# 6.200 Notes: Dependent Sources

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Up until now all device constitutive relations have been local. Namely, the only variables that appear in the relations are local to the branch. This situation is suitable to a wide range of simple electronic devices where the device physics acts locally (a current flows, forcing an electrostatic potential to develop across the resistor, as is the case in a resistor).

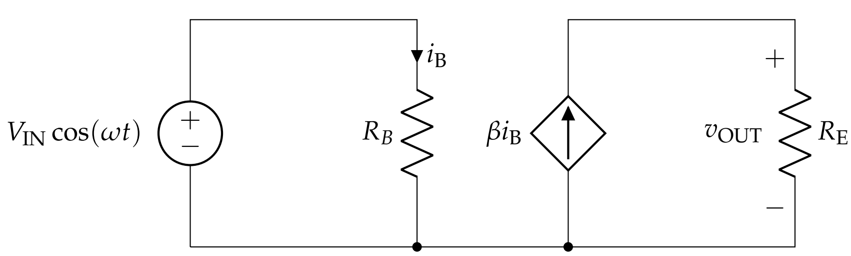
However, significantly more complicated devices can be developed in which electromagnetic fields, heat, and diffusion of electronic carriers can force one part of a circuit to impact another. Such behavior can cause signals to be transferred between ports and interesting switching behavior, where current or voltage in one part of the device can impact how current flows in a different part. Electrically, this can be modeled as a two-port device, in which current or voltage in one port is controlled by current or voltage in another. However, such a device is intrinsically non-local. We need to be able to create constitutive relations in which the branch variable is dependent on the value of a branch variable in a totally separate branch.

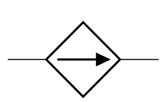
This need is filled by **dependent sources** (also called **controlled sources**), which break various assumptions we have made and thus need some special methods to handle in circuit analysis.

## Analysis with a Dependent Source

Before we talk about the theory, let's dive in with an example, because they actually can be fairly straight forward in the right context.[1]

[1] This particular example is based on a model of a bipolar junction transistor (BJT).




As the diagram suggests, the  symbol represents a current source with strength $\beta i_B$ where $i_B$ is the indicated branch current and $\beta$ is a device parameter.

Because the two loops are only connected by a single wire, no current can flow between them (there would be no return path for the current). That allows us to analyze the circuits separately, finding that $i_B = V_{IN}(t)/R_B$ and thus the current source has strength $\beta V_{IN}(t)/R_B$. From this we can calculate the voltage $v_{OUT} = \beta V_{IN}(t) R_E / R_B$.

If we were to imagine a purpose for this device, it could be a linear amplifier, namely it takes as its input $V_{IN}\cos(\omega t)$ and gives as its output $\beta R_E/R_B V_{IN}\cos(\omega t)$ which, if $\beta R_E/R_B > 1$ could have a larger amplitude, so this device is a kind of **amplifier**.

We call the multiplying coefficient $\beta R_E/R_B$ the **gain** of the amplifier, and give it the symbol $G$.

You may wonder where this extra signal "came from." After all, we're not used to signals getting bigger without some power being provided. The answer is that the dependent current source can supply power to the current (by our definition, it can have $p < 0$, and thus can have valid states in the bottom right and upper left quadrants of the $i$-$v$ axes.

The dependent source shown here has branch variables just like any other branch, and has a constitutive relation as well. The really unique property of this device is that its branch relation depends on $i_B$ which is the branch variable of another branch! We call this device a "current-controlled current source." This device, and its cousins that we will discuss shortly, are all particularly important in describing amplification.

## *Assumptions*

Just like any other circuit element, dependent sources all have branch variables $i$ flowing through them and $v$ across them, and the passive signs convention must always be obeyed. So in many ways, it looks just like a regular device.

Dependent sources also have a constitutive relation, but it is a constitutive relation unlike any we've see so far: these constitutive relations can include variables from outside the branch. We call these variables **non-local** by which we mean they are not local to the device.

