Electrical Engineering and Computer Science

6.200 – Circuits & Electronics
Spring 2023

Final Exam

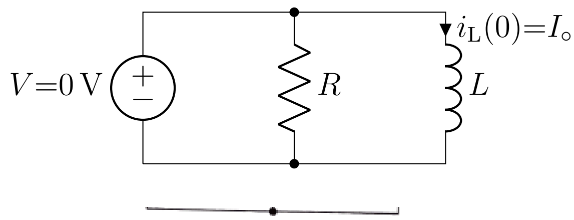
May 22, 2023

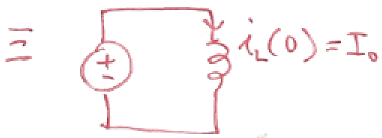
Name: ____

- There are 10 problems and 32 pages (including this cover page) in this exam. The last page of the exam is blank, in case extra space is needed.
- There are 141 possible points you can earn on this exam.
- Do not remove any pages from this exam.
- Do your work for each question within the boundaries of that question. If you use the additional pages for any work, indicate this fact in text in the valid answer space. Enter your answer to each question in the corresponding answer box provided.
- You may refer to one 8.5”× 11” pages of notes, double-sided.
- Calculators, smartphones, and laptops are not permitted. If you find yourself in need of a calculator, you may be going in the wrong direction. You may include expressions such as 2π or e in your answers, even if a numerical answer is called for.
- Show your work. Unless you write out your thought process clearly, partial credit will not be awarded for incorrect solutions. Some problems may require work to be shown to receive full credit.

Problem 1, 14 pts: Short Circuits

(1A) (1pts) The circuit below is observed at $t = 0$ to have current $i_L = I_\circ$ flowing through an ideal inductor. The voltage source is set to 0 V and does not change. What is the current i_L at time $t = L/R$?





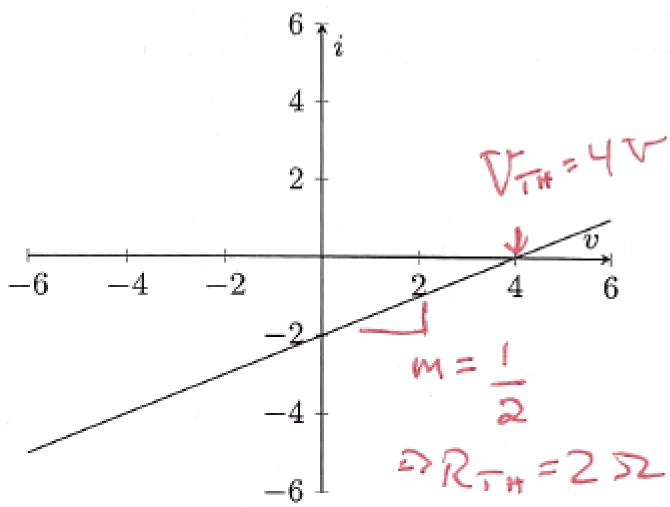
(R plays no role)

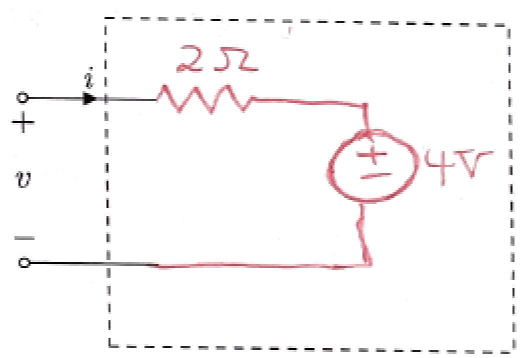
$\Rightarrow R_{eff} = 0 \Rightarrow \tau = L/R_{eff} = \infty$

$\Rightarrow$ no decay

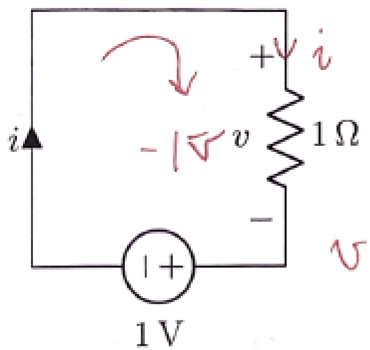
$i_L(t = L/R) = I_\circ$

(1B) (2pts) Fill in the box shown with a circuit that will provide the indicated i-v relation.

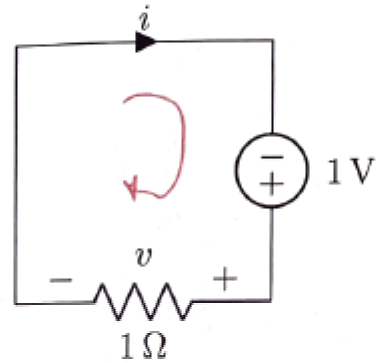




(1C) (4pts) Circle the correct variable values below each of the circuits.



$v + 1\text{V} = 0$



$-1\text{V} + v = 0$

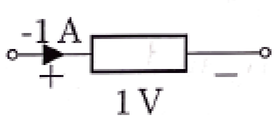
$\Rightarrow v = 1\text{V}$

$i =$	$+1\,\text{A}$	$-1\,\text{A}$
$v =$	$+1\,\text{V}$	$-1\,\text{V}$

$i =$	$+1\,\text{A}$	$-1\,\text{A}$
$v =$	$+1\,\text{V}$	$-1\,\text{V}$

(1D) (3pts) For each element below, assume it is connected to an external circuit network. Is power flowing *from the external circuit* to the element, or *from the element* to the external circuit, or is it impossible to say with the given information? Circle one option for each element:

$v = 1\text{V}$ $i = -1\text{A}$



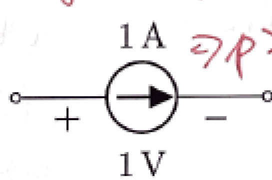
from ext. circuit

to ext. circuit

can't say

$v = 1\text{V}$ $i = 1\text{A}$

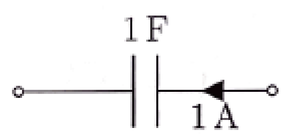
$\Rightarrow p > 0$



from ext. circuit

to ext. circuit

can't say



from ext. circuit

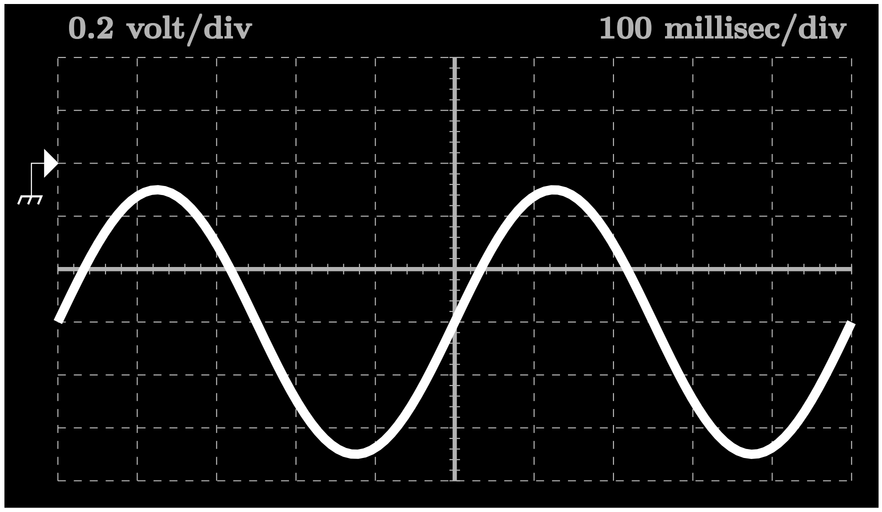
to ext. circuit

can't say

don't know state

$\Rightarrow$ don't know v

(1E) (4pts) Consider a signal on the triggered voltage-time oscilloscope trace shown below:



Write a numeric equation completely describing the signal. Pay close attention to the location of the ground.

