.

3 Sources

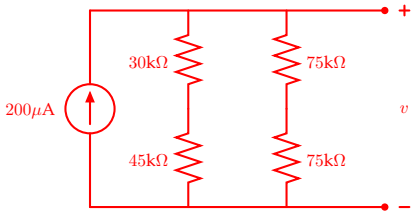
3.1 Currents

Solve for v in the circuit below using any method you prefer.



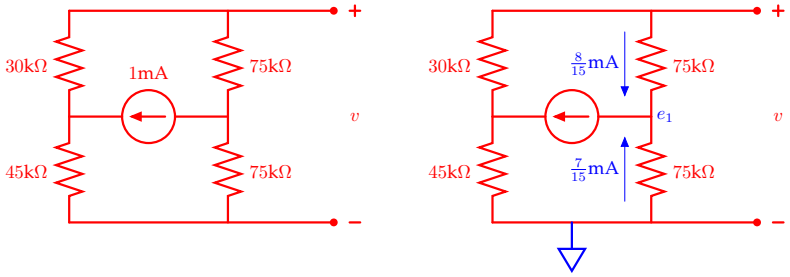
$v = 15\text{V}$

One approach is to use superposition. We can solve two circuits and then add up the results.



For this first circuit, we can squish everything down into a single resistor, and then v is that resistance times 200μA: $v = 200\mu\text{A} \times (75\text{k}\Omega \,||\, 150\text{k}\Omega) = 200\mu\text{A} \times 50\text{k}\Omega = 10\text{V}$

The second circuit is a current divider. We'll have $\frac{120}{225}\text{mA} = \frac{8}{15}\text{mA}$ of current flowing through the series combination of $30\text{k}\Omega$ and $75\text{k}\Omega$ up top, and $\frac{7}{15}\text{mA}$ of current flowing through the bottom series combination of $45\text{k}\Omega$ and $75\text{k}\Omega$:



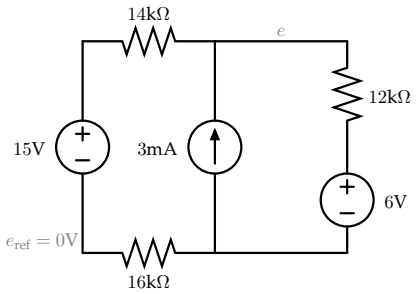
Then we can solve (and adding this to the result from the first circuit gives us our overall answer):

$$e_1 = -\frac{7}{15}\text{mA} \times 75\text{k}\Omega$$

$$\begin{aligned} v &= e_1 + \frac{8}{15}\text{mA} \times 75\text{k}\Omega \\ &= \frac{1}{15}\text{mA} \times 75\text{k}\Omega \\ &= 5\text{V} \end{aligned}$$

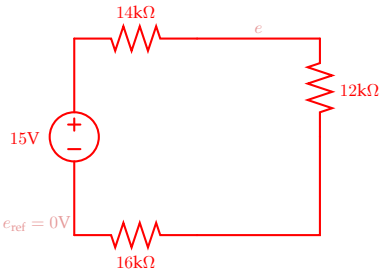
3.2 Multiple Sources

Find the node potential e (measured relative to the indicated reference node) in the circuit below. You do not need to find the other node potentials.



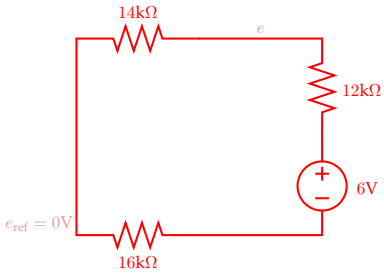
$e = 24\text{V}$

Again, we can use superposition, but now we'll have three circuits. Each one is going to give us a very common pattern so there shouldn't be much algebra here, but we do need to be careful about making sure we're measuring the right stuff.