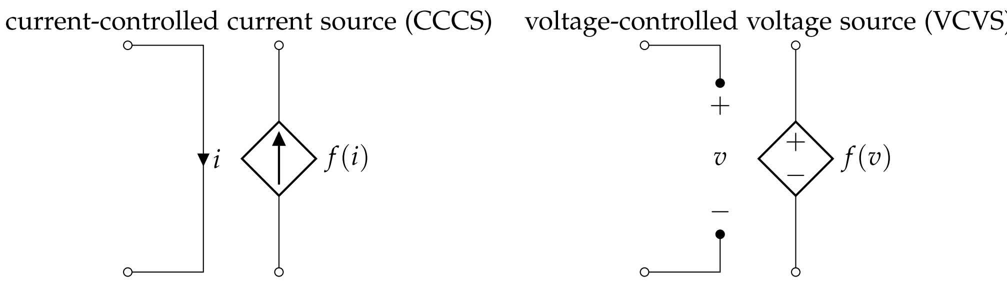
In addition, the devices have parameters, which typically appear as coefficients and offsets.

The non-locality permits dependent sources to model some remarkable physical effects, such as heat transfer, electromagnetic signal transfer, and many many other fascinating effects. They thus drastically broaden the scope of circuits beyond those one typically encounters in an introductory physics class.

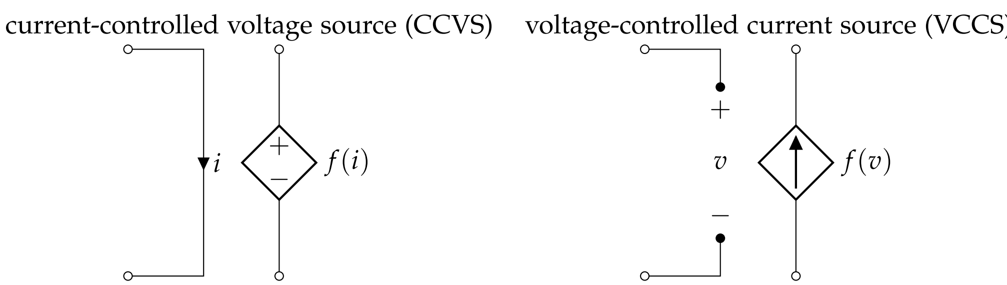
## *Bestiary of Dependent Sources*

There are essentially four different types of dependent sources. Each of these has two ports, one which specifies the control variable (an open in the case of a voltage, and a short in the case of a current). There are also two types of sources, current and voltage, thus there are a total of 4 devices.

1. Current-Controlled Current Source and its dual, the Voltage-Controlled Voltage Source. These devices involve using current to control a current source and voltage to control a voltage source.



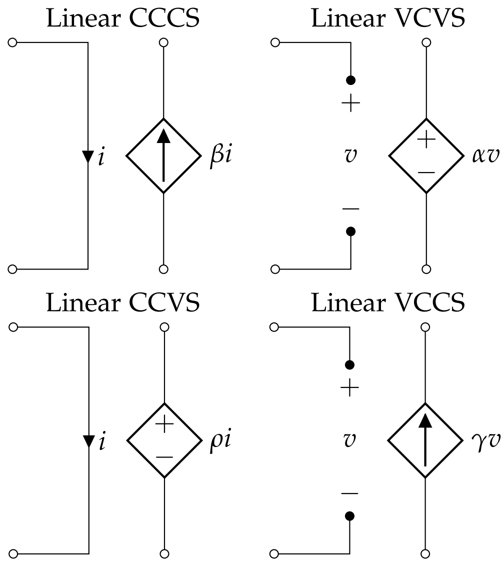

2. Current-Controlled Voltages Source and its dual the Voltage-Controlled Current Source. These devices use current to control a voltage source and a voltage to control a current source.




