

6.200 Midterm

Fall 2025

Name: **Answers**

Kerberos/Athena Username:

5 questions

1 hour, 50 minutes

- Please **WAIT** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5" × 11" sheet of handwritten notes (both sides) as a reference. This sheet must be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices (no computers, calculators, phones, etc.).
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.
- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam. **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

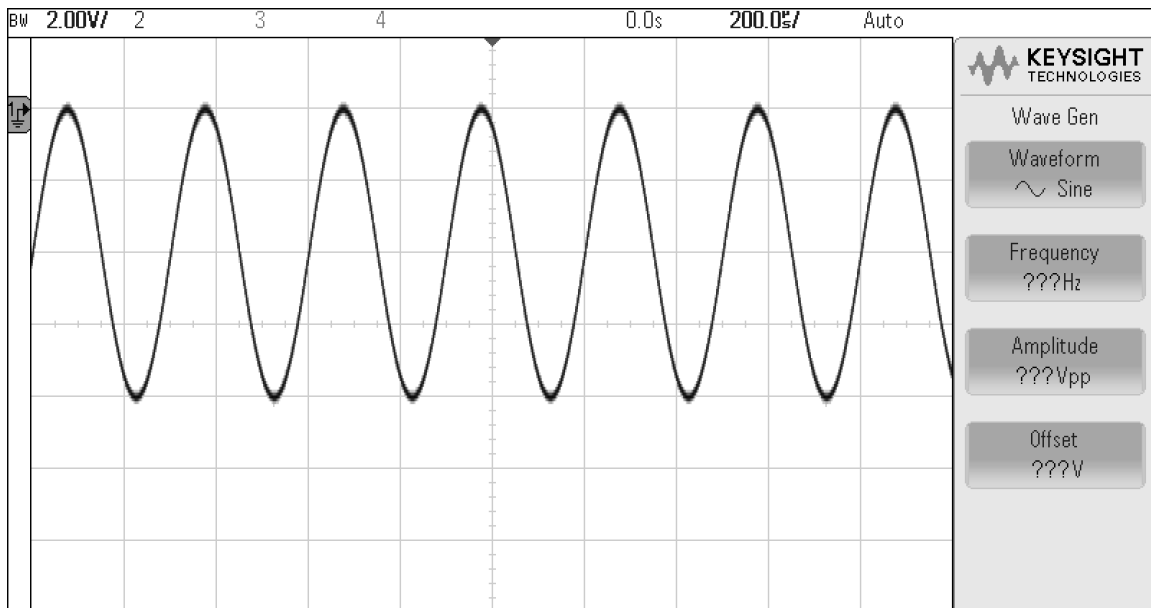
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1 Short Circuits

1.1 Oscilloscope

In lab, we've often used the wave generator on the oscilloscopes to generate sinusoidal signals of various kinds. Consider the following oscilloscope display, where channel 1 is hooked up to measure the output of the wave generator directly. Answer the questions below, and enter a single number (with units) in each box.



What is the approximate value of the Frequency setting that corresponds to this output, in Hz?

3.33kHz

The wave goes through 2 periods in around 3 divisions ($600\mu\text{s}$), or 1 period in around 1.5 divisions ($300\mu\text{s}$). Thus, the frequency is around $\frac{1}{300 \times 10^{-6} \text{s}}$.

What is the approximate value of the (peak-to-peak) Amplitude setting, in Volts?

8V

The wave goes through 4 divisions from peak to peak, each of which represents 2V.

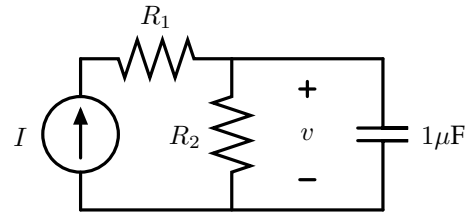
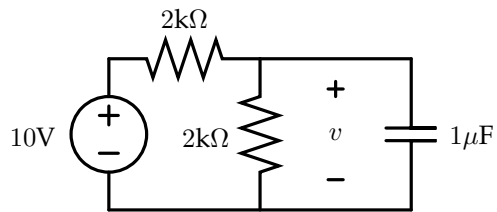
What is the approximate value of the Offset setting, in Volts?

-4V

The 0-Volt reference is indicated by the little ground symbol on the left-hand side of the screen. The center of the wave is about 2 divisions below the reference voltage.

1.2 Circuit Similarities

Consider the following two circuits:



Is it possible to choose a **finite, nonzero** value of I and **finite, positive** values of R_1 , and R_2 such that, for any possible initial condition of $v(t_0)$, the two circuits above will produce the same output $v(t)$ for all $t > t_0$?

