6.200 Notes: Introduction to Op-Amps

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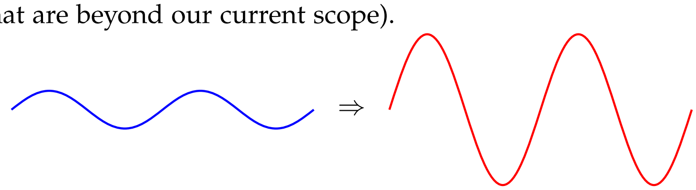
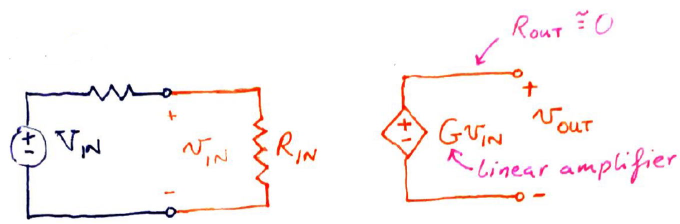
March 9, 2023

As discussed in the notes on dependent sources, signal amplitude is a key limitation of modern sensing and information processing. But with the simple resistors and ideal sources shown so far, sensor signals cannot be increased. Furthermore, the dependent sources we've shown are not particularly good amplifiers (the BJT behaves as a current-controlled current source, but with significant limitations that are beyond our current scope).



So how can we increase signal amplitudes without degrading the information carried in the signal? Let's suppose we have our signal in the form of a voltage (V_{IN}) and we wish to output an increased amplitude. Our ideal circuit might look something like this:



where we would like the input impedance of the amplifier R_{IN} to be infinite (i.e. an open circuit) so that the signal is maintained as it enters the amplifier, and we would like the output impedance (R_{OUT}) to be zero so that it can drive an arbitrary load at the next stage of the circuit.

Such a device doesn't exist as a single lumped element, but it can be constructed. The basic device is called an Operational Amplifier, or "Op-Amp."[1]

Although amplification may be the motivation we focus on, we shall see that op-amps have many uses beyond just amplification. We shall touch on those uses in our next set of notes, but for now we focus on deriving the basic properties of op-amps and explaining how to best work with them.

[1] A simple and not very good op-amp can be designed with a handful of the BJTs we discussed already, but unfortunately, is beyond our current scope. More advanced courses in electronics are required in order to design such circuits.

Op-Amp Intro: Comparator

An op-amp is typically drawn as a triangle with two inputs (labeled '+' and '-' and referred to as the '+' (non-inverting) and '-' (inverting) inputs). The op-amp senses the difference between the potentials at these inputs (i.e. the branch voltage between them) and amplifies the result. However, the amplification is so extreme, it can be considered infinite for practical purposes (at least for low signal frequencies). In fact, by the time the input is even only $\sim 10\,\mu\text{V}$ (a tiny voltage–typical voltage noise is in the mV range), the output voltage already exceeds the maximum possible.

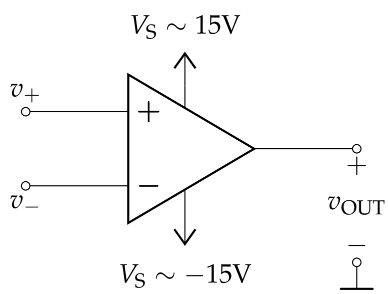


Figure 1: Schematic representation of an op-amp illustrating the power supply (V_S) and the inputs (v_+ and v_- and the output v_{OUT}. Some details, e.g. max and min values, depend vary from device to device—spec sheets should be consulted for each device use.

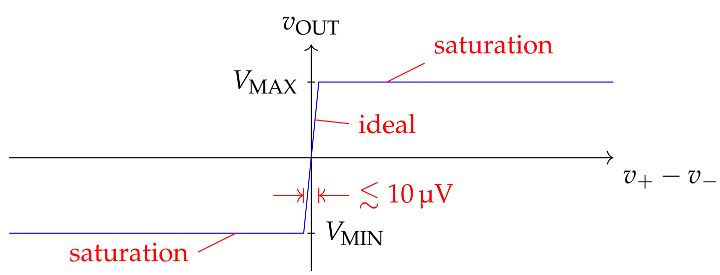


Figure 2: Plot of op-amp "transfer function" (see main text) relating output to input. The branch labeled "ideal" is not very stable or controllable, and is not really useful as is.

