6.200 Notes: Ideal Op-Amp Circuits

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In these notes, I present the method to be used when analyzing op-amp circuits. This is *not* intended to be a comprehensive introduction to op-amps, but rather a practical guide to their analysis, illustrating some useful examples.

Throughout this guide, we will generally assume the ideal op-amp approximation applies. We therefore are able to proceed without using the dependent source model of the op-amp. Generally, if you are resorting to the dependent-source model (unless specifically instructed to), you are making a mistake.

In future notes, we will consider cases where the op-amp is operating in its saturation states. Here, the ideal op-amp model cannot be applied, but that should be clear from either the statement of the problem, from the lack of negative feedback in the circuit, or from the calculated output voltage from the ideal op-amp model lying outside the voltage rails of the device (which is a contradiction, indicating that the ideal op-amp approximation was invalid).

For now, we will stick with examples where the ideal op-amp approximation applies.

The Basic Steps

There are four basic steps to analyzing an ideal op-amp:

1. Check that it is really ideal, i.e. that there is adequate negative feedback provided. Often in this course we will simply tell you that it is the case, or it will be obvious because only the negative terminal has feedback, but in more advanced classes you will learn how to do your own analysis in situations where it is less clear.

2. Assume that $v_+ \approx v_-$[1] and that $i_+ = i_- = 0$.

3. Solve for the circuit parameters of interest based on these assumptions.

4. If you are not told in advance or unsure if op-amp is ideal, you should check at the end of your analysis that the output is indeed not saturated, i.e. that $V_{\text{MIN}} < v_{OUT} < V_{\text{MAX}}$. If that is not the case, the assumptions that went into your analysis were flawed, so the analysis is invalid.

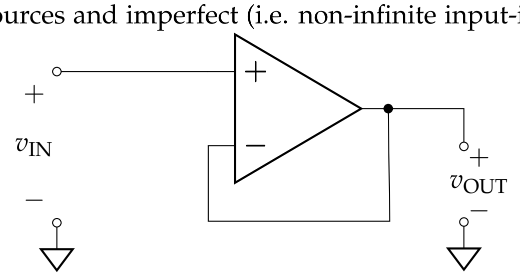
[1] It is important to respect the approximation symbol here! Students can run into big problems when they assume perfect equality. For example, one cannot simply connect a wire between the '+' and '-' terminals, and expect your circuit to work. The tiny difference between the two inputs maybe negligible for purposes of analysis, but it is critical to correct operation.

Voltage Buffer/Follower

It is very common in circuits to want to transfer a signal from a circuit with a finite output impedance to a circuit with a finite input impedance. In this situation, the signal is attenuated, and thus potentially the signal fidelity is reduced.

An op-amp can provide a simple solution to this problem, which is a **buffer**. A buffer is a circuit element that has a near-infinite input impedance and a near-zero output impedance. That means that it can accept an input and provide output without attenuation, thus compensating for imperfect (i.e. non-zero output-impedance) signal sources and imperfect (i.e. non-infinite input-impedance) loads.[2]

[2] Sometimes called a **voltage follower**, as the output follows the input.

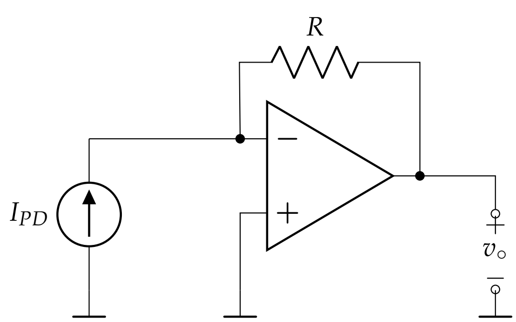


This circuit can be analyzed by observing that $v_- \approx v_+ = v_{IN}$ and that because of the feedback between the output and the - terminal, $v_{OUT} = v_- = v_{IN}$. This circuit thus simply provides the input signal at the output, but with a Thevenin equivalent resistance of $0\,\Omega$. This is a tremendously important and useful device throughout electronics as a result of this ability.

Photodiode Readout

A photodiode (a sensor that detects light) is often modeled as a current source whose strength depends on the light intensity applied to it.[3]

[3] The equivalent circuit model for a photodiode is more complex than a simple current source, but often, and for our purposes, a current source will suffice.