Is this possible? (circle one): **Yes** / **No**

If no, briefly explain why. If yes, clearly indicate values for I , R_1 , and R_2 for which this relationship holds. If there are multiple possible values, you only need to specify one such value.

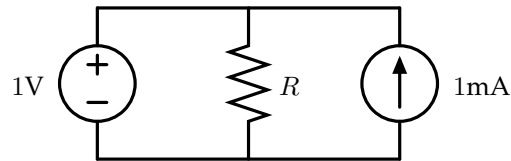
Yes, this is possible. We need the Thévenin equivalents of the circuit to the left of the capacitor to be equivalent, for which we need:

- $R_2 = 2\text{k}\Omega \parallel 2\text{k}\Omega = 1\text{k}\Omega$
- $I = 10\text{V}/2\text{k}\Omega = 5\text{mA}$

R_1 can have any value without affecting this equivalence.

1.3 Power to the People

Consider the following circuit, with an unspecified value of R :



The current running down through the resistor is $\frac{1V}{R}$. If that current is less than 1mA (the current coming from the current source), the voltage source will be providing power; if the current is exactly 1mA, it is neither producing nor consuming power (its current will be 0); otherwise, it will be consuming power.

Is it possible to choose a **positive, finite** value of R such that the voltage source is **providing** power? If yes, write an appropriate value of R in the box below; if no, write None instead. If there are multiple possible values of R , you only need to include one such value.

Value of R , or None:

any
 $R < 1k\Omega$

Is it possible to choose a **positive, finite** value of R such that the voltage source is **consuming** power? If yes, write an appropriate value of R in the box below; if no, write None instead. If there are multiple possible values of R , you only need to include one such value.

Value of R , or None:

any
 $R > 1k\Omega$

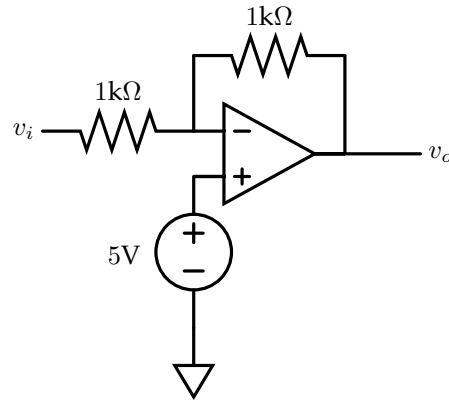
Is it possible to choose a **positive, finite** value of R such that the voltage source is **neither producing nor consuming** power? If yes, write an appropriate value of R in the box below; if no, write None instead. If there are multiple possible values of R , you only need to include one such value.

Value of R , or None:

$1k\Omega$

1.4 Op-Amps

Solve for the output voltage v_o in the following circuit, entering your answer as a single simple expression in terms of the input voltage v_i . Make the ideal op-amp assumption, and ignore limitations on the op-amp's output imposed by the power supply.



$$v_o = 10\text{V} - v_i$$

KCL at the inverting input of the op-amp:

$$\frac{v_i - v_-}{1\text{k}\Omega} = \frac{v_- - v_o}{1\text{k}\Omega}$$

By the ideal op-amp assumption, $v_- = v_+ = 5\text{V}$, so we have:

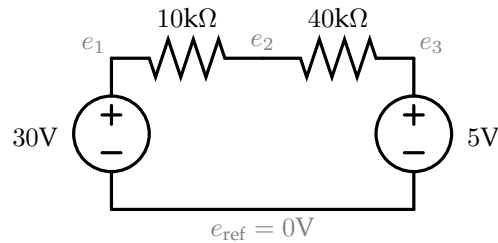
$$\frac{v_i - 5\text{V}}{1\text{k}\Omega} = \frac{5\text{V} - v_o}{1\text{k}\Omega}$$

Simplifying yields the answer above.

2 Solving Circuits

For each of the following circuits, solve for the indicated node potentials (relative to the indicated reference node). Enter your answers as exact numerical values (with units), not as expressions.

2.1 Circuit 1



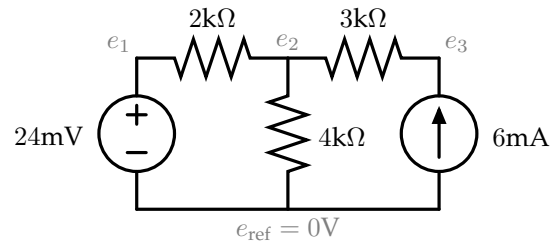
Here we have a voltage divider, so $e_2 = 5\text{V} + \frac{4}{5}(30\text{V} - 5\text{V})$

$$e_1 = 30\text{V}$$

$$e_2 = 25\text{V}$$

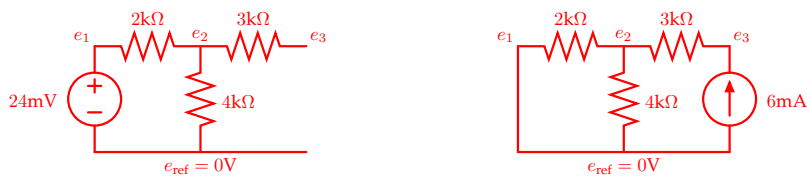
$$e_3 = 5\text{V}$$

2.2 Circuit 2



By the voltage source's constitutive equation, we have $e_1 = 24\text{mV}$. By Ohm's Law, we have $e_3 = e_2 + 18\text{V}$.