$0.5\text{V} \cdot \sin(2\pi f t + \varphi) - 0.6\text{V}$

arbitrary choice is OK b/c t=0 is not well defined

$T = 5 \text{ div} \cdot 100 \frac{\text{ms}}{\text{div}}$

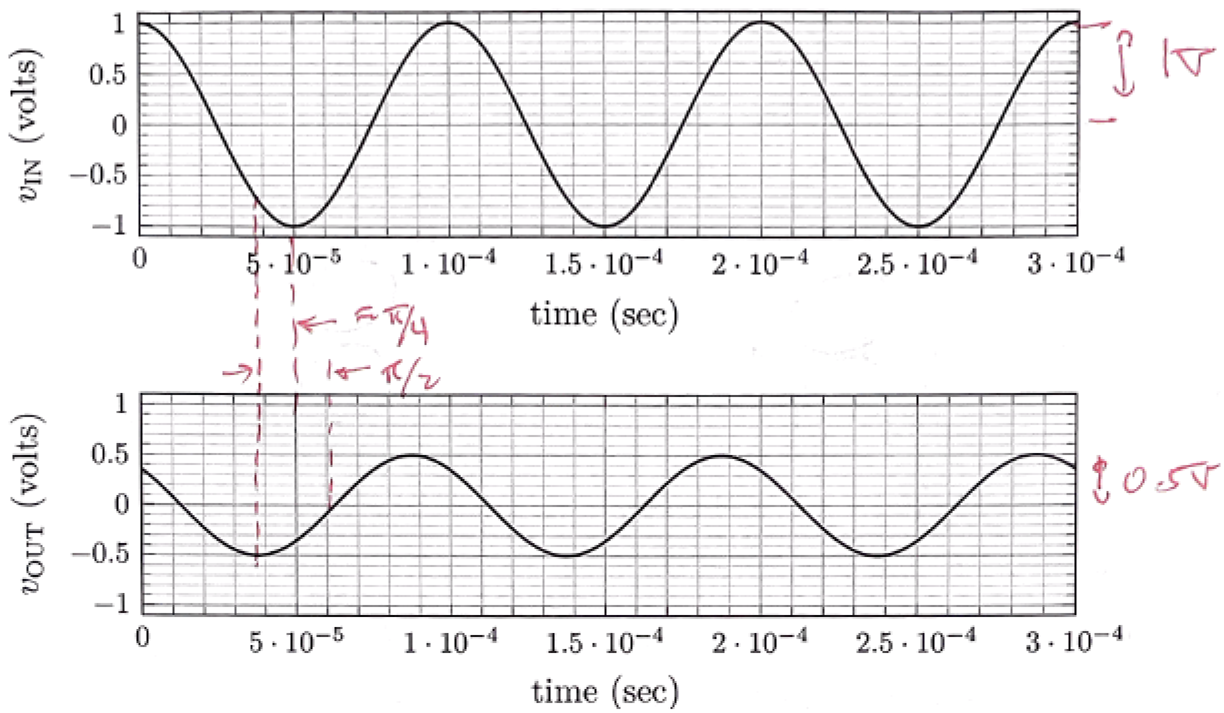
$= 0.5 \text{ s}$

$\Rightarrow f = 2 \text{ Hz}$

$v(t) =$ $0.5\text{V} \sin(2\pi \cdot 2 \cdot t) - 0.6\text{V}$

Problem 2, 6 pts: Phasors Set to Stun

Suppose you are given a black box, an input signal v_{IN}, and an output signal v_{OUT}.



(2A) (3pts) What is the gain of the circuit $|H| = \left|\frac{V_{out}}{V_{in}}\right|$?

$|H| = \frac{0.5}{1} = 0.5$

(2B) (3pts) What is the approximate phase shift $\angle H$ of the circuit? Be sure to specify your units (degrees or rads) and provide the correct sign.

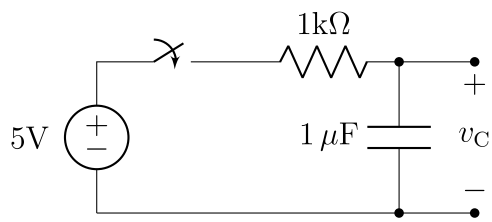
The output signal is advanced by $\frac{\pi}{4}$ rel. to i/p

$V_{out} = V_{in} e^{j\pi/4}$

$\angle H = \pi/4$

Problem 3, 12 pts: Charge Ahead

Consider the following circuit:



The voltage source shown on the left is a constant 5V source. The switch is opened and closed according to the following rules:

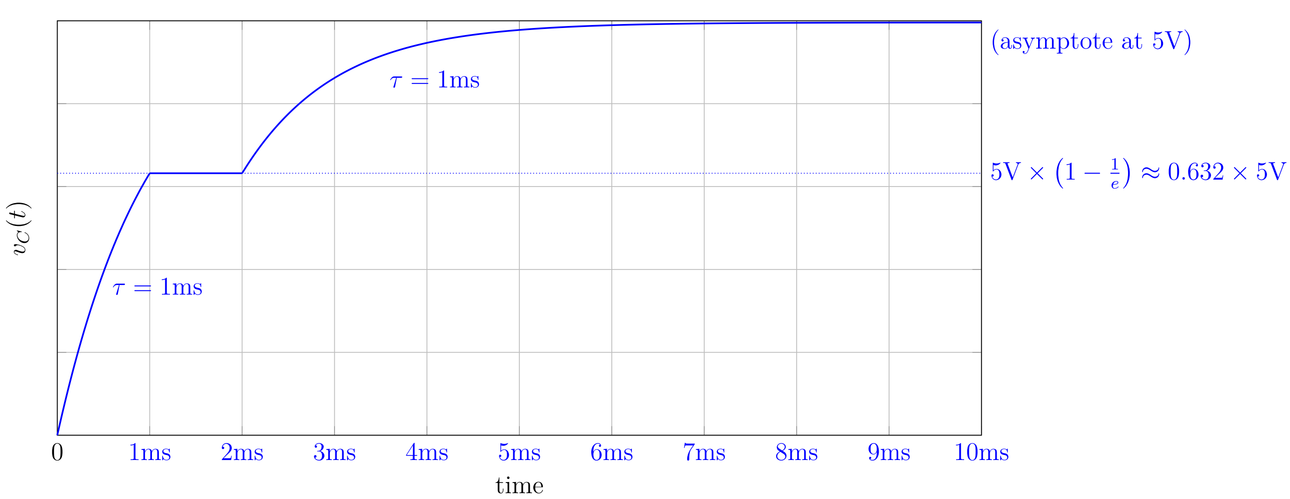
- Before $t = 0$, the switch is open (and $v_C(0_-) = 0$).
- From $0 \le t < 1\text{ms}$, the switch is closed.
- From $1\text{ms} \le t < 2\text{ms}$, the switch is open.
- For all $t \ge 2\text{ms}$, the switch is closed.

(3A) (6pts) On the axes below, sketch a plot of $v_C(t)$ versus t. Label all key values in your plot. For any exponential curves, also indicate the associated time constant.

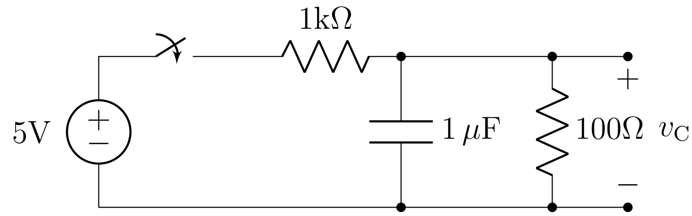
For $0 \le t < 1\text{ms}$, $v_C(t)$ follows an exponential curve with a time constant of $\tau = 1\text{ms}$. If we left the switch closed forever we would approach $\lim_{t\to\infty} v_C(t) = 5\text{V}$, but since we open the switch after exactly one time constant, we reach approximately $0.63 \times 5\text{V}$.

For $1\text{ms} \le t < 2\text{ms}$, $v_C(t)$ remains constant at the same value that that it had just before the switch was opened (with no path for current to flow, the capacitor can't discharge!).

For $t \ge 2\text{ms}$, $v_C(t)$ follows an exponential curve starting from that value, increasing to approach 5V.