## *Superposition with a Dependent Source*

If the constitutive relation of a dependent source is linear (meaning the source strength varies linearly with the control variable), then one can use superposition and Thevenin and Norton theorems with dependent sources. But this is a big “if,” so do make sure you’re confident that the source is linear before embarking.

Here is a table of all possible linear dependent sources:




When doing superposition, it is best not to turn off the dependent source. Because the dependent source has a constitutive relation that depends on a branch variable, it is not truly an input to the linear function. If you do turn it off, you'll have to do a nasty back-substitution step, and it will make a mess of your problem. Better to avoid.

## *Thevenin/Norton with a dependent source*

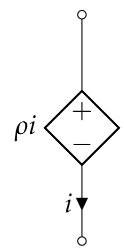
As mentioned above, Thevenin and Norton's theorems only apply to circuits containing linear dependent sources.

The test-source method of Thevenin's theorem will always work with dependent sources, and needs no further discussion.

Another method that will almost always work is determining $i_{sc}$ and $v_{oc}$, setting $i_{sc}$ to $I_N$ and $v_{oc}$ to $V_{TH}$ and calculating $R_{TH}$ from $V_{TH}/I_N$. This approach will only fail if $I_N = V_{TH} = 0$ or if $R_{TH} = 0$ or $\infty$.

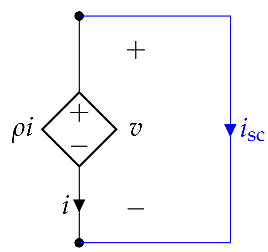
Finally, there is a method of determining $R_{TH}$ wherein all the **independent sources** (i.e. regular current and voltage sources) are turned off and the equivalent resistance at the port is determined. Because this method neglects the possible influence of the external circuit on the dependent sources, it will not work with networks that contain dependent sources. I suggest you don't use it for anything but the simplest of circuits.

Here is an example which clarifies why dependent source should not be set to zero.



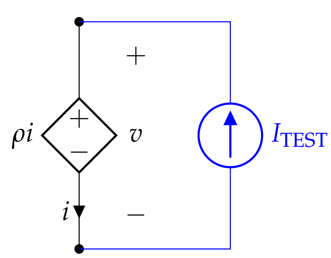

What is the Thevenin equivalent circuit of this device at its terminals? If we were to naively (and incorrectly) turn off the dependent source to analyze it, we would conclude $R_{TH} = 0$ and $V_{TH} = 0$ (i.e. it is a short circuit).

In our next attempt, we might try to short the device and determine $i_{sc}$.

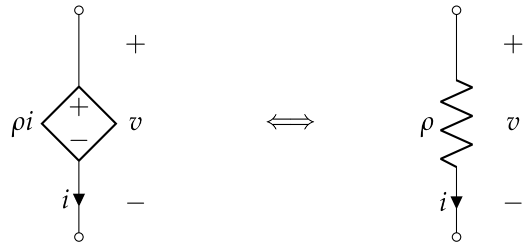



In such a case we know the voltage across the device is zero, thus $-\rho i_{sc} = 0$ which implies $i_{sc} = 0$. This might be ok–afterall, plenty of circuits have $I_N = 0$. Unfortunately, as soon as we try to do something similar for the open-circuit voltage, we realize that an open circuit would mean $i = 0$ which implies $\rho i = v_{oc} = 0$. Now we have discovered the pathological case where $I_N = 0$ and $V_{TH} = 0$ and as a result the relation we normally use to discover $R_{TH} = V_{TH}/I_N$ cannot be used.