The transfer function of an op-amp v_{OUT} vs. $(v_+ - v_-)$ relates the output to the input. The massive amount of gain referenced in the previous paragraph means that the transfer function appears to be nearly a step. This type of transfer function is characteristic of a device called a **comparator**. A comparator operates by referencing an input value to a fixed threshold. If the value is above that threshold, it provides one output, if the value is below that threshold, it provides another.

The op-amp shown above compares the v_+ value to the v_- value and outputs V_{MAX} if $v_+ > v_-$ and V_{MIN} if $v_+ < v_-$. When the two inputs are equal, the output is undefined (because of noise, it will flop back and forth rapidly. We will improve on this basic comparator design in later chapters.[2]

[2] Op-amps in general should not be used as comparators, although they can operate as such in a pinch—bespoke comparators are available that provide better stability and speed performance than a generic op-amp can provide.

If you zoom in near the origin of the transfer function drawn above, the step would not be linear (although it looks like it in our sketch), in practice it has curvature and moves a lot based on tiny changes in temperature and is generally useless as in amplifier on its own. However, this situation can be fixed using **feedback**, resulting in an amplifier that is both linear and stable.

Negative Feedback

The key to getting good results out of an op-amp is **negative feedback**: when signals go up, negative feedback acts to reduce that increase.

Negative feedback is what it sounds like—an effect that acts to suppress an input to reduce a given signal. Imagine you have a rambunctious toddler named "Oppy" in your household who likes to yell at the top of their lungs. Every time they talk too loud, you take away some screen time.[3] The louder they go, the more screen time they lose. That's negative feedback. If Oppy is our op-amp, and the loud voice results from the huge gain, then the negative feedback can make our op-amp output more manageable.

[3] Feedback to young children should probably be more immediate than taking away screentime, BTW, but the analogy is still useful.

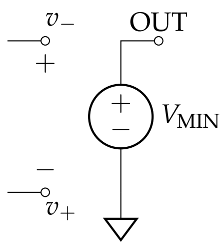
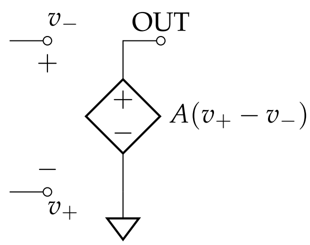
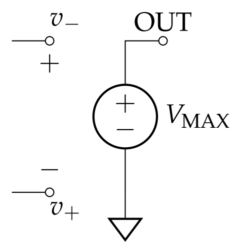
In our case, we achieve the negative feedback by taking our amplified output and using some means (e.g. a voltage divider) to apply a fraction of it back to our input. Because the gain is so enormous, the strength of the feedback is huge, and the voltage difference at the input remains near zero (Oppy is barely whispering).

We'll give another, more quantitative, picture of feedback later in our notes, but for now we have what we need to build our first op-amp model.

Op-Amp Circuit Model

An op-amp has three possible circuit models. Which model should be used depends on the output.[4]

[4] As a result, it is difficult to know which model to use before having solved the problem, leading to an apparent contradiction. The way this is typically addressed is by assuming a given output, solving the problem, and then verifying that your assumption was correct. This approach is known as the **method of assumed states** and amounts to a **self-consistency** check of your solution.

Neg. Saturation	Ideal Regime	Pos. Saturation
$v_{OUT} = V_{MIN}$	$V_{MIN} < v_{OUT} < V_{MAX}$	$v_{OUT} = V_{MAX}$
		

A is the open-loop amplifier gain, and is typically $\sim 10^7$.

The $v_{OUT} = V_{MAX}$ and $v_{OUT} = V_{MIN}$ regimes are always present if negative feedback is insufficient, or if the input is too large. V_{MIN} and V_{MAX} are called the rails, and represent the minimum and maximum output voltages of the device. However, these regimes are more difficult to analyze, so we will start by focusing in on the ideal op-amp region in the middle.

1. Whenever this model applies, $v_+ \approx v_-$, i.e. the two inputs have approximately equal potential.

   This result can be derived from the properties of the model. Firstly, the model only applies when $V_{MIN} < v_{OUT} < V_{MAX}$. V_{MAX} and V_{MIN} are typically $\pm \sim 5 - -15$ V. As a result, the output voltage of an op-amp is in a reasonable range of a few volts around 0.