(3B) (6pts) Now consider the following circuit (slightly modified from the previous one):



In this circuit, the switch is changed **in the same way as in the previous part**:

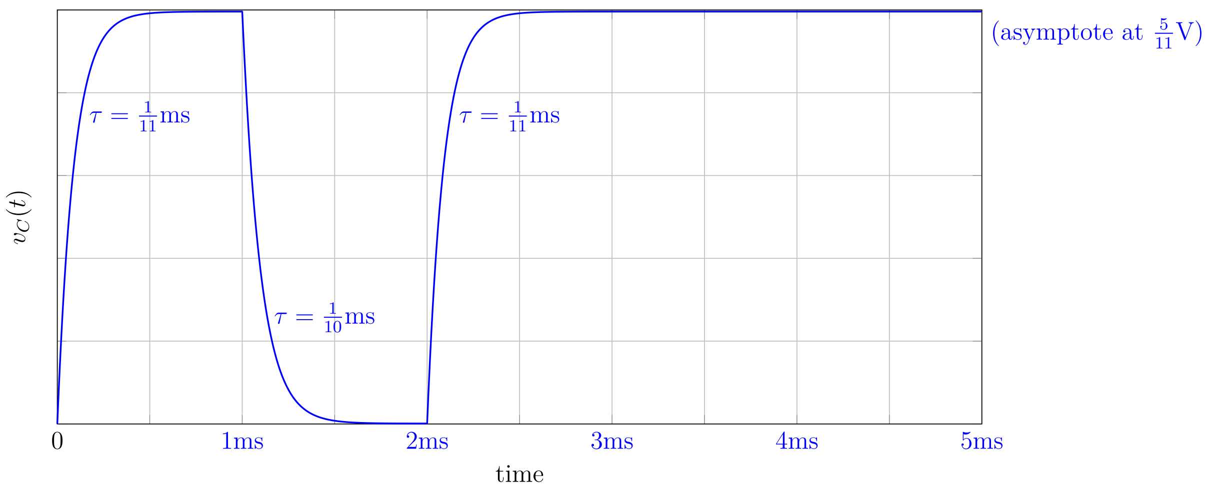
- Before $t = 0$, the switch is open (and $v_C(0_-) = 0$).
- From $0 \leq t < 1$ms, the switch is closed.
- From 1ms $\leq t < 2$ms, the switch is open.
- For all $t \geq 2$ms, the switch is closed.

On the axes below, sketch a plot of $v_C(t)$ versus t. Label all key values in your plot. For any exponential curves, also indicate the associated time constant.

For $0 \leq t < 1$ms, $v_C(t)$ still follows an exponential curve, but now with a much smaller time constant ($\tau = (100\Omega || 1000\Omega) \times 1\mu F$, less than 1/10 the original time constant). This is small enough that the circuit has effectively converged by the time 1ms rolls around; but it no longer converges to 5V, but rather to $\frac{5V}{11}$.

For 1ms $\leq t < 2$ms, the capacitor now discharges during this time, with a time constant of 0.1ms. The small time constant (relative to the length of this region) means that the capacitor is effectively completely discharged by the end of this period).

For $t \geq 2$ms, $v_C(t)$ still converges to a constant, but now that constant is $\frac{1}{11} \times 5V$ instead of 5V.



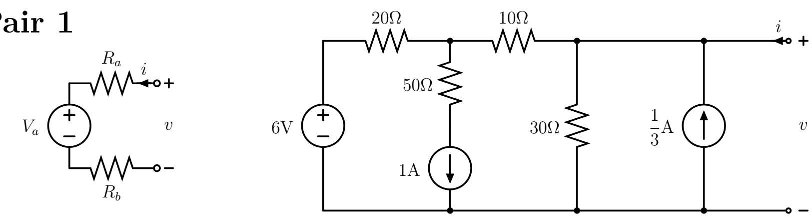
Problem 4, 18 pts: Peaches and Pairs

For each pair of circuits below, find constraints on the parameter values of the unspecified components that would make the two circuits equivalent to one another (in terms of the i-v relation at the indicated port). If any of the unspecified component parameter values are irrelevant, indicate that explicitly in your answer.

You should solve for numeric values exactly, but you may use $+$ and $||$ in your answers to represent series and parallel combinations of symbolically-specified resistors, respectively, rather than expanding those expressions out fully.

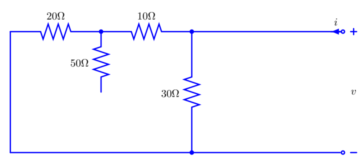
You may assume that all op-amps are ideal, and you may ignore power supply limitations of the op-amps.

(4A) (6pts) **Pair 1**



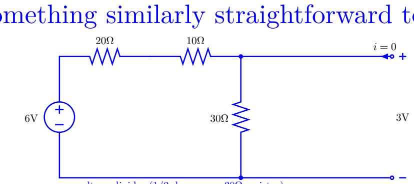
Using whichever method you prefer, we can find the Thévenin equivalent of the circuit on the left to be $V_{th} = V_a$ and $R_{th} = R_a + R_b$. So we won't be able to solve for R_a or R_b as individual values, but we can find some relationships by finding the Thévenin equivalent of the circuit on the right. Then its Thévenin voltage will be equal to V_a and its Thévenin resistance will be equal to $R_a + R_b$.

We can first solve for the resistance by setting all the source values to 0 and then finding the resistance between the two terminals. Setting the source values to zero gives us a circuit like this:

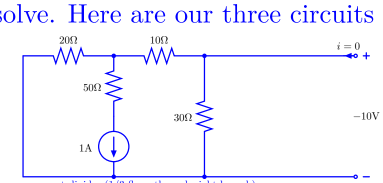


We can then solve for the resistance between the two terminals. From that perspective, we see the 30Ω resistor in parallel with the series combination of 20Ω and 10Ω, so we have $R_{th} = 30\Omega||(20\Omega + 10\Omega) = 15\Omega$.

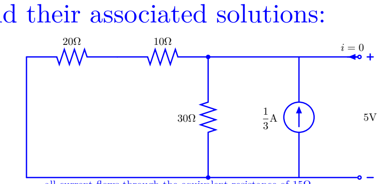
Next, we can find the Thévenin voltage. The easiest way to do that for this circuit is to use superposition. This will give us three circuits to solve, but each circuit will be a simple voltage or current divider, or something similarly straightforward to solve. Here are our three circuits and their associated solutions:



voltage divider (1/2 drops over 30Ω resistor)



current divider (1/3 flows through right branch)

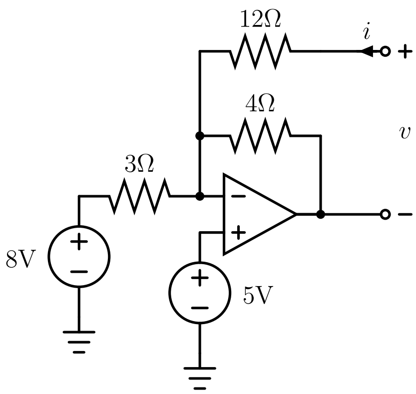


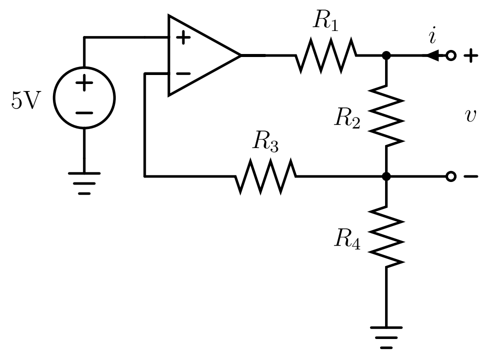
all current flows through the equivalent resistance of 15Ω

Summing, we find our overall Thévenin voltage is −2V. So for these to be equivalent, we need:

$V_a = -2\text{V}$, $R_a + R_b = 15\Omega$

(4B) (6pts) **Pair 2**





The strategies from the last part won't really work here because of the op-amp, but hope is not lost! We can make these two circuits have equivalent v/i relationships by equating their short-circuit currents and open-circuit voltages, respectively.

Solving for the open-circuit voltage of the circuit on the left, we find that the voltage at the output terminal of the op-amp (the $-$ terminal of the port we're interested in) is at 1V with respect to our reference voltage. Since no current flows through the 12Ω resistor, the $+$ terminal must be at the same potential as the inverting input of the op-amp, which is at 5V relative to our reference. Thus, $v_{\text{OC}} = 5\text{V} - 1\text{V} = 4\text{V}$.