e_2 might be most easily found by superposition, which involves solving the following two circuits:



The circuit on the left is a voltage divider, so $e_2 = \frac{2}{3} (24\text{mV})$.

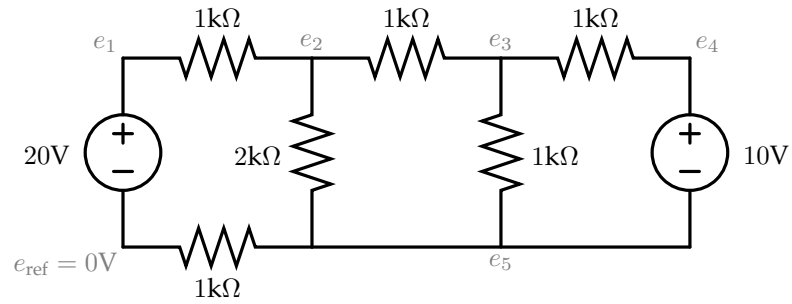
In the circuit on the right, we can combine the $2\text{k}\Omega$ and $4\text{k}\Omega$ resistors in parallel, giving $e_2 = (2\text{k}\Omega \parallel 4\text{k}\Omega) 6\text{mA}$.

$$e_1 = 24\text{mV}$$

$$e_2 = 8.016\text{V}$$

$$e_3 = 26.016\text{V}$$

2.3 Circuit 3



Answers on next page.

$$e_1 = 20V$$

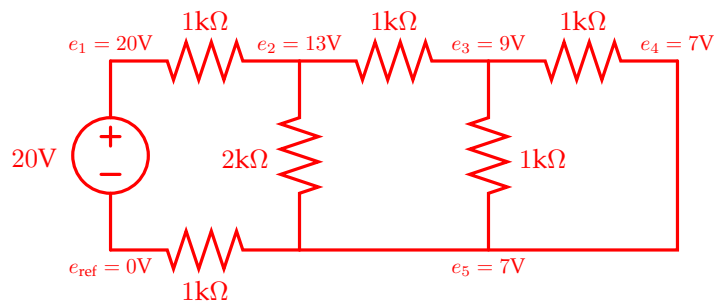
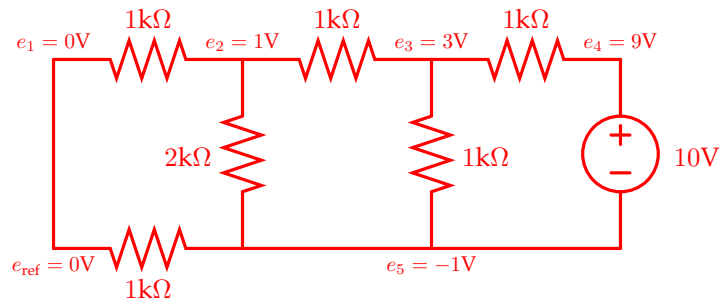
$$e_2 = 1V + 13V = 14V$$

$$e_3 = 3V + 9V = 12V$$

$$e_4 = 9V + 7V = 16V$$

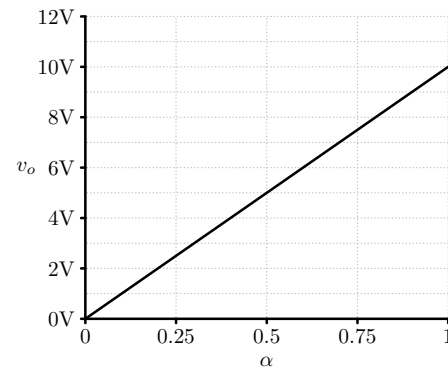
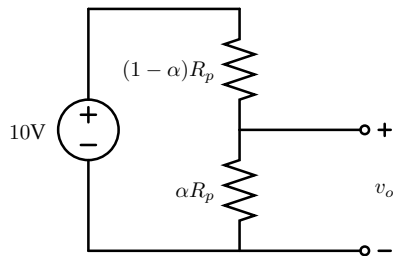
$$e_5 = -1V + 7V = 6V$$

As in the last part, this is perhaps most easily approached via superposition. Because we have two independent sources, we'll again need to solve two circuits. Each of the circuits below can then be simplified using series/parallel combinations of resistors and voltage/current dividers.