1. Observing that there is only feedback to the negative terminal, it seems likely that the system can be treated as ideal[4]

2. Assuming that $v_+ \approx v_-$ we conclude that $v_- \approx 0$

[4] Be careful to verify that there is negative feedback in the circuit before assuming $v_+ \approx v_-$. Circuits with positive feedback will often appear to provide the same analytic result as circuits with negative feedback, but due to the positive feedback we know the solution will be unstable–any slight fluctuation in $v_+ - v_-$ will immediately be amplified and the detector will end up at its rails. In this class, we will generally tell you in advance when it is safe to assume the opamp is ideal.

3. We perform nodal analysis by summing the inputs at the negative input terminal.

$$I_{PD} + (v_\circ - 0)G = 0$$
$$\Rightarrow v_\circ = I_{PD}/G = -I_{PD}R.$$

A more conventional approach would be to use the "intuitive" method of analysis. By this method, one looks at the circuit and starts making simplifications one at a time until a solution presents itself.

In this case, we observe that I_{PD} must flow up towards and through the feedback resistor (because it cannot enter the - terminal). As a result, the voltage drop from v_- to $v_\circ$ must be $-I_{PD}R$.

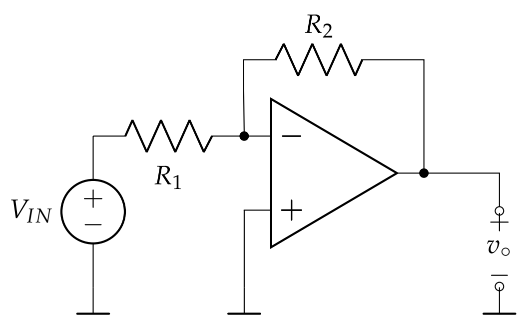
We showed you the nodal analysis approach first,[5] to help you see the connection between the two approaches. In future examples, we will only illustrate the intuitive method.

[5] You may have noticed that we did not do the node method at the output terminal. This was deliberate: the node method cannot be used at the output because the dependent source hidden inside each op-amp (remember the fundamental op-amp model) can source an arbitrary current, and thus is not sufficiently constrained to add useful information. Unfortunately, it is common for students to try this anyway, assume the output current of the opamp is zero (which it is generally not!), and end up in a mess.

4. Because in this case we were not given actual numbers, if we hadn't been informed in advance that the system was ideal, we would state that this treatment is valid only when $-V_{\mathrm{MIN}}/R > I_{PD} > -V_{\mathrm{MAX}}/R$.

Inverting Amplifier

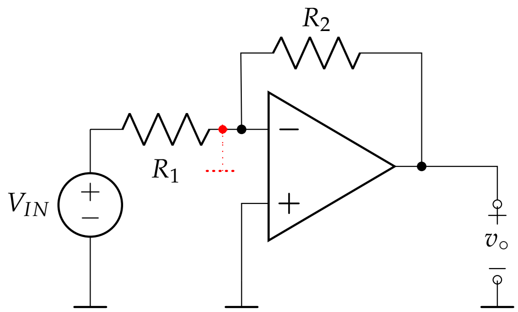
One of the most common and useful op-amp circuits is the inverting amplifier. This circuit has negative gain, meaning the output voltage has opposite sign to the input. This is what the "inverting" refers to in the title.



We will solve this circuit by using the same four-step process:

1. We first check that there is feedback to the negative terminal, which there is.

2. We next notice that the + terminal is connected to the reference ground, and so set the node potential at the - terminal to 0 V.



Notice that we draw the ground symbol on the - terminal with a dotted line to remind us that this is not physically connected to ground, it simply happens to be at 0 V potential due to the action of the feedback resistor, thus no current can flow directly to the reference ground from this point.

3. We finally analyze the circuit by performing KCL at the + terminal.

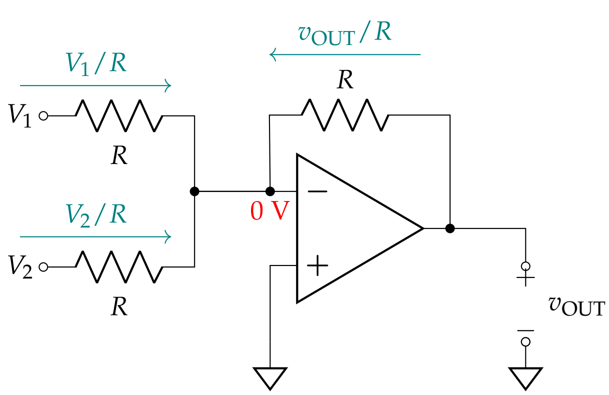
$$V_{\text{IN}}/R_1 + v_\circ/R_2 = 0$$
$$\Rightarrow v_\circ = -\frac{R_2}{R_1}V_{\text{IN}}.$$

From which we see that the magnitude of the gain is R_2/R_1 and that, as promised, the gain is negative, i.e. the device is inverting.