   In this regime, $v_{OUT} = A(v_+ - v_-)$ where A is the gain of the op-amp (typically called the **open loop** gain). We can then calculate that $v_+ = v_- - v_{OUT}/A$. But as we've already seen, A is huge (typically $\sim 10^6$ – 10^8), so $v_{OUT} \sim 10^{-7}$ V. From this, we conclude that $v_+ \approx v_-$.

   Note however that, although $v_+ \approx v_-$, $v_+ \neq v_-$. The tiny difference is conceptually important—without it the output would be identically zero.

2. Remember that this case will only be possible if negative feedback is applied to the op-amp in the form of applying some component of the output voltage back to the input—specifically the negative input.

Using an Ideal Op-Amp

Let's see how we can use negative feedback to make a good linear amplifier.[5]

[5] The purpose of this approach is to derive the properties of an op-amp in the presence of negative feedback. Once these properties are derived, we will not use this method for working with actual op-amp circuits.

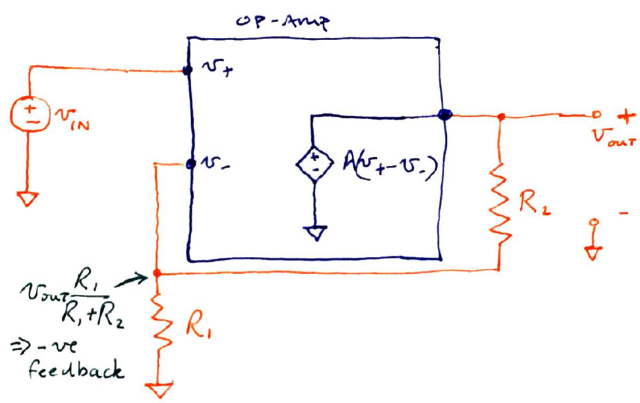


Figure 3: Op-amp model with negative feedback applied. The op-amp is drawn in the box, and external circuit components are drawn in orange. The op-amp supply voltages (V_S) are now drawn as they do not interact with the circuit in meaningful ways—they act like a "wall plug," supplying power so that the op-amp can work its magic, but otherwise irrelevant.

We are going to assume an ideal op-amp model. Because we do not have numbers for this model, we will not be able to check this assumption at the end, but in general one could do so by making sure v_{OUT} was within the V_{MAX}, V_{MIN} range. Under these assumptions, we have:

$$v_{OUT} = A(v_+ - v_-)$$
$$= A(v_{IN} - v_{OUT}\frac{R_1}{R_1+R_2}).$$

Solving for v_{OUT} we find

$$(v_{OUT} + Av_{OUT}\frac{R_1}{R_1+R_2}) = Av_{IN}$$
$$v_{OUT}(1 + \frac{AR_1}{R_1+R_2}) = Av_{IN}$$
$$\Rightarrow v_{OUT} = v_{IN}\frac{A}{1+\frac{AR_1}{R_1+R_2}}. \quad (1)$$

Assuming

$$\frac{R_1+R_2}{R_1} \ll A$$

(which is pretty reasonable because $(R_1+R_2)/R_1$ is typically $\lesssim 10$, and A is typically $\sim 10^7$,

$$\Rightarrow \frac{AR_1}{R_1+R_2} + 1 \approx \frac{AR_1}{R_1+R_2}$$

because $\frac{AR_1}{R_1+R_2} \gg 1$.

Returning with this result back to (1)

$$\Rightarrow v_{OUT} \simeq \frac{v_w A(R_1+R_2)}{AR_1} \simeq \frac{R_1+R_2}{R_1}v_{IN}$$

where $\frac{R_1+R_2}{R_1}$ is now a reasonable (~ 10) gain which doesn't depend on A.

Let's consider what the new modified transfer function of our amplifer has become.

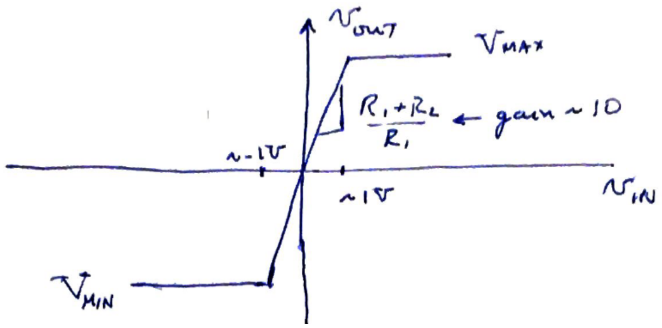


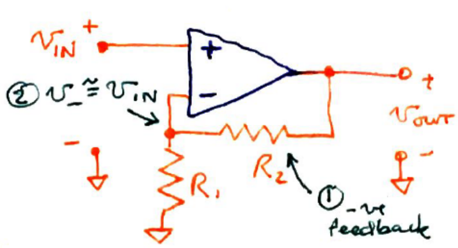
Figure 4: Transfer function of non-inverting op-amp amplifier generated by using negative feedback. The input range is now much more reasonable (~ 1 V, depending on the circuit parameters), and the gain does not depend on the details of the op-amp open-loop gain.