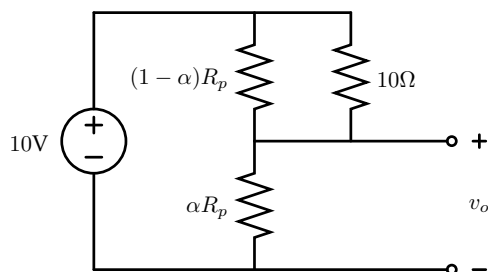
3 Gateway Circuits

In the recitations in weeks 1 and 6, we connected a potentiometer as a voltage divider. Consider the following circuit, where $R_p = 10k\Omega$. In this configuration, v_o and α are related as shown in the graph on the right.

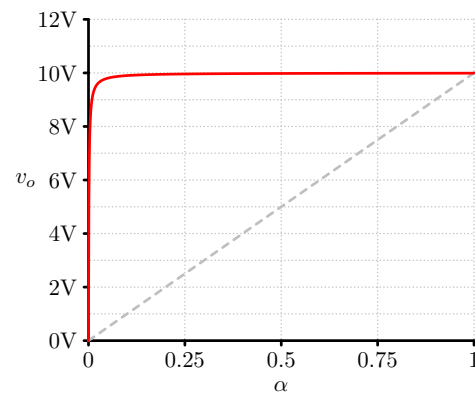


In this problem, as in that recitation, we'll consider how that graph changes (if at all) as we modify the circuit in a few ways. For each of the following circuit configurations, sketch the relationship between v_o and α in that circuit. The original curve has been reproduced in grey on each graph, to be used as a reference.

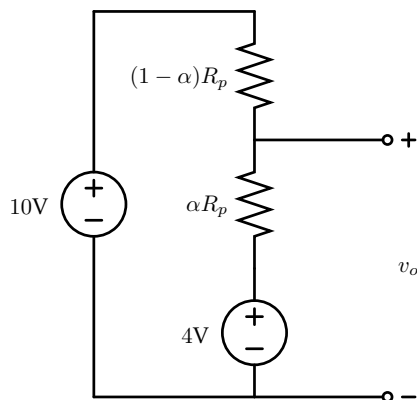
3.1 Configuration 1



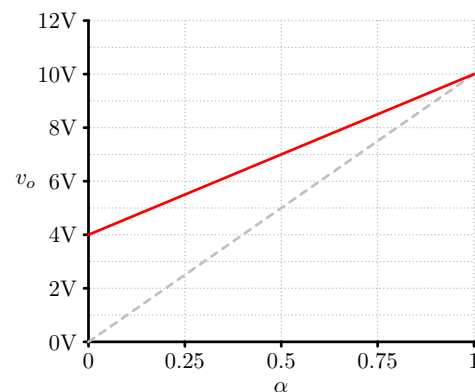
$$v_o = \left(\frac{\alpha R_p}{\alpha R_p + ((1-\alpha)R_p \parallel 10\Omega)} \right) 10V$$



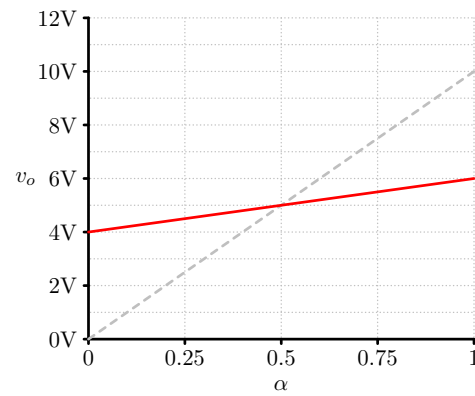
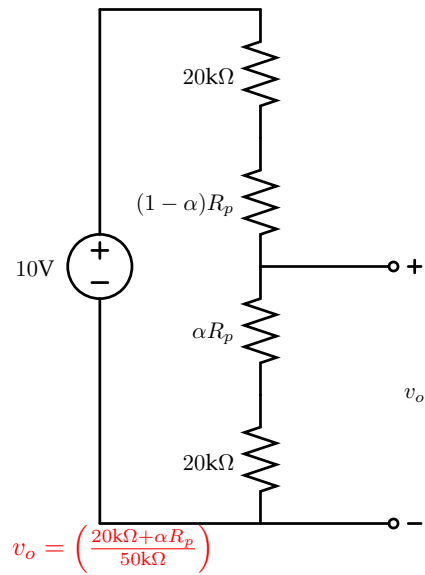
3.2 Configuration 2