For the circuit on the right, we know that no current flows into/out of the input terminals of the op-amp, so the node in between R_2 and R_4 (the $-$ terminal of the port we're interested in) must be at 5V. Thus, the current flowing down through R_4 must be $\frac{5\text{V}}{R_4}$, and, by KCL and taking into account the constraints the op-amp imposes, this same current must be flowing through R_2. Since v is just the drop across R_2, this tells us the $v_{\text{OC}} = \frac{R_2}{R_4}5\text{V}$. This isn't enough to solve everything yet, but combining this with our result from the left circuit tells us that $\frac{R_2}{R_4} = \frac{4}{5}$.

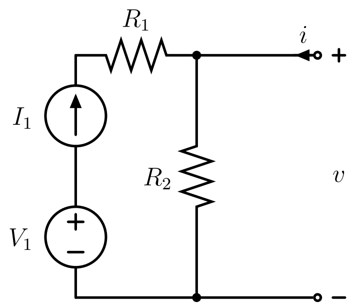
Now what remains is to find the short-circuit current for each. Let's start with the circuit on the left again. Here, if we short the terminals together, the 12Ω and 4Ω resistors form a current divider. We have 1A flowing through the combination from the left (via the 3Ω resistor), and 1/4 of that current will flow through the top branch. So our $i_{\text{SC}} = -0.25\text{A}$.

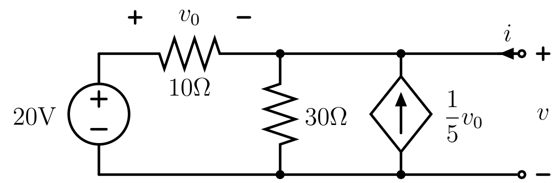
Back to the circuit on the right, if we whort the terminals to find i_{SC}, we still have the constraint from before that the current flowing down through R_4 must be $\frac{5\text{V}}{R_4}$. But now, that same current must be exactly what is flowing through our short, so we have $i_{\text{SC}} = -\frac{5\text{V}}{R_4}$. Combining with our result from the left circuit, this tells us that $-0.25\text{A} = \frac{5\text{V}}{R_4}$, so we must have $R_4 = 20\Omega$.

Finally, we can combine this with the constraint we got from equating the v_{OC} values, we find the total answer. These circuits will be equivalent if:

$R_2 = 16\Omega$, $R_4 = 20\Omega$, R_1 and R_3 are irrelevant

(4C) (6pts) **Pair 3**

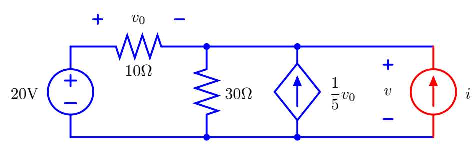




Let's start by finding a Thévenin equivalent for the left. Using your favorite method of choice, we should find that $R_{th} = R_2$ and $V_{th} = I_1 R_2$ (note that V_1 and R_1 are irrelevant).

Let's go ahead and find the equivalent circuit for the circuit on the right, then! This one is complicated by the dependent source in the mix, but perhaps the most straightforward thing to do is to hold i constant and solve for v in terms of i (from which we can pick out the Thévenin voltage and Thévenin resistance).

One way of holding i constant is to pretend that we're hooking up a current source between the terminals:



Now if we solve for the voltage v in terms of i, we should be able to pick our the values we need. From here, it's just math. We'll use the node method and assume that the bottom node is our ground (0V reference). Then we'll write KCL at the top-right node, which should allow us to solve for everything. Here we go (and we'll go ahead and sub $20\text{V} - v = v_0$ as well):

$$\frac{20\text{V} - v}{10\Omega} + \frac{1}{5}(20\text{V} - v) + i = \frac{v}{30\Omega}$$

$$2\text{A} - \frac{v}{10\Omega} + 4\text{A} - \frac{v}{5\Omega} + i = \frac{v}{30\Omega}$$

$$6\text{A} + i = \frac{10v}{30\Omega}$$

$$18\text{V} + 3\Omega \times i = v$$

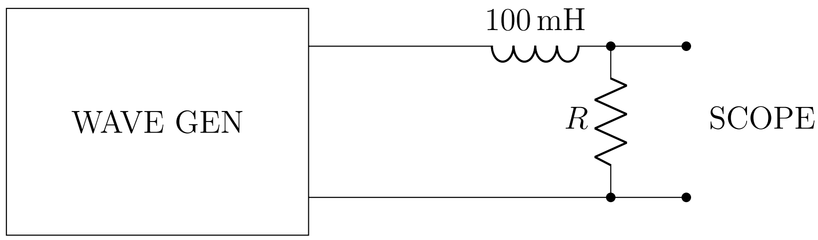
From this, we can see that $V_{th} = 18\text{V}$ and $R_{th} = 3\Omega$. Combining with the results from the left-hand circuit, we arrive at our final result, that these two circuits are equivalent if:

$I_1 = 6\text{A}$, $R_2 = 3\Omega$, R_1 and V_1 are irrelevant

Worksheet (intentionally blank)

Problem 5, 10 pts: The Resistance is Coming from *Inside the House*

Consider the following filter being tested in lab:



For this filter, R is chosen such that the -3dB point of the filter is around 10 kHz.

(5A) (2pts) What is the value of R?

$$\frac{R}{L} = 2\pi \cdot 10 \text{ kHz} = \frac{R}{100 \text{ mH}}$$

$$\Rightarrow R = 2\pi \cdot 10^4 \cdot 0.1 = 2\pi \cdot 10^3 \Omega$$

$R = 2\pi \times 1\text{k}\Omega \approx 6.283\text{k}\Omega$

(5B) (2pts) What is the gain G of this filter as $f \to 0\,\text{Hz}$? Do **not** use your numerically estimated value of R. Instead use R as a symbol in your answer.

$$\text{As } f \to 0,\ jwL \to 0$$

$$\Rightarrow v_{IN} = v_{OUT} \Rightarrow G = 1$$

$G(f \to 0) = 1$

(5C) (2pts) With this value of R, what is the gain of this filter as $f \to \infty$? Do **not** use your numerically estimated value of R. Instead use R as a symbol in your answer.

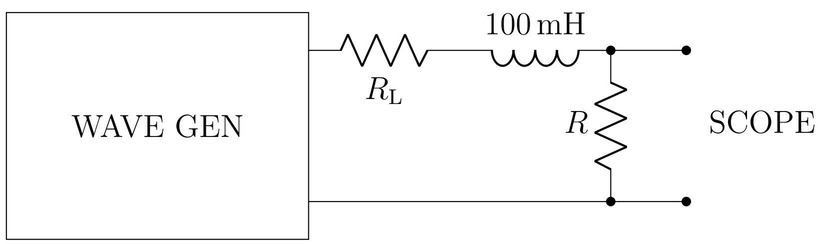
$$\text{As } f \to \infty,\ jwL \to \infty$$

$$\Rightarrow v_{OUT} = 0 \Rightarrow G = 0$$

$G(f \to \infty) = 0$

When the circuit proposed above is measured in the lab, the values do not agree with what you found on the previous page. Instead, $G(f \to 0) = \frac{1}{2} \equiv -6\text{dB}$.

We account for this by modeling the inductor as having some series ("parasitic") resistance (from the long coil of wire) as well as the desired inductance:



(5D) (4pts) Determine the value of R_L and find the frequency at which the gain of the filter will be $\frac{1}{2\sqrt{2}} \equiv -9\text{dB}$.

$$H(s) = \frac{R}{R + R_L + sL} \qquad H(0) = \frac{1}{2} = \frac{R}{R+R_L} \Rightarrow R_L = R$$

$$\frac{1}{2\sqrt{2}} = \left| \frac{R}{2R + j\omega L} \right| \Rightarrow \frac{R}{R\sqrt{4 + \omega^2 \frac{L^2}{R^2}}} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{4 + \omega^2 \frac{L^2}{R^2}} = \frac{1}{8} \Rightarrow 4 + \omega^2 \frac{L^2}{R^2} = 8 \Rightarrow \omega^2 \frac{L^2}{R^2} = 4$$

$$\Rightarrow \omega \cdot L = 2 \Rightarrow \omega = \frac{2R}{L} = 2 \cdot 10\text{kHz}$$

As $\omega \to 0$, the inductor looks like a short, so we just have R_L and R in series with each other. They form a voltage divider, and in order for the voltage to be cut in half (our gain of -6dB), we need $R_L = R$.