Instead, the right way to have analyzed this circuit was to apply a test source $I_{TEST}$




We can analyze this circuit fairly easily by noticing that $i = I_{TEST}$ (that is always by construction when using the test source method), and the output $v = \rho I_{TEST} = \rho i$; thus, the constitutive relation actually is just Ohm's law for a resistance $\rho$. From this, we conclude indeed $V_{TH} = I_N = 0$, and $R_{TH} = \rho$ so the equivalent circuit is just:




## *Amplification*

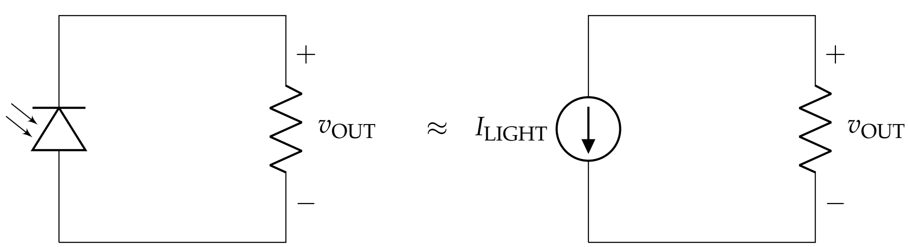
There are many situations where small signals must be increased in order to be usefully processed. Many analog to digital converters add noise or can't discriminate small levels, and if small signals are input, they will be lost in that noise. In these situations, amplification (increasing the signal amplitude) is essential.

Even more significant, access to amplification allowed robust transmission of information across great distances and in complex multi-component and high-noise environments. These achievements in turn ushered in technology such as electronic computation and communication. It is not an exaggeration to say that we owe the end of the industrial age and the start of the information age of human history to amplification.

The term "amplification" comes from "amplitude", which refers to the deviation of a sinusoidal signal from zero, but amplification generally refers to any increase in a signal's magnitude (whether from zero, or from some offset that is agreed to represent zero).

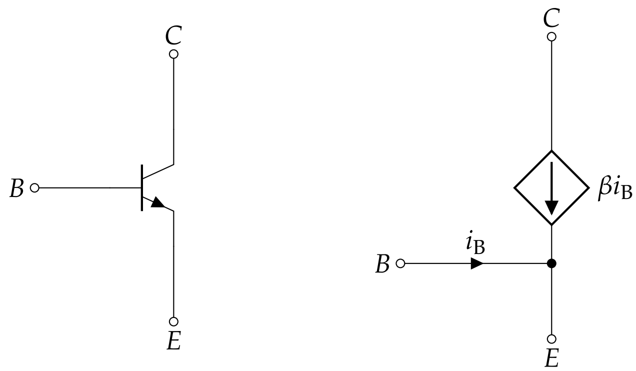
A classic example is the photodiode.[2] We can model a photodiode as a current source whose strength is a function of the brightness of the light incident on it. The simplest way to envision reading out the signal of a photodiode is through a resistor as shown here:

[2] Think of the remote controls for your TV. These all send signals to a photodiode in your TV and/or DVR.

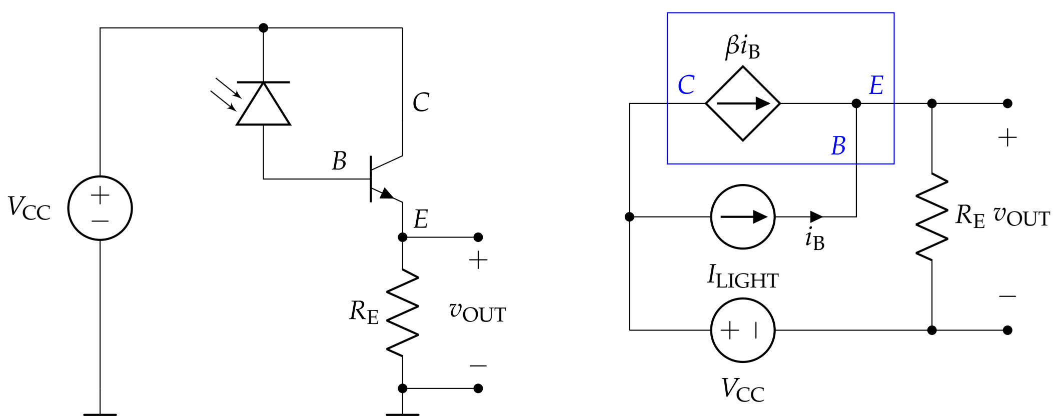



There are a lot of problems with this readout, one being that the readout resistor needs to be quite large to get a substantial voltage, which means circuits that use it will tend to be slow (for reasons we'll discuss in future lectures). To make a fast amplifier, we can use a device called a Bipolar Junction Transistor, which is a three-terminal device and that we are going to model as a linear CCCS.