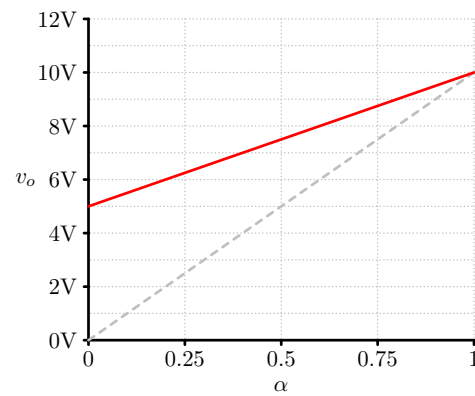
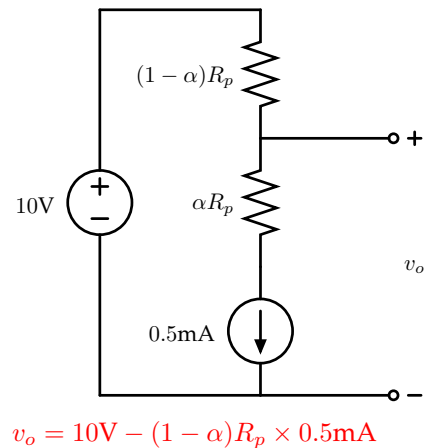
$$v_o = 4V + \alpha \times 6V$$



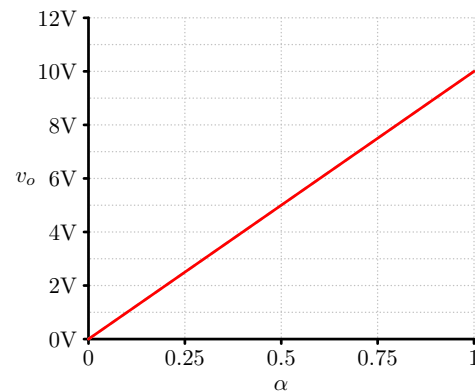
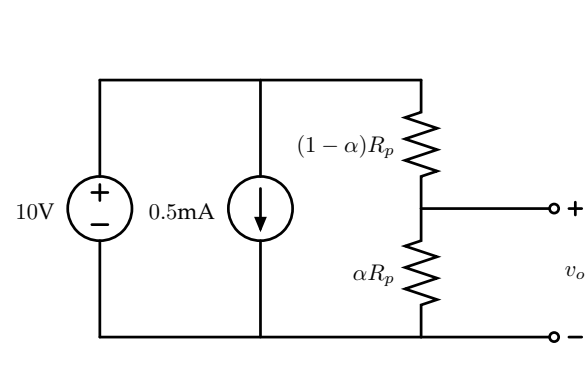
3.3 Configuration 3



3.4 Configuration 4



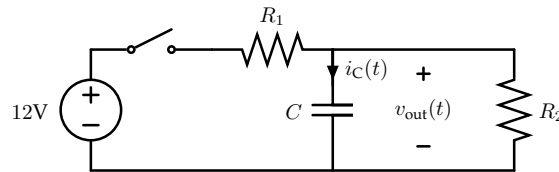
3.5 Configuration 5



The current source is irrelevant for this question, so the output is the same as the original circuit.

4 Changing Values

In this problem, we'll consider the following circuit:

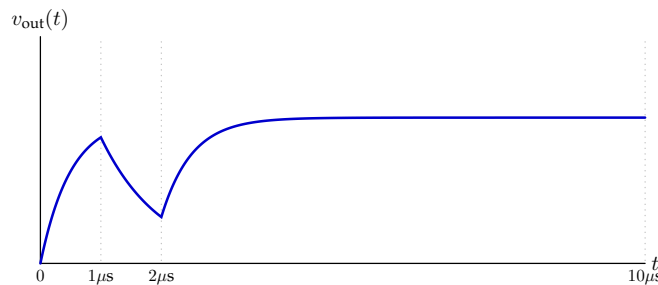


Specifically, we'll consider the following situation:

- $v_{\text{out}}(t = 0^-) = 0\text{V}$
- The switch is open for all time $t < 0$.
- The switch is closed for $0 \leq t < 1\mu\text{s}$.
- The switch is open for $1\mu\text{s} \leq t < 2\mu\text{s}$.
- The switch is closed for $t \geq 2\mu\text{s}$.

4.1 Original Values

For this part, consider the circuit above with $R_1 = R_2 = 1\text{k}\Omega$ and $C = 1\text{nF}$. A graph of v_{out} as a function of time is shown below. The vertical axis is intentionally not labeled.