Since $R_L = R$, this cuts the time constant of our filter in half from what it was before, and the knee frequency doubles. Since we originally targeted 10kHz, the new cutoff frequency will be at 20kHz instead.

$R_L = R$	$f_{-9\,\text{dB}} = 20\text{kHz}$

Worksheet (intentionally blank)

Problem 6, 18 pts: Motor Races

Each circuit on the facing page contains several motors. Each motor can be modeled as a resistor with a positive, finite, non-zero resistance R_m, and **all motors have the same R_m value**. Assume that the speed of a motor is proportional to the voltage drop across it.

(6A) (18pts) For each of the circuits below, sort the motors in order of increasing speed (regardless of direction). Enter your answer for each circuit as a sequence of numbers from slowest to fastest, with $<$ or $=$ in between. For example, if M_2 is the slowest motor in some circuit, then M_3 and M_1 are moving at the same faster speed, and M_4 is the fastest motor, you should write the following in the box:

$$M_2 < M_1 = M_3 < M_4$$

Circuit-1: $M_1 = M_2 = M_3$

Slowest → Fastest

Circuit-2: $M_1 = M_2 < M_3 = M_4$

Slowest → Fastest

Circuit-3: $M_2 < M_1 = M_3$

Slowest → Fastest

Circuit-4: $M_5 = M_4 < M_3 < M_2 < M_1$

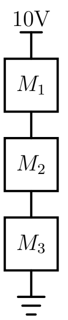
Slowest → Fastest

Circuit-5: $M_1 = M_2 = M_3 = M_4$

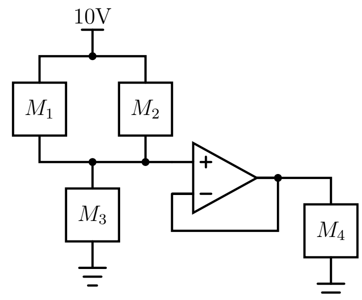
Slowest → Fastest

Circuit-6: $M_1 = M_2 = M_3 = M_4 = M_5 < M_6$

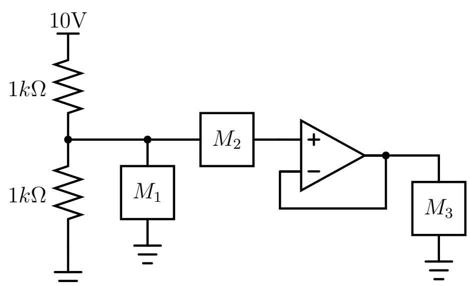
Slowest → Fastest



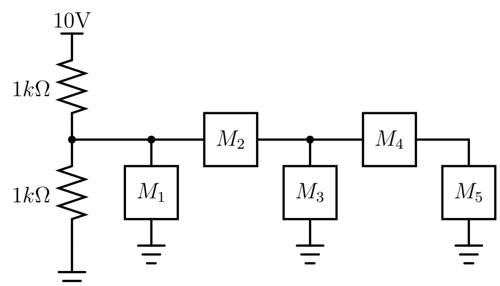
Circuit 1



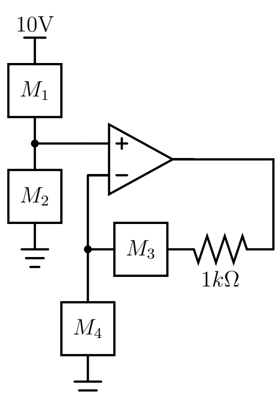
Circuit 2



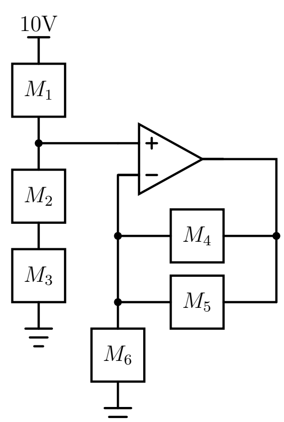
Circuit 3



Circuit 4



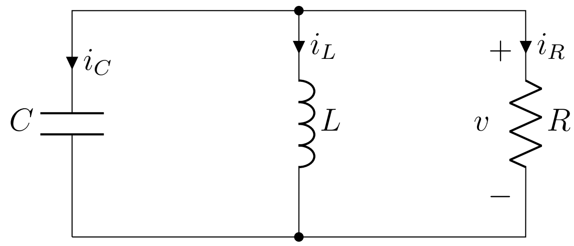
Circuit 5



Circuit 6

Problem 7, 21 pts: Step Up

Shown below is an underdamped second-order parallel-resonator network with three labeled currents.



(7A) (8pts) For each of the six statements given below, circle the correct completion. *Briefly explain your reasoning in the space below the statement completion.*

- If the resistance R is increased, the quality factor Q will $\cdots$ $Q = R/\sqrt{L/C}$

  $\cdots$ increase. $\cdots$ decrease. $\cdots$ remain unchanged.

- If the capacitance C is increased, the quality factor Q will $\cdots$ $Q = R/\sqrt{L/C}$

  $\cdots$ increase. $\cdots$ decrease. $\cdots$ remain unchanged.

- If the inductance L is increased, the quality factor Q will $\cdots$ $Q = R/\sqrt{L/C}$

  $\cdots$ increase. ***$\cdots$ decrease.*** $\cdots$ remain unchanged.

- If the resistance R is increased, the period of oscillation T will $\cdots$ $T \approx 2\pi\sqrt{LC}$

  $\cdots$ increase. $\cdots$ decrease. ***$\cdots$ remain unchanged.***

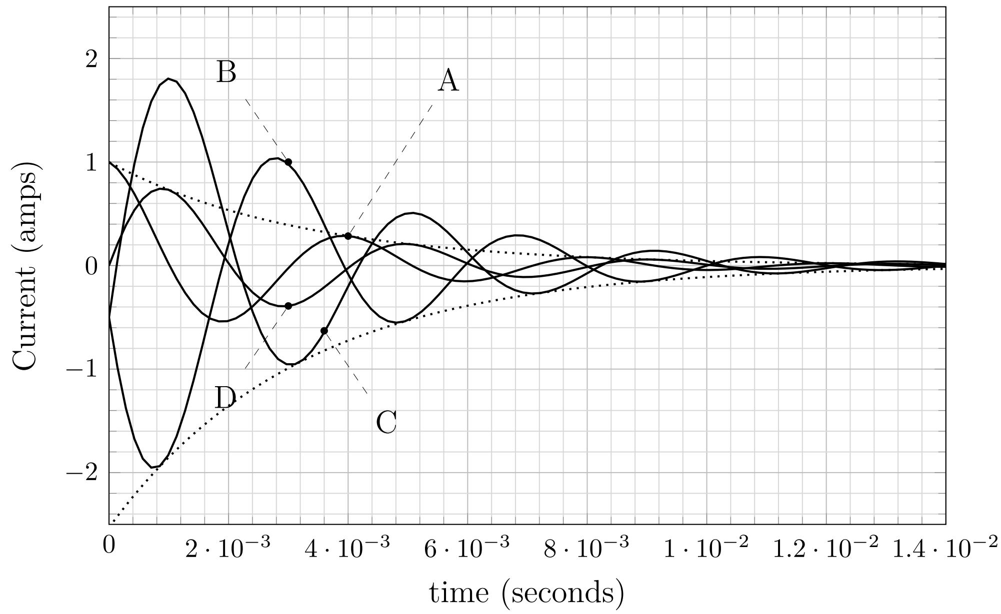
- If the capacitance C is increased, the period of oscillation T will $\cdots$ $T \approx 2\pi\sqrt{LC}$

  $\cdots$ increase. $\cdots$ decrease. $\cdots$ remain unchanged.

- If the inductance L is increased, the period of oscillation T will $\cdots$ $T \approx 2\pi\sqrt{LC}$

  $\cdots$ increase. $\cdots$ decrease. $\cdots$ remain unchanged.

Shown below are four current waveforms displayed as functions of time. Three of the waveforms correspond to the labeled resonator currents.



