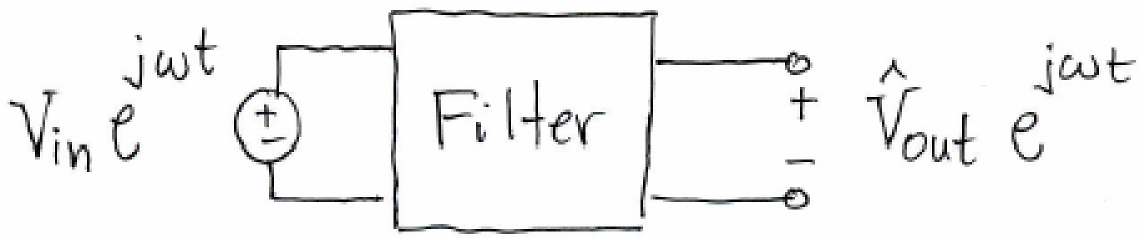
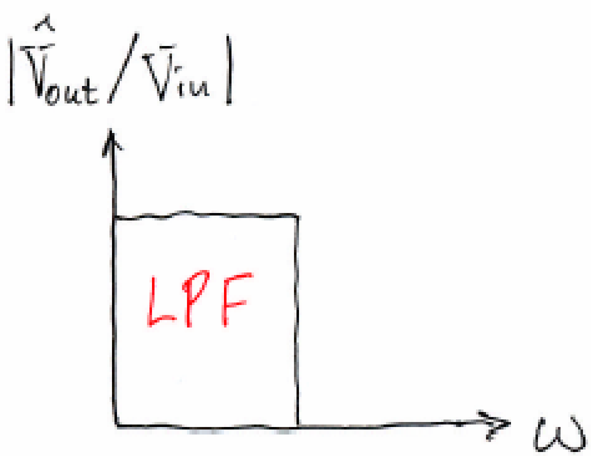
6.002 - Lecture 11B

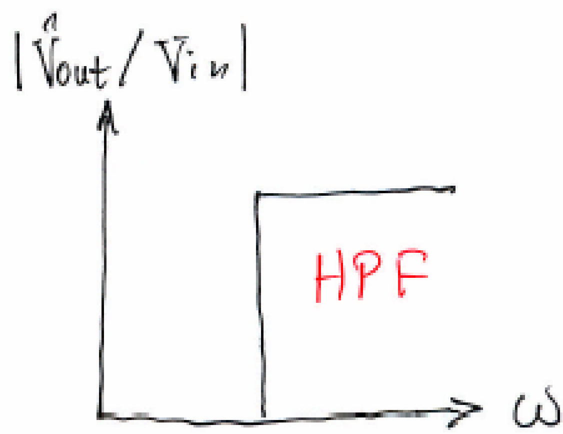
Filtering:

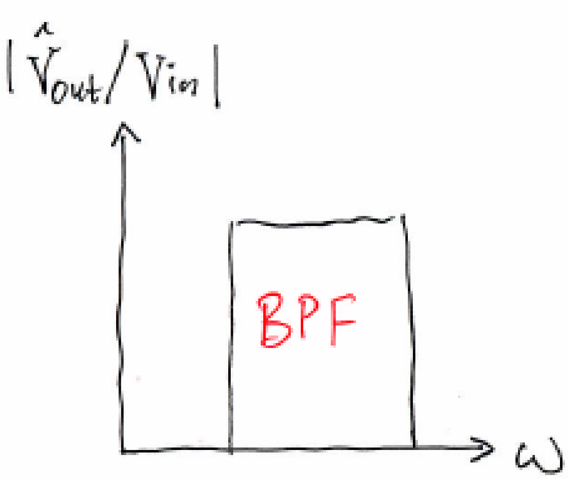
- Review Simple Objectives (LPF , HPF , BPF , BSF)
- Review Simple Synthesis
- LC Resonators
- Tesla Coil