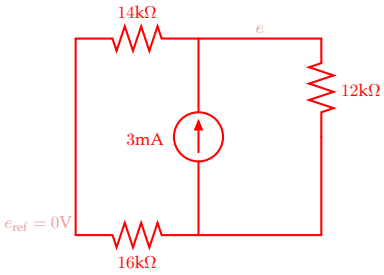
Voltage Divider

$$e = 15\text{V}\left(\frac{28}{42}\right) = 10\text{V}$$



Voltage Divider

$$e = 6\text{V}\left(\frac{14}{42}\right) = 2\text{V}$$



Current Divider + Ohm's Law

$$e = 3\text{mA}\left(\frac{12}{42}\right)(14\text{k}\Omega) = 12\text{V}$$

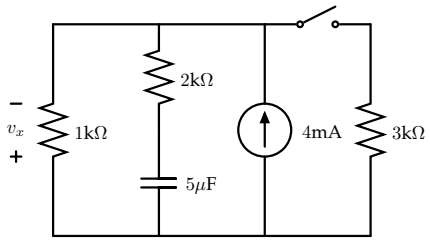
Adding these up gives us our overall answer.

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4 Time After Time

4.1 No Cap

Consider the following circuit, where the switch is closed for $t < 0$ and opened for $t \geq 0$.



Determine the following values.

$v_x(t = 0^-) =$ -3V

$v_x(t = 0^+) =$ $-\frac{11}{3}\text{V}$

$v_x(t = \infty) =$ -4V

At 0^-, the switch has been closed for a long time, so we can assume we have reached DC steady state. We can find $v_x(0^-)$ by treating the capacitor as an open and solving. In this case, the 2kΩ resistor is irrelevant, and the 3kΩ and 1kΩ resistors are in parallel with the 4mA current flowing through that combination, so:

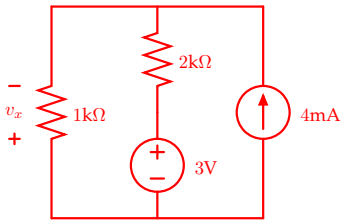
$$v_x(0^-) = -4\text{mA}\,(3\text{k}\Omega \,||\, 1\text{k}\Omega) = -3\text{V}$$

.

For $t = \infty$, we can do the same analysis, but now all of the current is flowing through the 1kΩ resistor, so:

$$v_x(\infty) = -4\text{V}$$

At 0^+, we can model the capacitor as a voltage source, with a value corresponding to the voltage across it at $t = 0^-$:



We can solve this any way we prefer. Nodal analysis (with reference on bottom, e on top) gives:

$$4\text{mA} = \frac{e_1 - 3\text{V}}{2\text{k}\Omega} + \frac{e_1}{1\text{k}\Omega}$$

Solving, we find $e = 11/3\text{V}$, which gives us $v_x = -11/3\text{V}$.

(answers continued on next page).

Give the general expression for $v_x(t > 0)$, with numerical values from the circuit:

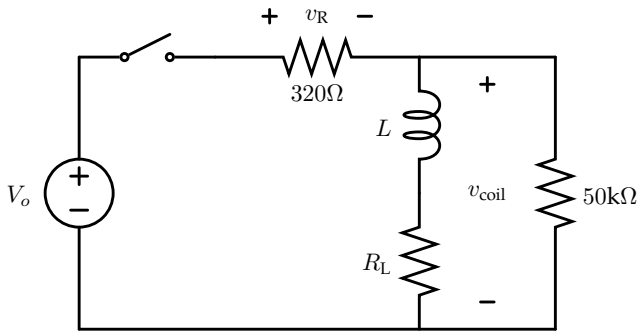
$$v_x(t > 0) = -\frac{11}{3}\text{V} - \frac{1}{3}\text{V}\left(1 - e^{-t/15\text{ms}}\right)$$

We know that every current and voltage in this circuit is going to follow our characteristic exponential curve connecting our initial value to our final value, with time constant $\tau = RC$. We find the relevant R by Thévenizing the circuit around the capacitor and find $R = 3\text{k}\Omega$, so $\tau = (3\text{k}\Omega)\,(5\mu\text{F}) = 15\text{ms}$.