(7B) (2pts) Estimate the quality factor Q for the resonator.

$Q =$ 2.5

Following the dashed envelope for waveform D, it takes about 1.25 cycles for the waveform to drop to 20% of its original amplitude. It therefore takes about 2.5 cycles for the waveform to drop to 4% of its original amplitude. So, $Q \approx 2.5$.

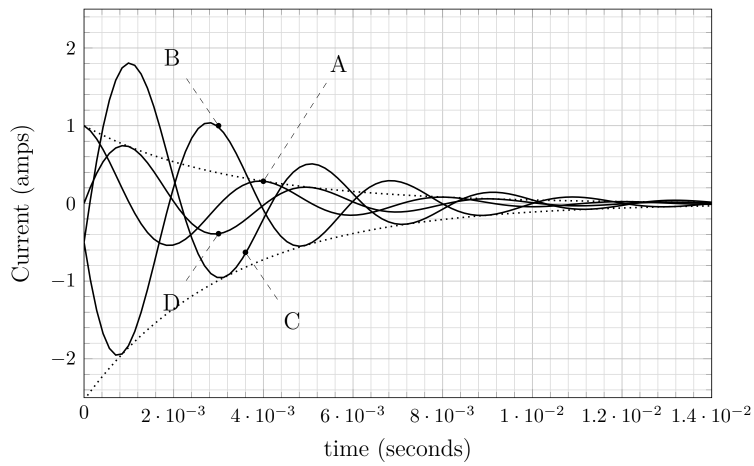
(7C) (2pts) Assuming that $R = 4\ \Omega$, estimate C and L.

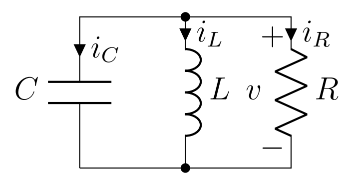
$C =$ 400 μF

$L =$ 1 mH

Note that $T \approx 0.004$ s. Combining the expressions for T and Q given above, $C = (QT)/(2\pi R) =$ 400 μF. From the expression for the period, $L = T^2/(4\pi^2 C) = 1$ mH.

(7D) (9pts) For each current listed below, identify the corresponding waveform in the figure of currents by circling the waveform letter, and provide a brief explanation of your reasoning in the box below. Both diagrams are repeated here for your convenience.





i_C → Waveform   A   ***B***   C   D

i_L → Waveform   A   B   ***C***   D

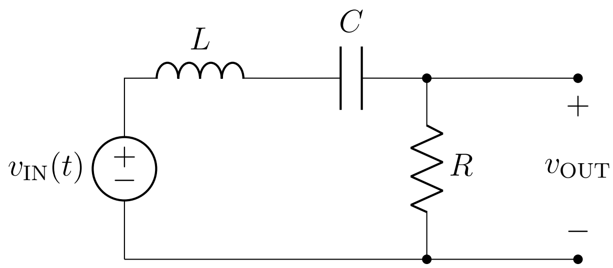
i_R → Waveform   ***A***   B   C   D

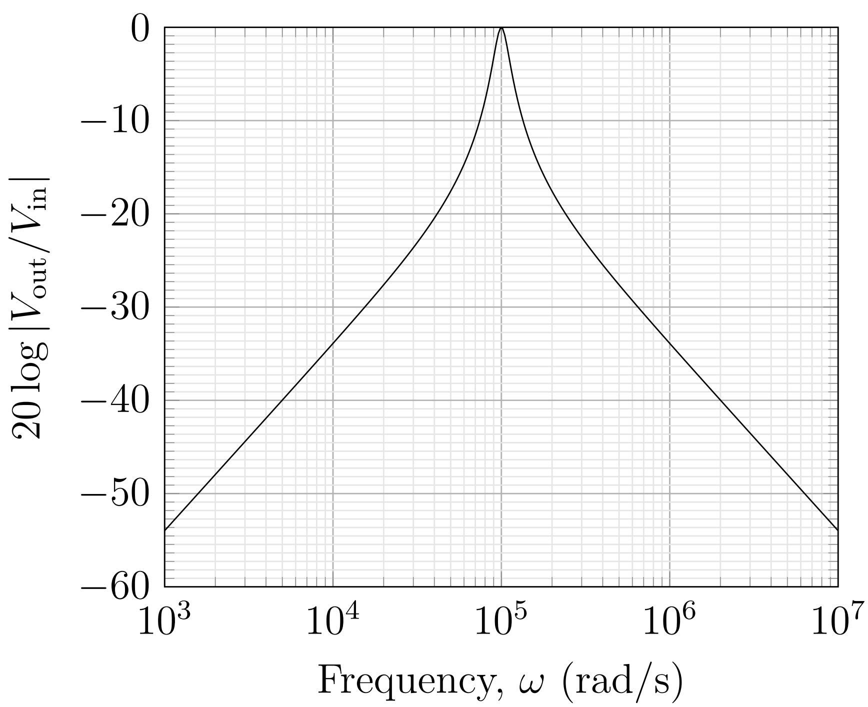
Explanation: Since the resonator is reasonably underdamped its two largest currents are i_C and i_L; i_R is small so as to have low loss. The two largest currents are B and C, so i_R must be either A or D. The positive peak of i_R occurs when v exhibits a positive peak, which in turn occurs at the end of the positive portion of i_C, that is, at the time of the maximum charge in the capacitor. Given that i_C must be either B or C, and that i_R must be either A or D, the only possible combination is for i_C to be B and i_R to be A. Then, i_L must be B. Alternatively, since the resonator is lightly damped, its operation can be viewed as being in a quasi-sinusoidal steady state. In the sinusoidal steady state, v, and hence i_R, would lead i_L and lag i_C, both by approximately 90 degrees. The only such arrangement of current waveforms is B leading A leading C. This yields the same assignment of labels to currents.

Worksheet (intentionally blank)

Problem 8, 16 pts: Ringing and Stepping

The frequency response of $|V_{\text{out}}/V_{\text{in}}|$ for a series RLC circuit is given below where V_{in} is the complex amplitude of a sinusoidal steady-state voltage v_{IN}, and V_{out} is the corresponding amplitude at the indicated output port.





(8A) (4pts) Use the graph on the previous page to estimate the Q factor of this circuit.

$$Q = \frac{\omega_\circ}{\Delta\omega}$$
$$\omega_\circ = 10^5 \text{rad/s}$$
$$\Delta\omega \approx 2.0 \times 10^4 \text{rad/s}$$
$$Q = \frac{10^5 \text{rad/s}}{2 \times 10^4 \text{rad/s}} = 5$$

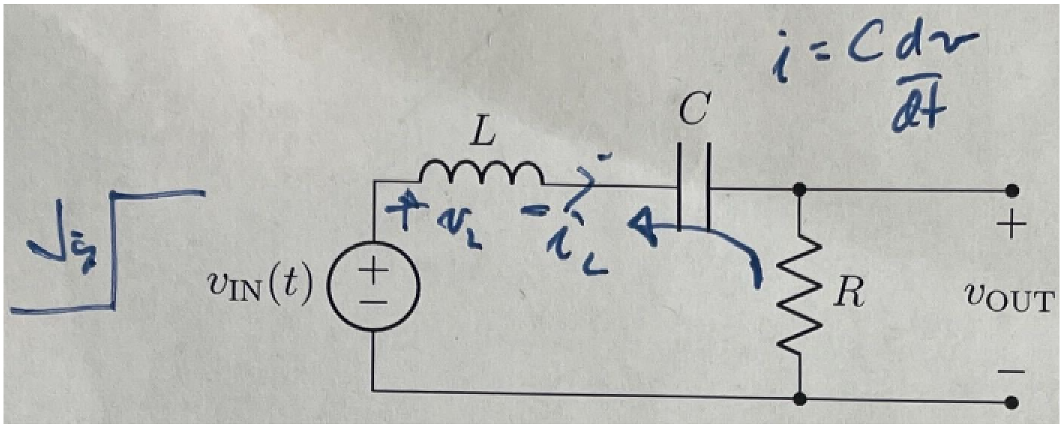
$Q = 5$

(8B) (4pts) Given that $R = 10\,\Omega$, estimate L, and C.