Find *exact* expressions for the following values of $v_{\text{out}}(t)$, simplifying your answers to the extent possible:

$$v_{\text{out}}(t = 1\mu\text{s}) = 6\text{V} (1 - e^{-2})$$

$$v_{\text{out}}(t = 2\mu\text{s}) = 6\text{V} (1 - e^{-2}) (e^{-1})$$

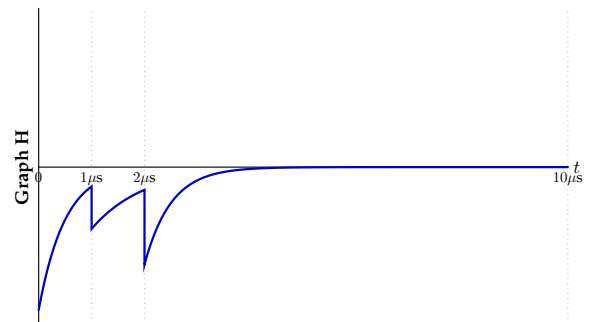
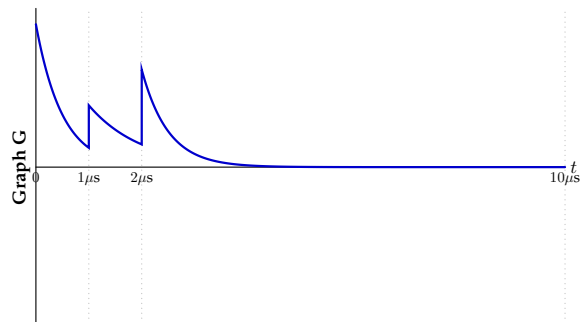
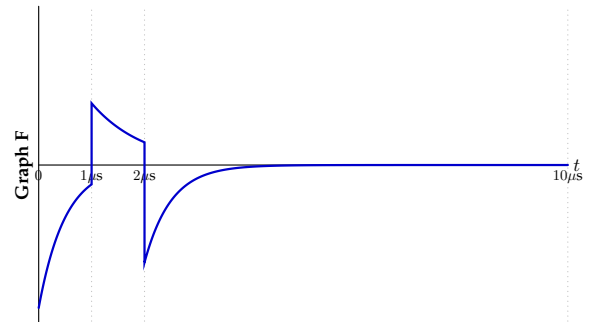
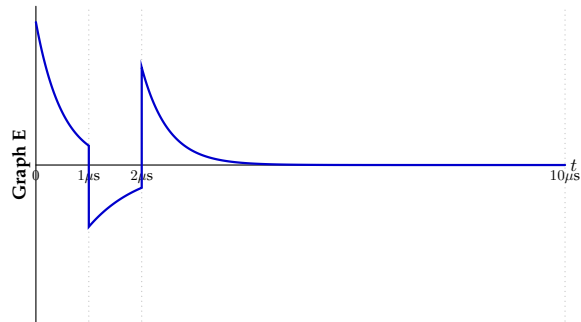
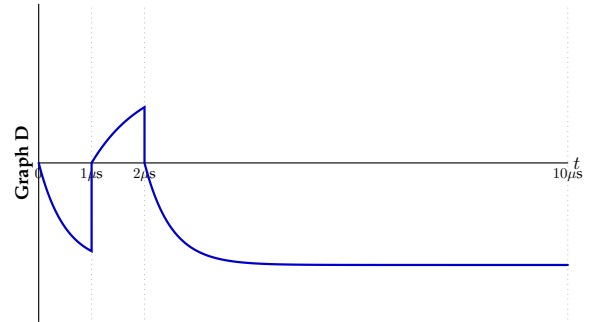
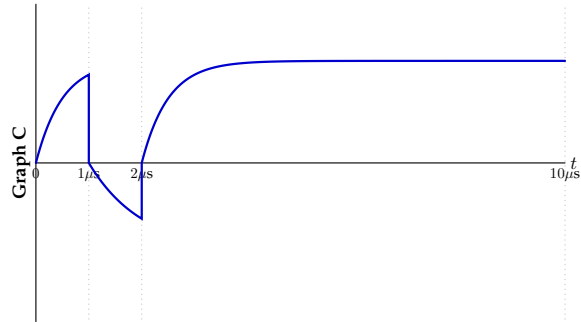
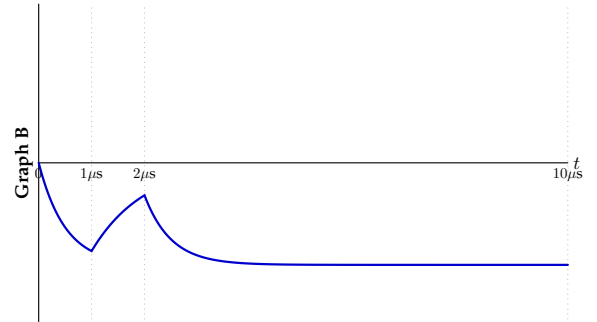
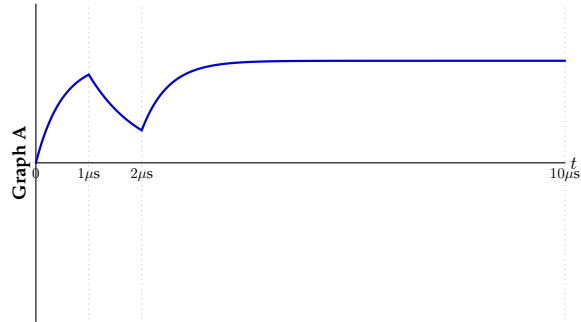
$$\lim_{t \rightarrow \infty} v_{\text{out}}(t) = 6\text{V}$$