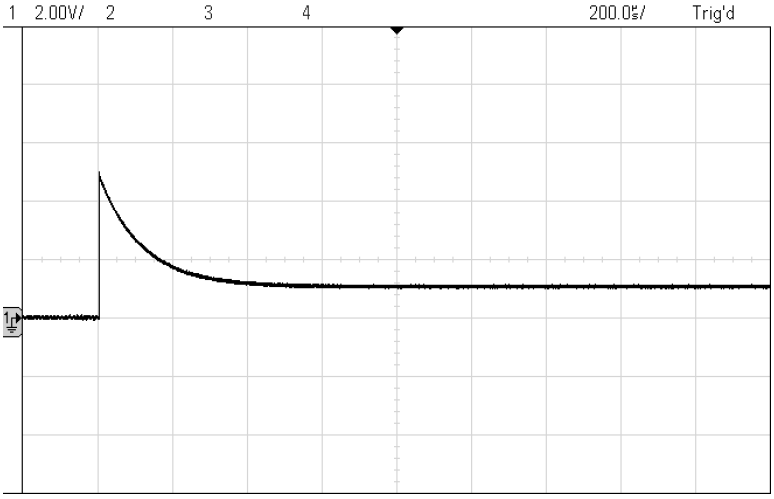
Plugging initial value, final value, and τ into our standard formula, we find the answer above.

4.2 Scope Creep

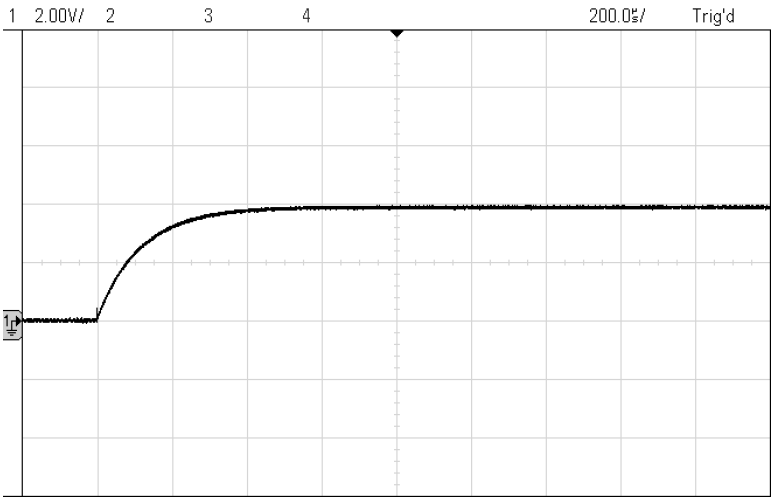
Your friend was working in the lab late one night and they built the following circuit using a carefully wound coil (here modeled as an inductor and a resistor in series), and after a long while with the switch open, they closed the switch.



They saved screenshots of their scope's outputs during this experiment, measuring v_R and v_{coil}. However, the way they organize the files on their computer is a mess, so they forgot which was which. Here are their two screenshots:



`circuit_graph_final_final_real_A.png`



`circuit_graph_final_final_real_B.png`

Which graph is which? Enter "A" or "B" in each box:

Graph [B] is the graph of v_{R}. Graph [A] is the graph of v_{coil}.

The voltage across the resistor is almost exactly proportional to the current through the inductor, which must be a continuous function of time.

Your friend also did some work to characterize the circuit, but oh no, they lost the paper they did their work on. Help them out. Using the graph(s), estimate the values of V_o, L, and R_{L}. Work in the space below the boxes will be considered for partial credit.

$V_o \approx$ [5V] $L \approx$ [48mH] $R_{\text{L}} \approx$ [80Ω]

One piece of information comes from looking at v_{coil} right after throwing the switch (let's call that $t = 0$). Immediately upon throwing the switch, the inductor looks like an open. So our output voltage looks like:

$$v_{\text{coil}}(0^+) = V_o \left(\frac{50\text{k}\Omega}{50.32\text{k}\Omega} \right) \approx V_o$$

Since we measure 5V there, we can say $V_o \approx 5\text{V}$.

Then we can look at the steady-state value of v_{coil}. In DC steady-state, the inductor looks like a short, so at that point, we have:

$$v_{\text{coil}}(\infty) = 5\text{V} \left(\frac{R_L \,||\, 50\text{k}\Omega}{320\Omega + (R_L \,||\, 50\text{k}\Omega)} \right)$$

Since the measured voltage ($\approx 1\text{V}$) is not particularly close to 5V, we can say that $R_{\text{L}} << 50\text{k}\Omega$, so $R_{\text{L}} \,||\, 50\text{k}\Omega \approx R_{\text{L}}$. Thus we can approximate things as:

$$v_{\text{coil}}(\infty) \approx 5\text{V} \left(\frac{R_L}{320\Omega + R_L} \right)$$

Since our steady-state voltage is about $1/5$ of V_o, we know that $R_L \approx 320\Omega/4$, so $R_L \approx 80\Omega$.