4. Were we given numeric values, and were we in doubt about the validity of the ideal op-amp approximation, we would check that $-\frac{R_1}{R_2}V_{\text{MIN}} > V_{\text{IN}} > -\frac{R_1}{R_2}V_{\text{MAX}}$.

Summing Amplifier

In circuits, it is quite easy to add two currents—one need simply apply them both to a single short, and the current in the short will equal the sum of the input currents by KCL. However, adding two voltages referenced to ground is a bit trickier. To accomplish this, we can use an op-amp.



We will again follow the algorithm given above:

1. Confirm the existence of negative feedback. In this case we see feedback from the output to the - terminal, so this is confirmed.

2. Set the input terminal potentials to be (approximately) equal to each other. In this case, that means that the potential at the - terminal will be 0 V because the + terminal was connected to the reference ground node.

3. Analyze the circuit. We will use the "intuitive method" but should point out that this is essentially simply performing KCL on the - terminal of the op-amp.

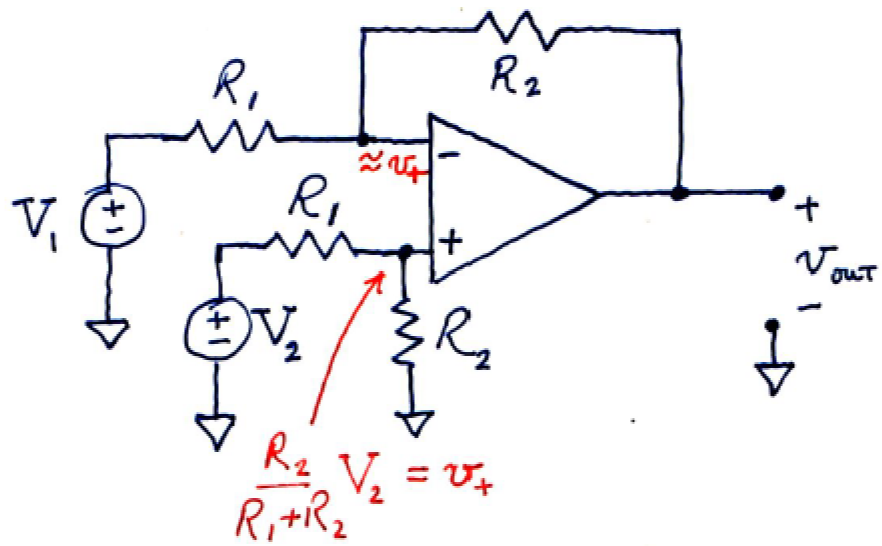
$$\frac{V_1}{R} + \frac{V_2}{R} = \frac{v_{\text{OUT}}}{R}$$
$$\Rightarrow v_{\text{OUT}} = V_1 + V_2.$$

   If we wish to pre-weight any of the inputs, we can adjust their resistor values down (to increase the weight) or up (to decrease the weight). Furthermore, to add an overall multiplier, we can increase resistance of the **feedback resistor**.

4. We again know the op-amp is ideal by construction, but had we not known that in advance, we would check now that $V_{\text{MIN}} < V_1 + V_2 < V_{\text{MAX}}$ to confirm that indeed the ideal op-amp assumption applied.

Differential Amplifier

Often, one finds oneself trying to find the difference between two signals. This comes up when, for example, a small sensor is picking up a signal but in the background some large noise signal is affecting the overall system. Luckily, the noise on the two ends of the sensor branch is often highly correlated, so by amplifying the difference between the sensor electrodes, the signal can be extracted from the noise.



The routine one follows should be getting familiar by now...

1. Verify that feedback is coming to the - terminal, as it is.

2. Set $v_+ \approx v_-$. Observing that the circuit connected to the + terminal is a simple voltage divider, one can calculate $v_+ = \frac{R_2}{R_1+R_2}V_2$ which is of course then the same as v_-.