The circuit has resulted in a much more reasonable and stable amplifier. Notice that A does not appear in the final transfer function. As a result, if A varies due to small drifts, the transfer function won't move. Notice also that we clip the linear portion of our model at the voltage rails where the device will act as a simple voltage source.

Ideal Op-Amp Analysis

As mentioned above, when actually analyzing an op-amp circuit with negative feedback, one does not have to use the full dependent-source model for the op-amp. The clue that this may be unnecessary is the fact that A does not appear in the final transfer function of the amplifier. Here we show a much faster and easier way to analyze exactly the same circuit we showed earlier that will apply generally to ideal op-amp circuits. Remember, if numeric values are given, one should typically check at the end that v_{OUT} lies between the output rails, as required by the ideal op-amp model. If numeric values are not given, it is impossible to determine whether the assumption is valid, and so one must simply rely on the directions given in the problem.

In this case, we will assume the ideal op-amp model applies.



1. First, locate the negative input and confirm that negative feedback exists. For negative feedback to apply, a fraction of the output must appear at the negative terminal. If there is feedback to both terminals, the fraction on the negative terminal must be larger than the fraction on the positive terminal. In this case, clearly negative feedback is applied.

2. Apply $v_+ \approx v_-$ to establish v_- input terminal potential based on the output and/or v_+. Note that although they are equal, current cannot flow between them. Also note that no current can flow into or out of either the + or - terminals. In this case, we can conclude that $v_- \approx v_{\text{IN}}$.

3. Use circuit analysis to calculate v_{OUT}.

$$\frac{v_{\text{OUT}} R_1}{R_1 + R_2} = v_{\text{IN}}$$

$$\Rightarrow v_{\text{OUT}} = \frac{R_1 + R_2}{R_1} v_{\text{IN}}.$$

Note because $V_{\text{MIN}} < v_{\text{OUT}} < V_{\text{MAX}}$ (ideal op-amp assumption) this solution only works for

$$\frac{V_{\text{MIN}} R_1}{R_1 + R_2} < v_{\text{IN}} < \frac{V_{\text{MAX}} R_1}{R_1 + R_2}.$$

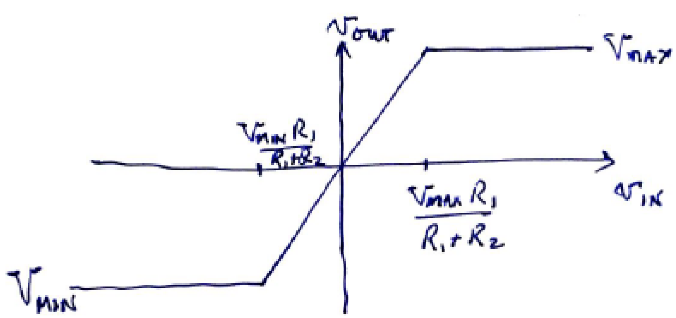
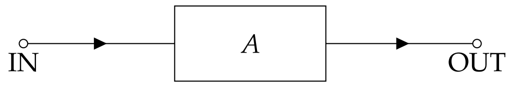


Figure 5: Transfer function for non-inverting op-amp amplifier based on ideal op-amp analysis.

How Is All This Possible

It is very legitimate at this point to ask how is all this possible? After all, negative feedback seems to be a bit of a cheat—an answer to all our problems that is hard to quantify and understand.

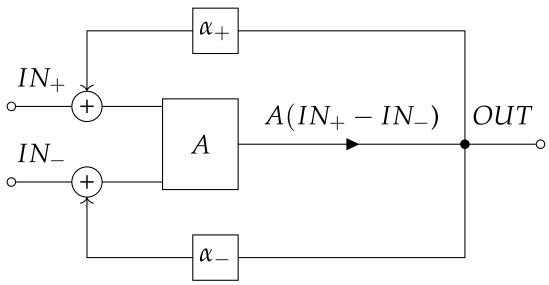
Feedback is genuinely hard to wrap your head around. Basically, we're used to thinking in a more stimulus-response style, where actions happen and we see consequences, as shown below.