Finally, knowing this, we can estimate τ to find L. The curves seem to converge after around 3 divisions (600μs), so we can say $5\tau \approx 600\mu$s, or $\tau \approx 120\mu$s. For an inductor connected to a Norton/Thévenin, we have $\tau = L/R$. Here, that amounts to:

$$120\mu\text{s} = \frac{L}{80\Omega + (320\Omega \,||\, 50\text{k}\Omega)} \approx \frac{L}{400\Omega}$$

Solving out, $L \approx 48\text{mH}$.

Your friend did one final experiment. After leaving the switch closed for a long time, they opened the switch at time t_0 and noticed changes in both v_{R} and v_{coil}. But alas, they can't remember what changed, so they once again ask you for help. Determine the values of v_{R} and v_{coil} in the instant after the switch is opened.

$v_{\text{R}}(t_0^+) =$ [0] $v_{\text{coil}}(t_0^+) =$ [−625V]

In steady-state (just before the switch opens), the current flowing through the inductor will be $i_{\text{L}} \approx \frac{5\text{V}}{400\Omega}$. When we open the switch, that current must flow somewhere. It all flows around the loop through the 50kΩ resistor, causing an instantaneous voltage $v_{\text{coil}}(t_0^+) \approx -12.5\text{mA} \times 50\text{k}\Omega$.

5 Mystery Circuit

In the pink envelope at your table (which should be labeled with the same number that is printed on the bottom of this exam near the QR codes), you will find a breadboard containing the circuit we'll use for this problem. The breadboard contains an RP2040 microcontroller (the same one we've used in lab), as well as a green PCB with two output wires on it. There is a switch on the battery case that you can use to turn things on, and an orange LED on the PCB should light up when you power it on.

Even though there are some complex things in this circuit, we can model the whole thing (from the perspective of those two exposed wires) as consisting only of linear components. But note that turning off the power is *not* equivalent to zeroing out the sources in the circuit.

Your goal in this problem is to determine: if we were to connect a 700Ω resistor between those two output terminals and measure the voltage drop across it, what would that voltage drop be? Of course, you will not be able to solve for this value exactly and you don't have a calculator, so reasonable approximations are OK.

You may make any measurements you wish, but note that solving the problem does not require making any measurements on the RP2040, nor any measurements internal to the circuit on the green PCB.

There is also a resistor in the breadboard, which is currently not connected to the rest of the circuit (it's just there as an extra component in case you want it). This resistor's resistance is **not** 700Ω.

Enter your answer below (as a single number with units), and then in the box on the facing page (page 18), include a description of what measurements you made (and the values you found), as well as how you used those results to arrive at your answer.

When you are finished, please **turn off the power**, put your circuit back in the envelope, and **leave the envelope at your table**.

Predicted voltage drop across a 700Ω resistor connected between the terminals:

0.25V

Measurements and reasoning:

There are a bunch of useful measurements we could make, but we need two i/v pairs to characterize our line. Two useful ones are:

- Measuring the voltage drop between the terminals (v_{oc}).
- Measuring the current flowing between the terminals when they're approximately shorted by the meter (i_{sc}).

For most, these measurements would come out to approximately:

- $v_{\text{oc}} \approx 0.75\text{V}$
- $i_{\text{sc}} \approx 0.55\text{mA}$

We can use these to find $V_{\text{TH}} \approx 0.75\text{V}$ and $R_{\text{TH}} \approx \frac{0.75\text{V}}{0.55\text{mA}} \approx 1.4\text{k}\Omega$.
Then if we imagine hooking up our 700Ω resistor, we get a voltage divider:

$$v_{\text{out}} \approx V_{\text{TH}} \left(\frac{700\Omega}{700\Omega + R_{\text{TH}}} \right) = \frac{0.75\text{V}}{3} = 0.25\text{V}$$

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