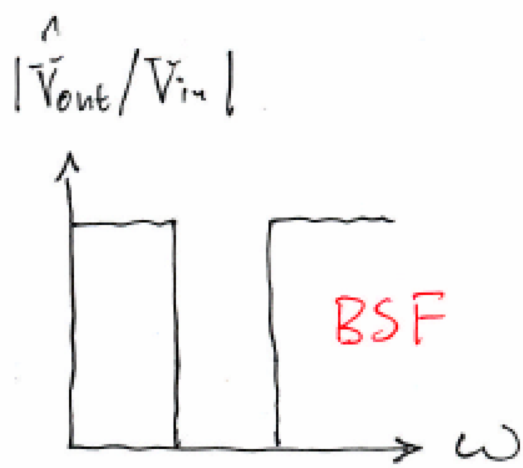
Simple Filtering Objectives



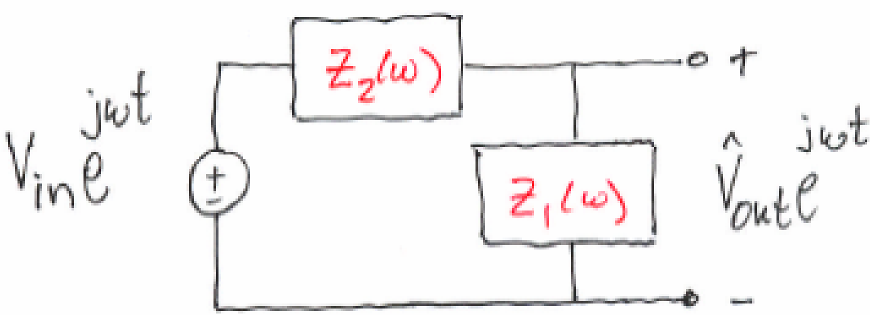




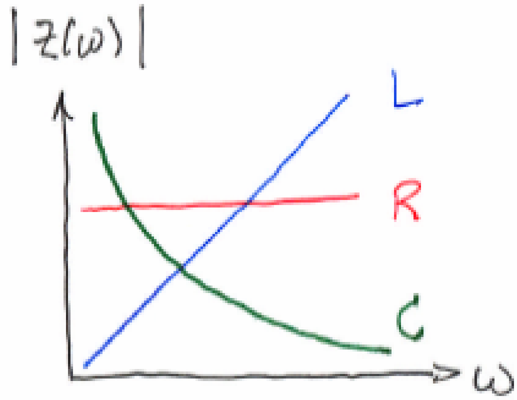


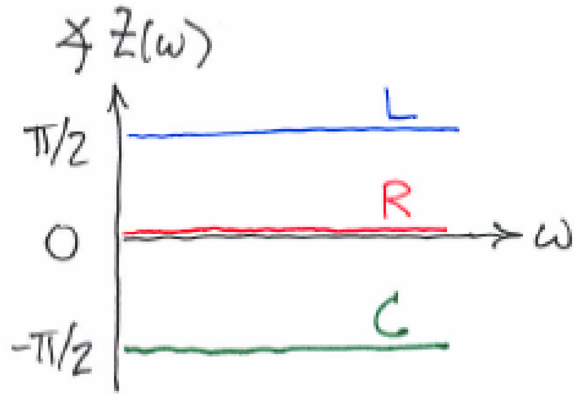


Simple Filtering Synthesis

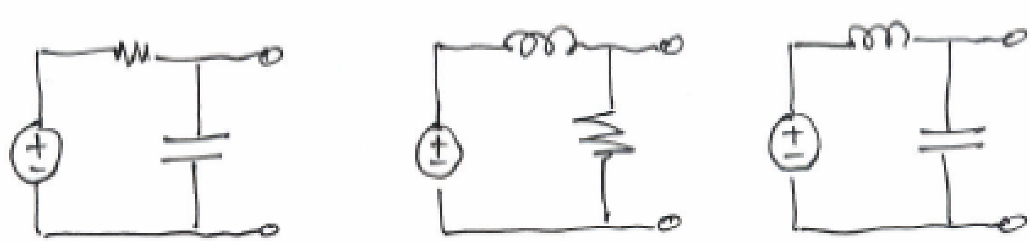


$$\frac{\hat{V}_{out}}{V_{in}} = \frac{Z_1(\omega)}{Z_2(\omega)+Z_1(\omega)}$$

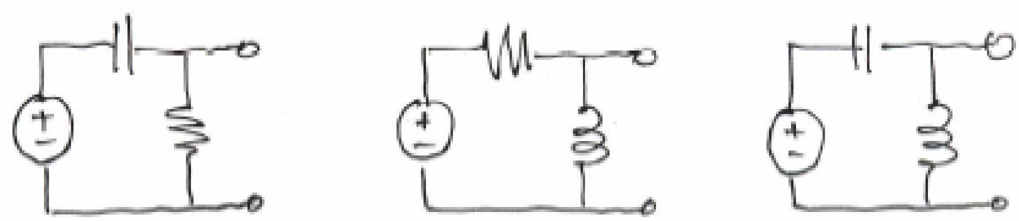