3. We analyze the circuit. Doing KCL at the - terminal, we find

$$
\begin{aligned}
(V_1 - v_+)\frac{1}{R_1} + (v_{\mathrm{OUT}} - v_+)\frac{1}{R_2} &= 0 \\
\Rightarrow v_{\mathrm{OUT}} &= -R_2(V_1 - v_+)\frac{1}{R_1} + v_+ \\
\Rightarrow v_{\mathrm{OUT}} &= -V_1\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right)v_+ \\
\Rightarrow v_{\mathrm{OUT}} &= -V_1\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right)\frac{R_2}{R_1+R_2}V_2 \\
\Rightarrow v_{\mathrm{OUT}} &= -V_1\frac{R_2}{R_1} + \left(\frac{R_1+R_2}{R_1}\right)\frac{R_2}{R_1+R_2}V_2 \\
\Rightarrow v_{\mathrm{OUT}} &= \frac{R_2}{R_1}(V_2 - V_1).
\end{aligned}
$$

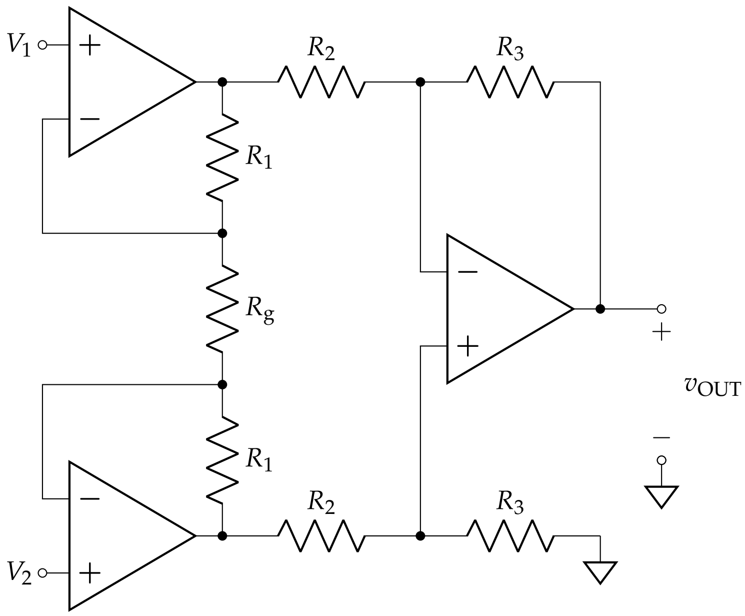
4. And finally (and not relevantly in this example, because the problem statement asserted ideal op-amp behavior), we confirm that for this treatment to be valid, $V_{\mathrm{MIN}}\frac{R_1}{R_2} < (V_2 - V_1) < V_{\mathrm{MAX}}\frac{R_1}{R_2}$.

Instrumentation Amplifier

In a laboratory setting, one often is interested in a very flexible differential amplifier with a guarantee of an infinite input impedance. Notice that the differential amplifier shown above has a finite input impedance (imagine that instead of a simple V_1 or V_2 source one had a source with a finite output impedance—this impedance would affect the amplifier significantly).

To resolve this problem, we can combine two of the designs we've seen so far to create a buffered differential amplifier called an instrumentation amplifier.

There's a typo on the topmost equation, R_2 should be R_1.

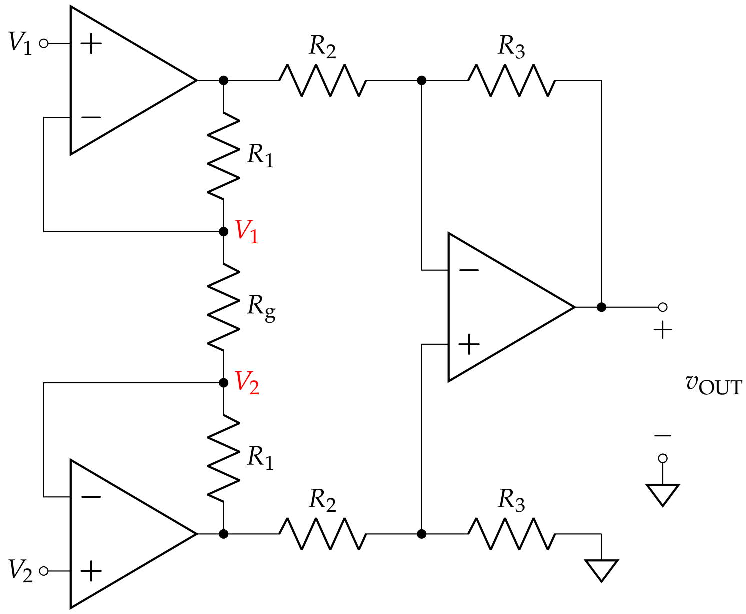


You can possibly observe that the input of this amplifier (V_1 and V_2) can draw no current, and thus is said to have infinite input impedance.