A bipolar junction transistor is a 3-terminal device where one of the terminals (the "Emitter") is shared between the control port and the source. A rough equivalent circuit diagram is provided here:




This circuit can be applied directly to the problem of amplifying a photodiode basically by applying the current from the photodiode to the base terminal of the transistor. The result is that instead of $i_B$ traveling through the readout resistor, $\beta i_B$ travels through the readout.




where we have modeled the photodiode as a simple ideal current source whose strength $I_{LIGHT}$ is proportional to the light intensity. Our model of the BJT is now contained in the blue box, with the terminals labeled (notice it has been rotated 1/4 turn counterclockwise relative to thee original layout).

Let's look at $v_{OUT}$ in this case, compared to the case with only the resistor above. Performing KCL at the '+' terminal, we find $\beta i_B + i_B - v_{OUT}/R_E = 0$. Solving for $v_{OUT}$ we find

$$v_{OUT} = (\beta + 1) i_B R_E.$$

For a typical BJT, $\beta \sim 200$; thus, the resulting output voltage is significantly increased relative to the case of a simple resistor readout. This increase can be used in a number of ways (e.g. to decrease $R_E$ while maintaining an acceptable output signal, and thus obtain a lower output impedance for the circuit–recall that lower output impedances are preferred for voltage signals).

## *Conclusions*

Dependent sources permit circuit models that describe non-local interactions across devices. The consequence is a device that can be used to provide signal amplification.

Techniques we've learned in analysis so far such as superposition and equivalent circuits have to be modified somewhat to account for dependent sources. In particular, **when performing superposition, dependent sources should not be manipulated in any way, they need to be included in the analysis of each sub-circuit. Similar, when performing Thevenin analysis, dependent sources should not be set to zero.**

One word of caution: a device cannot be sensitive to a node potential because the choice of ground (0V reference node) is arbitrary in a circuit. As a result, node potential is not a valid control variable. Only branch voltages (i.e. potential differences) or branch currents can be used as control variables.

## *Glossary*

*Amplifier* Circuit network designed to accept an input signal and increase its amplitude (typically a time-dependent signal like a sine or cosine).

*Base* BJT terminal which carries control-current signal for dependent current source.

*Bipolar Junction Transistor (BJT)* Three-terminal device that behaves in certain circumstances as a current-controlled current source.

*Collector* BJT terminal where dependent-current-source input originates.

*Controlled Source* Synonym of dependent source.

*Current-Controlled Current/Voltage Source* As it sounds, a dependent current/Voltage source that is controlled by the current flowing in another branch in the circuit network.

*Dependent Source* A source whose strength depends on a branch variable of the circuit (not necessarily local to the device).

*Emitter* BJT terminal where dependent-source output is directed.

*Gain* Unitless ratio of output signal to input signal of a system. In an amplifier, gain is always greater than one.

*Independent Source* A source whose strength is set locally (i.e. a "normal" ideal voltage or current source)

*Local Branch Variable* A branch variable defined on the given branch.

*Non-local Branch Variable* A branch variable that is not on the same branch as the given device.

*Photodiode* Two-terminal device that produces current when light is shined on it. It can be modeled as a current source for our purposes here.

*Voltage-Controlled Current/Voltage Source* As it sounds, a dependent voltage source that is controlled by the current flowing in another branch in the circuit network.

## *Acknowledgements*

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