$$\omega_\circ = \frac{1}{\sqrt{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$
$$C = \frac{1}{R\omega_\circ Q} = 2 \times 10^{-7}\text{F}$$
$$L = \frac{1}{C\omega_\circ^2} = 5 \times 10^{-4}\text{H}$$

$L = 5 \times 10^{-4}\text{H}$ $\qquad$ $C = 2 \times 10^{-7}\text{F}$

(8C) (4pts) Assume that this circuit is driven by the unit step $v_{\text{IN}}(t) = V_{\text{S}}u(t)$ at $t = 0$ where $V_{\text{S}} = 1\,\text{V}$, and the capacitor and inductor are fully discharged for $t < 0$. Write an analytical expression for $v_{\text{OUT}}(t)$ including R, L, and C (as symbols, **NOT** using the numbers from the last part) for $t > 0$ in the form $v_{\text{OUT}} = V_{\text{out}}e^{-\alpha t}\cos(\omega_{\text{D}}t + \phi) + K$ (i.e. determine $V_{\text{out}}, \alpha, \omega_{\text{D}}, \phi$, and K). The circuit schematic is repeated from part A for your convenience:



$i_L(0_+) = 0$

$v_c(0_+) = 0$

$v_L(0_+) = V_S = L\dfrac{di_L}{dt}$

$i_L(t) = I_\circ e^{-\alpha t}\cos(\omega_\circ t + \phi_L) + K_L$

$i_L(0_+) = 0 \Rightarrow I_\circ \cos\phi_L = 0 \Rightarrow \phi_L = \pm\dfrac{\pi}{2}$

$\alpha = \dfrac{R}{2L}$

$\omega_\circ = \dfrac{1}{\sqrt{LC}}$

$\omega_\circ = \sqrt{\omega_\circ^2 - \alpha^2}$

$i_L(\infty) = 0 \Rightarrow K_L = 0$

$$\frac{di_L}{dt} = \frac{V_S}{L} = I_\circ(-\alpha\cos\phi_L - \omega_D\sin\phi_L)$$

$$\Rightarrow \frac{V_S}{I_\circ L} = -\alpha\cancelto{0}{\cos\phi_L} - \omega_D\cancelto{\pm 1}{\sin\phi_L}$$

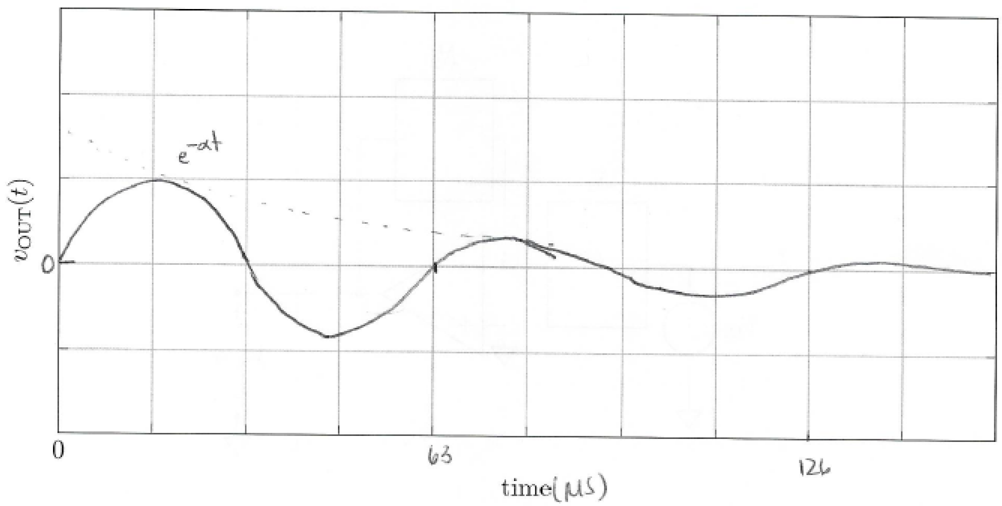
$$\Rightarrow \frac{V_S}{I_\circ L} = \mp\omega_D \Rightarrow I_\circ = \frac{V_S}{\mp\omega_D L}$$

$$v_{\text{out}} = i_L \cdot R = I_\circ \cdot Re^{-\alpha t}\cos\left(\omega_D t + \cancelto{\pm\frac{\pi}{2}}{\phi_L}\right)$$

Sign choice because $\frac{dv_\circ}{dt}(t = 0) > 0$ by current direction.

$v_{\text{OUT}}(t) = I_\circ Re^{-\alpha t}\cos\left(\omega_D t - \frac{\pi}{2}\right)$

(8D) (4pts) Sketch the time domain response of $v_{OUT}(t)$. Make sure to label your sketch with the initial value of v_{OUT}, final value of v_{OUT}, and any periods of oscillation or timescales of exponential decay that appear in the problem. For full credit show all the relevant calculations.



$Q = 5$

$Q = \frac{\omega_0}{2\alpha} \rightarrow \alpha = \frac{\omega_0}{2Q} = \frac{10^5}{2(5)} = 10^4$ rad/s

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.9 \times 10^4$ rad/s

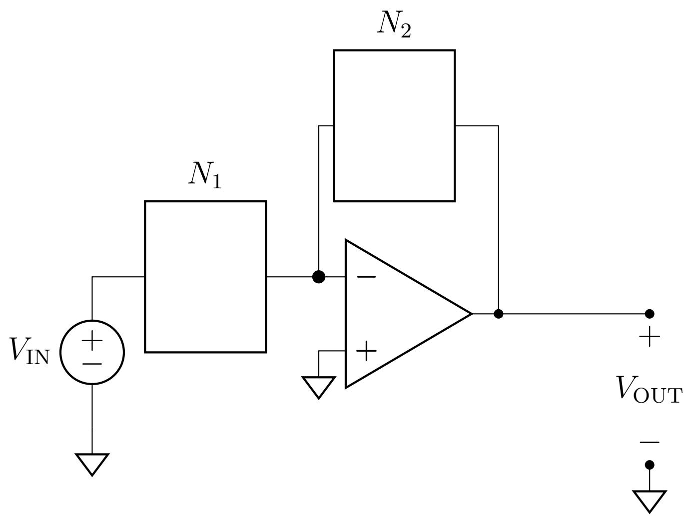
$T = \frac{2\pi}{\omega_d} = 63\ \mu s$

$v_{OUT}(t=0) = 0$ inductor looks like open circuit

$v_{OUT}(t \rightarrow \infty) = 0$ capacitor looks like open circuit

Problem 9, 14 pts: New First Order

Shown below is the topology of an op-amp-based filter having sinusoidal steady-state input with complex amplitude V_{IN} and output amplitude V_{OUT}. The filter has two "to-be-designed" networks, N_1 and N_2.



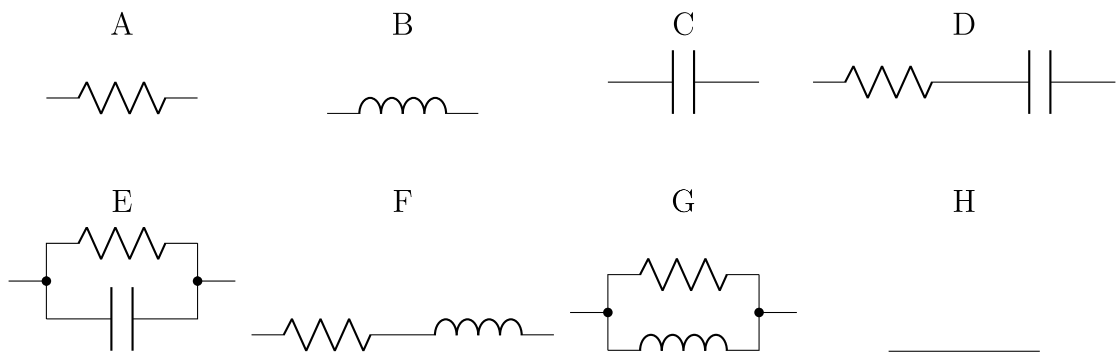
(9A) (2pts) Write an expression for the transfer function, $H = \frac{V_{\text{out}}}{V_{\text{in}}}$, assuming that the impedances of networks N_1 and N_2 are Z_1 and Z_2, respectively.