4.2 Current Events

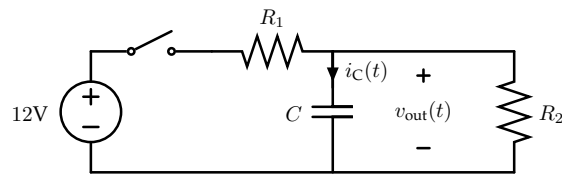
Which of the graphs below most closely resembles the shape of the current i_C as a function of time, using the component values and initial conditions described on the previous page?

Graph of $i_C(t)$ (enter a single letter):

E

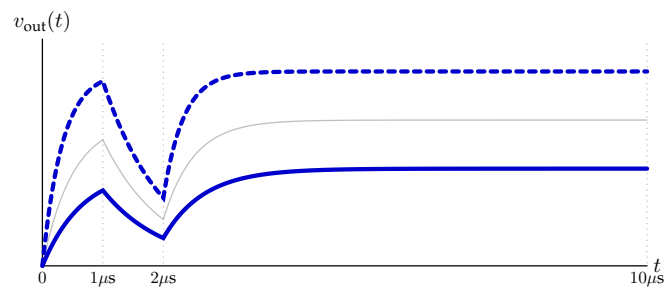


The original circuit is repeated here for reference:



In each of the graphs below, the original voltage curve (with the values of R_1 , R_2 , and C from the previous part of this problem) is shown in grey as a point of comparison, along with two curves, one of which results from *increasing* the given value by a factor of 2, and the other of which results from *decreasing* the given value by a factor of 2. For each graph, indicate which curve is which.

4.3 Changing R_1



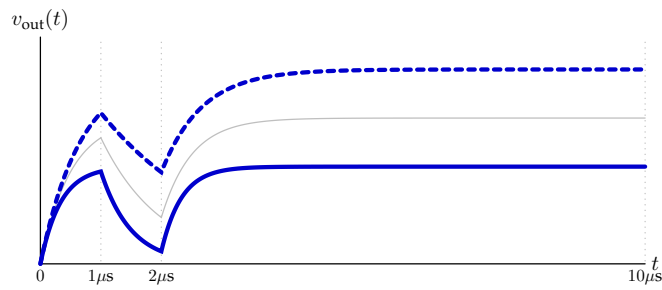
Does the solid curve correspond to **increasing** or **decreasing** R_1 ?

increasing

Does the dashed curve correspond to **increasing** or **decreasing** R_1 ?

decreasing

4.4 Changing R_2



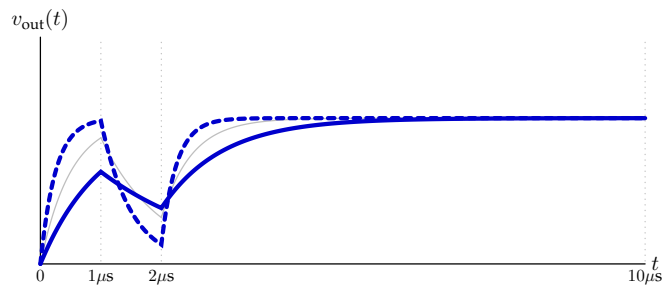
Does the solid curve correspond to **increasing** or **decreasing** R_2 ?

decreasing

Does the dashed curve correspond to **increasing** or **decreasing** R_2 ?

increasing

4.5 Changing C



Does the solid curve correspond to **increasing** or **decreasing** C ?

increasing

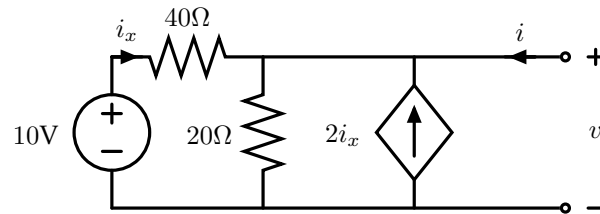
Does the dashed curve correspond to **increasing** or **decreasing** C ?

decreasing

5 Equivalence

5.1 Part 1

Find the Thévenin and Norton equivalents of the following circuit, viewed from the port labeled with i and v :



Draw your equivalent circuits in the boxes on the facing page, clearly labeling all values.