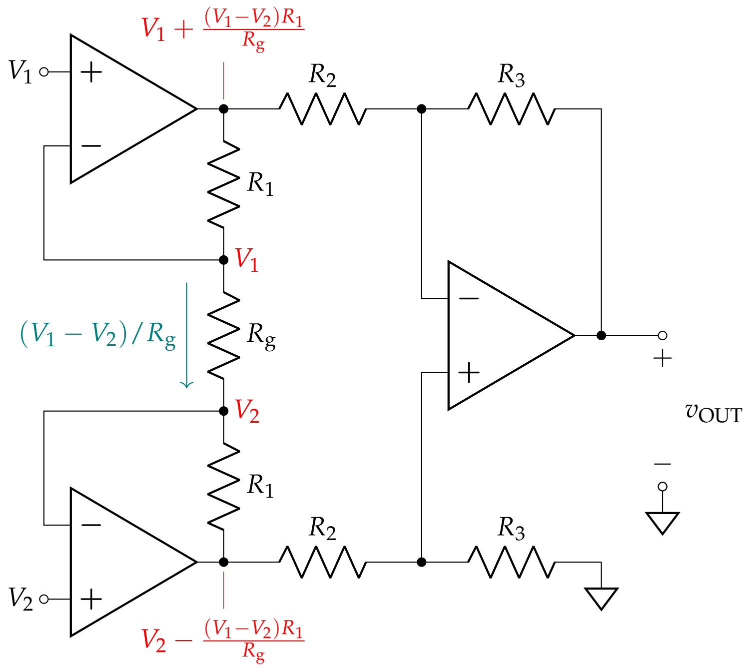
Despite its somewhat daunting topology, this circuit is not overly difficult to analyze if you realize that the outputs of the first two op-amps are simply the inputs to a differential amplifier. The left-hand-most op-amps thus play little role other than to assure the differential amplifier of a low-output-impedance input.

1. We see that all three of the op-amps have negative feedback.

2. We assign the - terminals of the first two op-amps according to the standard process, yielding V_1 for the top and V_2 for the bottom op-amp, as indicated in red on the diagram above.



3. We then calculate the current in the direction indicated in R_g (the gain resistor). This current is $i_g = (V_1 - V_2)/R_g$. Because we know the - terminals draw no current, this current must be the same in the R_1 resistors, and thus using Ohm's law and KVL, we can calculate the output node potentials of the initial op-amp buffers, as labelled in green. For the V_1 op-amp, the output voltage is $V_1 + (V_1 - V_2)R_1/R_g$ and for the V_2 op-amp, the output voltage is $V_2 - (V_1 - V_2)R_1/R_g$.



Recognizing that these output voltages are input voltages into the final, differential amplifier op-amp, we can write down

$$v_{\text{OUT}} = \frac{R_3}{R_2}\left(V_2 - \frac{R_1}{R_g}(V_1 - V_2) - V_1 - \frac{R_1}{R_g}(V_1 - V_2)\right)$$
$$\Rightarrow v_{\text{OUT}} = \frac{R_3}{R_2}\left(\frac{-2R_1 - R_g}{R_g}(V_1 - V_2)\right).$$

4. We would have to check that this output, as well as all the intermediate op-amp outputs, are within the rails of the respective op-amps to assure ourselves that the ideal op-amp assumption indeed applies.

Conclusions

Opamps are primarily useful because they provide powerful functional capability in a circuit while remaining fairly simple to analyze.

Until now, we had to just postulate that things like current sources and dependent sources were possible. Now, we know how to make these circuit elements using op-amps.

The analytic simplicity comes about as a result of negative feedback. There are useful op-amp circuits that do not use negative feedback, and we may look at some of those later in the class, but when negative feedback applies, the assumption of ideality provides massive simplification.

Glossary and Definitions

Buffer Also known as Voltage Buffer or Voltage follower, this circuit provides an large input impedance and small output impedance, thus enabling a signal from an imperfect voltage source (e.g. with high output impedance) to be passed on to an imperfect receiving circuit (e.g. one with low input impedance).

Inverting Amplifier Amplifier that inverts the sign of a signal while amplifying it.

Summing Amplifier Amplifier that sums two or more input voltages with varying weights and then amplifies the result.

Differential Amplifier Amplifier that amplifies the difference between two signals. Often useful when trying to discern a signal in the presence of common noise shared between the devices.

Feedback Resistor Resistor connecting the output back to one of the input terminals of an amplifier (e.g. an op-amp).

Instrumentation Amplifier Amplifier with high input impedance that provides high gain and differential amplification for use with a wide range of inputs, such as one might encounter in a laboratory.

Voltage Buffer See Buffer.

Voltage Follower See Buffer.

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