Inverrting amplifier topology:

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = -\frac{Z_2}{Z_1}$$

$H = -Z_2/Z_1$

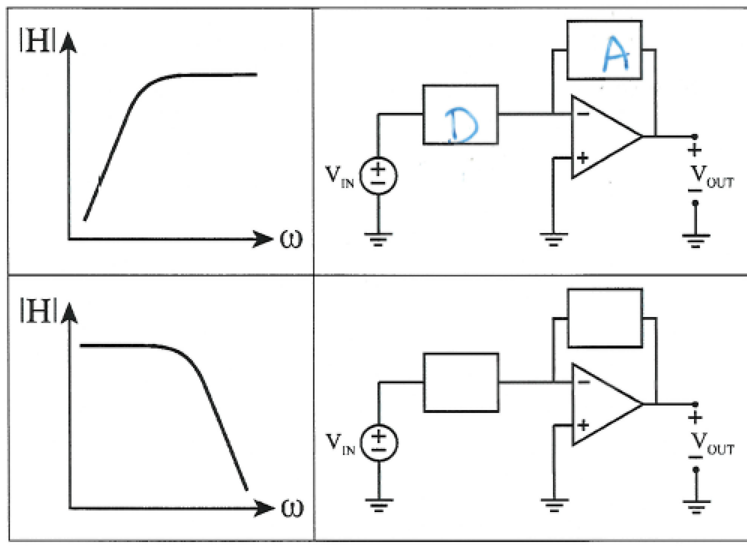
(9B) (12pts) For each of the magnitude ($|H| = \left|\frac{V_{OUT}}{V_{IN}}\right|$) plots provided, select an N_1 and N_2 from the list of options that will implement the desired response. Note that there may be more than one correct design.



Fill in each box indicated below with the appropriate letter label from the table above.

(a)

(b)



(a): $\Rightarrow H \propto \frac{\omega\tau}{1+j\omega\tau} = \frac{\omega L/R}{1+j\omega L/R}$

(b): $\Rightarrow H \propto \frac{1}{1+j\omega\tau}$

if $\tau = RC$

$\Rightarrow \frac{1}{1+j\omega RC}$

(a) cont'd

$= \frac{j\omega L}{j\omega L+R} \Rightarrow (F, B)$

or $H \propto \frac{\omega RC}{1+\omega RC}$

$= \frac{R/(1/\omega C)}{1+R/(1/\omega C)}$

$= \frac{R}{\frac{1}{\omega C}+R} \Rightarrow (D, A)$

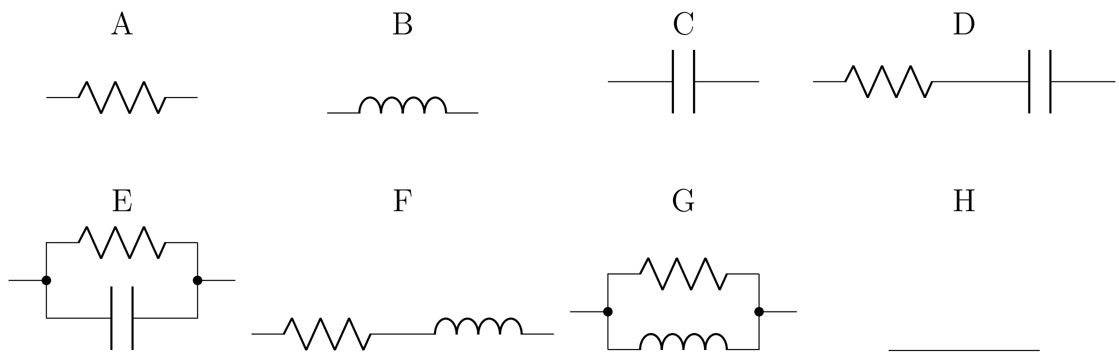
(b) cont'd

$= \frac{1/j\omega C}{1/j\omega C + R} \Rightarrow (D, C)$

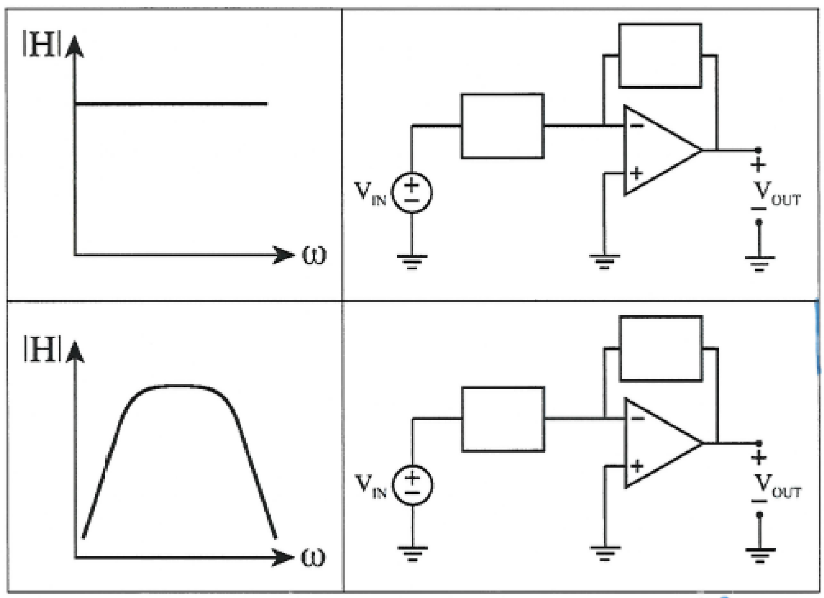
or if $\tau = L/R$

$\Rightarrow = \frac{1}{1+j\omega L/R} = \frac{R}{R+j\omega L}$

$\Rightarrow (F, A)$



Fill in each box indicated below with the appropriate letter label from the table above.



H = constant

$\Rightarrow |Z_1| = |Z_2|$

$\Rightarrow$ (A,A), (B,B)

etc.

|H| must be in the form $\left|\frac{Z_2}{Z_1}\right|$ where $|Z_2|$ either goes down at high frequency (1) or low freq (2) & $|Z_1|$ goes up at low freq. (1) or high freq (2).

$\Rightarrow Z_2 \equiv$ (1)  (2) 

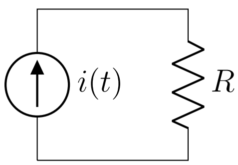
$Z_1 \equiv$ (1)  (2) 

$\Rightarrow$ case (1) $\equiv$ (D, E) case (2) $\equiv$ (F, G)

Problem 10, 12 pts: Fight the Power

Shown below are three networks involving the same current source, and separately a resistor, a capacitor and an inductor. All three networks are at rest prior to $t = 0$. For $t \geq 0$, the current source in all three networks sources a ramping current $i(t) = It/T$. For each network, determine the power $P(t)$ produced by the source for $t \geq 0$, and the total energy $E(T)$ delivered by the source by the time $t = T$.

(10A) (4pts)

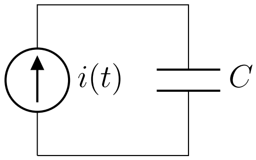


$P(t) = Ri^2 = RI^2t^2/T^2$

$E = \int_0^T P(t)\ dt = RI^2T/3$

$P(t) = RI^2t^2/T^2$	$E(T) = RI^2T/3$

(10B) (4pts)



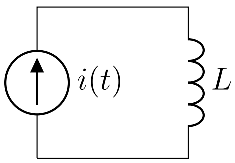
$v(t) = \frac{1}{C}\int_0^t i(s)ds = (It^2)/(2CT)$

$P(t) = v(t)\ i(t) = (I^2t^3)/(2CT^2)$

$E = \int_0^T P(t)\ dt = (I^2T^2)/(8C)$

$P(t) = (I^2t^3)/(2CT^2)$ $E(T) = (I^2T^2)/(8C)$

(10C) (4pts)



$v(t) = L\ di/dt = LI/T$

$P(t) = v(t)\ i(t) = LI^2t/T^2$

$E = \int_0^T P(t)\ dt = LI^2/2$

$P(t) = LI^2t/T^2$ $E(T) = LI^2/2$

Worksheet (intentionally blank)

Worksheet (intentionally blank)