KCL at the top-right node:

$$\frac{10\text{V} - v}{40\Omega} + 2i_x + i = \frac{v}{20\Omega}$$

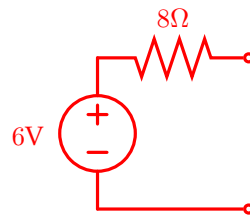
$$3 \left(\frac{10\text{V} - v}{40\Omega} \right) + i = \frac{v}{20\Omega}$$

$$30\text{V} - 3v + (40\Omega \times i) = 2v$$

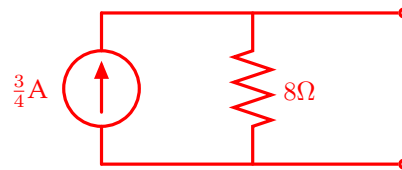
$$30\text{V} + (40\Omega \times i) = 5v$$

$$6\text{V} + (8\Omega \times i) = v$$

Thévenin Equivalent:

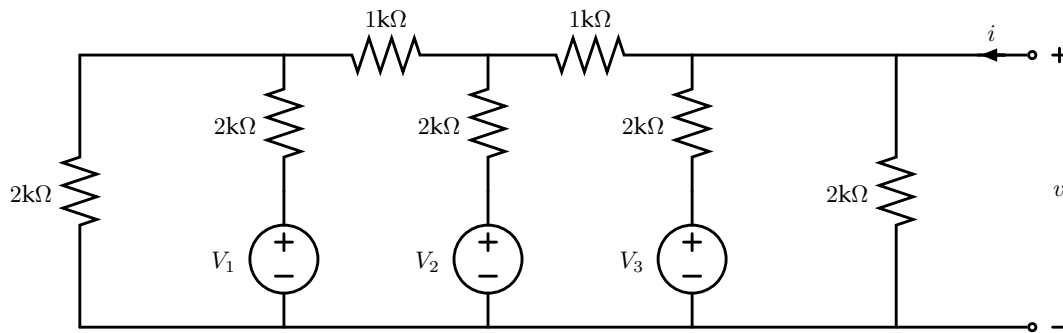


Norton Equivalent:

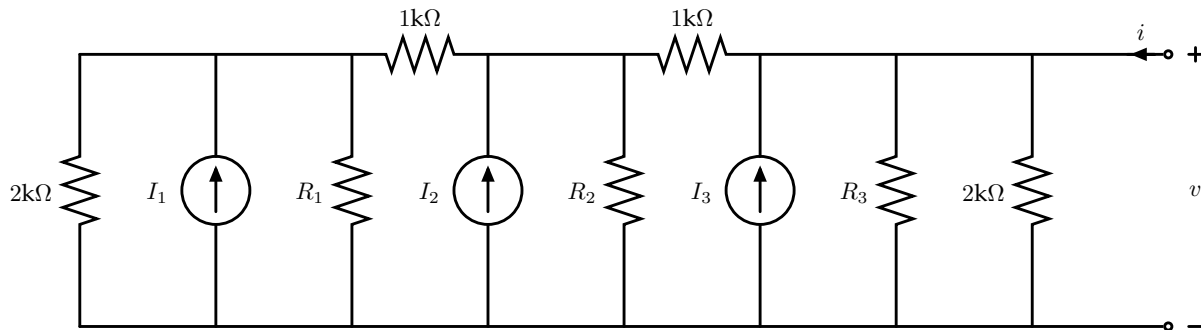


5.2 Part 2

Consider a 3-bit version of the DAC circuit we've seen in lab:



As well as a closely-related design using current sources instead of voltage sources:



It is possible to make these two circuits equivalent (in terms of terminal behavior at the port labeled with v and i) by choosing appropriate values of I_1 , I_2 , I_3 , R_1 , R_2 , and R_3 , where:

- I_1 depends on V_1 but not on V_2 or V_3 .
- I_2 depends on V_2 but not on V_1 or V_3 .
- I_3 depends on V_3 but not on V_1 or V_2 .

Find these values, and enter your answers in the boxes on the facing page as simple expressions.

Note that there are several Thévenins as subparts of the first circuit, and we construct the second circuit by replacing each of these Thévenins with a Norton; the easiest approach here is not to find the overall Thévenin equivalent of the whole ladder, but rather to make sure that the Nortons in the second circuit are equivalent to the corresponding Thévenins in the first circuit.

$$I_1 = \boxed{V_1/2\text{k}\Omega}$$

$$I_2 = \boxed{V_2/2\text{k}\Omega}$$

$$I_3 = \boxed{V_3/2\text{k}\Omega}$$

$$R_1 = \boxed{2\text{k}\Omega}$$

$$R_2 = \boxed{2\text{k}\Omega}$$

$$R_3 = \boxed{2\text{k}\Omega}$$

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