In this case, a signal travels from IN to OUT and is multiplied by a factor A. There is no feedback in this system. Now let's see what happens when we add feedback.

But with feedback, the consequence influences the action. As such, the behavior of the system can't be reasoned out in a linear path.

The following sketch more accurately models an op-amp, including feedback to both the terminals.



Here, the feedback is summed with IN+ and IN- before the signal goes to the opamp inputs. This circuit will only be stable (i.e. will exhibit negative feedback) if $\alpha_- > \alpha_+$, i.e. if the feedback to IN- dominates over the feedback to IN+. Imagine you are sitting at IN- and the signal goes up slightly (say by Δ), the output will start to drop, say in the initial phase of the feedback, it drops by δ initially. Before the output can drop more, the feedback kicks in and lowers the negative input to the amplifier A by $\alpha_- A\delta$ (and raises the positive input by $\alpha_+ A\delta$). But now those changing inputs will change the output! So we have to repeat this analysis. If we assume $\alpha_+ = 0$, soon $A\alpha_-\delta = \Delta$, the two inputs to the amplifier will be equal, the output will be 0, and the feedback signal will stop. As a result, the negative feedback acts to shut down any changes in the inputs, and stabilizes the system.[6]

[6] Of course, as mentioned above, this is a simplified picture, and not intended to fully satisfy your curiosity. You can take a class like 6.302 Feedback Systems, to learn it properly. Suffice it to say that feedback and control are some of the most important concepts used in technology in the modern era

Conclusions

Opamps are primarily useful because they provide powerful functional capability in a circuit while remaining fairly simple to analyze.

The analytic simplicity comes about as a result of negative feedback. There are useful op-amp circuits that do not use negative feedback, and we may look at some of those later in the class, but when negative feedback applies, the assumption of ideality provides massive simplification.

A simple model of an ideal op-amp can be created from a voltage-controlled voltages source (VCVS), but this model is generally useful only for purposes of derivation of the basic principles of operation. After this derivation is complete, one typically assumes $v_+ \approx v_-$ and uses this to assist in anlyzing and designing op-amp circuits.

Glossary and Definitions

Amplifier A circuit element that increases any signal parameter (generally voltage in this class). Generally, power must be supplied from an external source.

Amplifier Transfer Function Output signal vs. input signal for an amplifier. For a simple linear amplifier this is characterized by a simple line through the origin whose slope is called the "gain". Nonlinear amplifiers are more complicated, and the transfer function is sometimes plotted.

Comparator Device that compares an input voltage level to a threshold and outputs a different value depending on whether it exceeds or is smaller than the threshold. An op-amp can be thought of as a simple comparator.

Gain The amplification factor by which a signal is multiplied in an amplifier.

Ideal Op Amp A concept used in circuit analysis of an op-amp without saturation rails, with exactly zero current into the input terminals, and with infinite gain.

Lumped Element A simple component (i.e. single element) in which the electric and magnetic fields are assumed to be entirely local to the device.

Method of Assumed States Analysis method for devices which have a variety of states (relevant models depending on circuit parameters). A model is assumed for a device based on unknown circuit conditions (but sometimes informed by the analyzers intuition about the problem). The circuit is then analyzed and the circuit conditions required for the model are either verified, or contradicted. If contradicted, or if multiple solutions are possible, a new state is assumed and the analysis repeated.

Open-loop gain Gain of op-amp when feedback is not present. Typically denoted with the letter A. Open-loop gain of an op-amp is typically in the range of 10^6 to 10^8.

Op-Amp Short form for Operational Amplifier.

Operational Amplifier Amplifier characterized by high input impedance, high open-loop gain, and low output-impedance, used for a variety of purposes.

Saturation A level (either positive or negative) which an amplifier cannot exceed even though inputs are increased. Saturation in an op-amp is often near the power supply levels (positive and negative).

Self-Consistency A check for whether the assumptions of the problem are satisfied by the solution. If not, the solution is not self-consistent, and cannot be used.

Rail Saturation levels for an op-amp or other electronic system. Often used to refer to the power supply levels.

Acknowledgements

Thank you to Rinske Wijtmans for assistance with formatting, and Cecelia Chu for helpful comments.