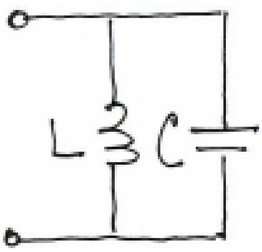
LPF:



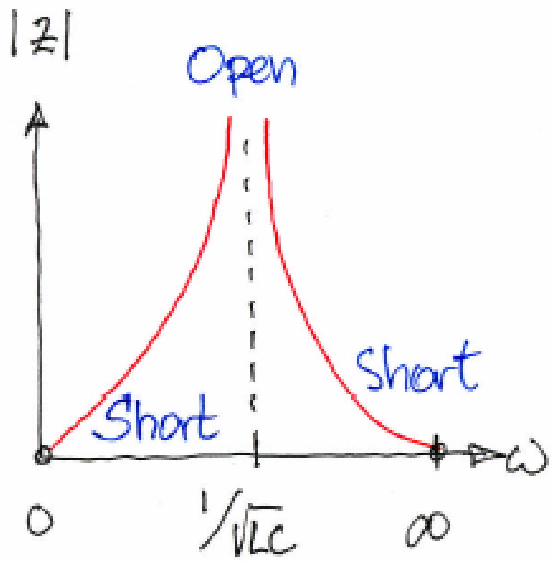
HPF:

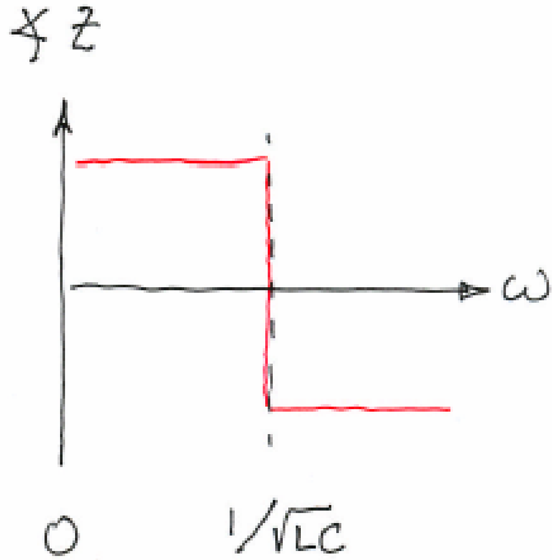


Parallel LC Resonator

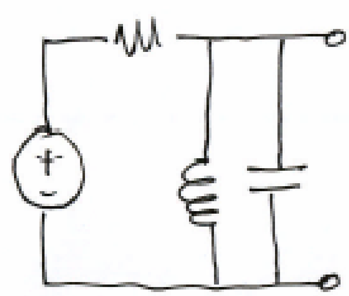


$$Z = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

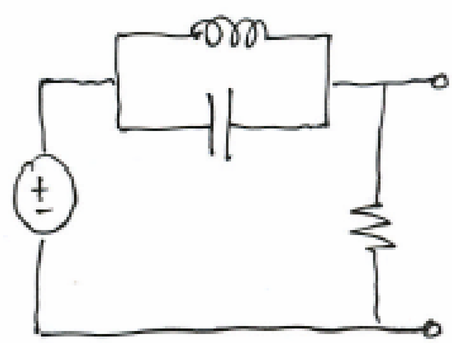




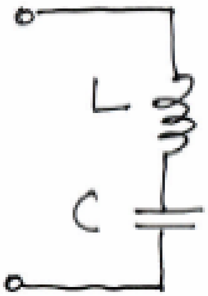
BPF:



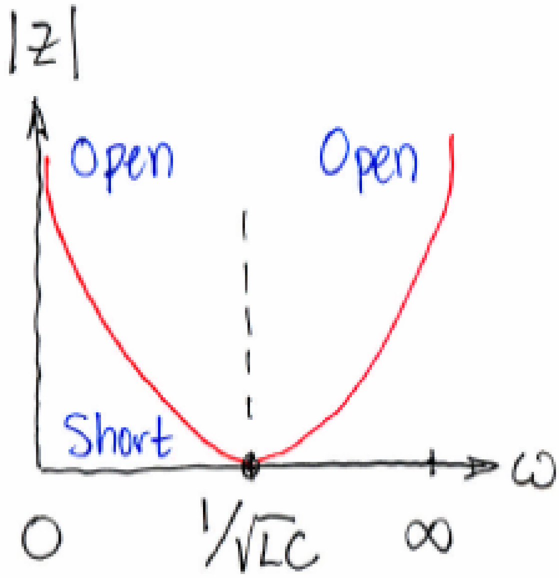
BSF:

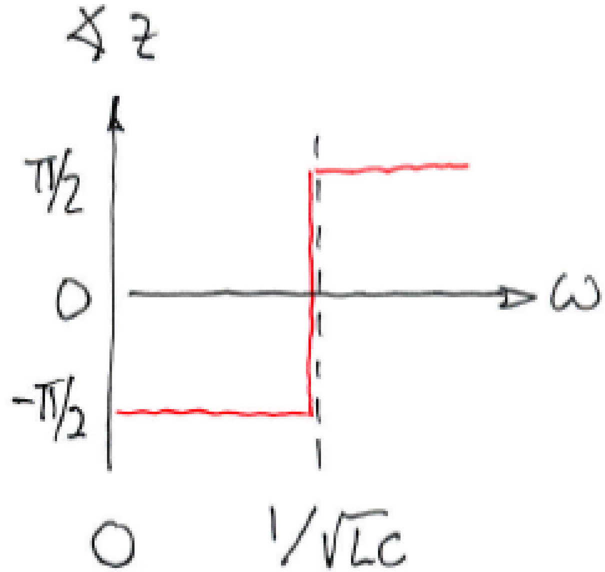


Series LC Resonator

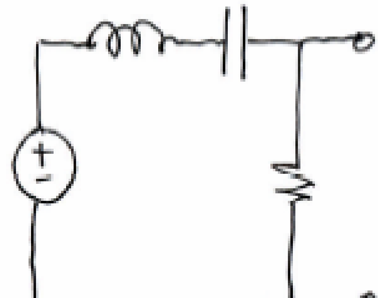


$$Z = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

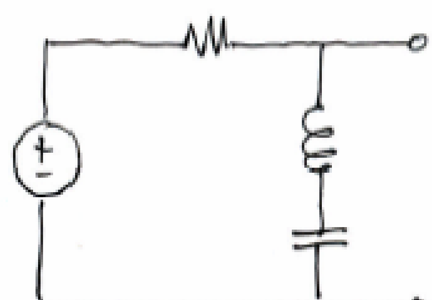




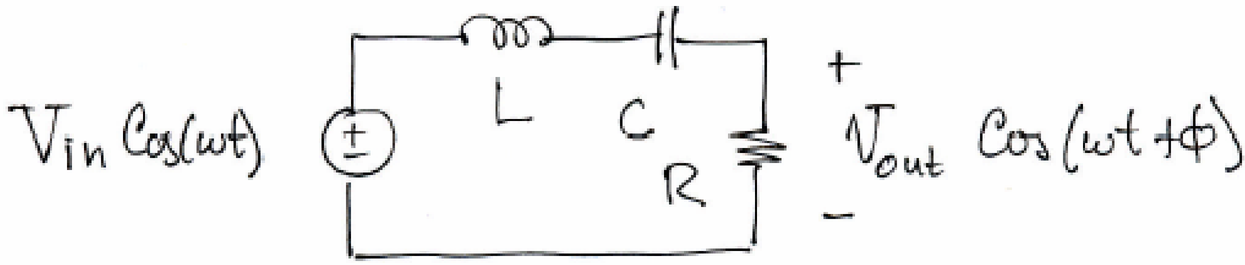
BPF:

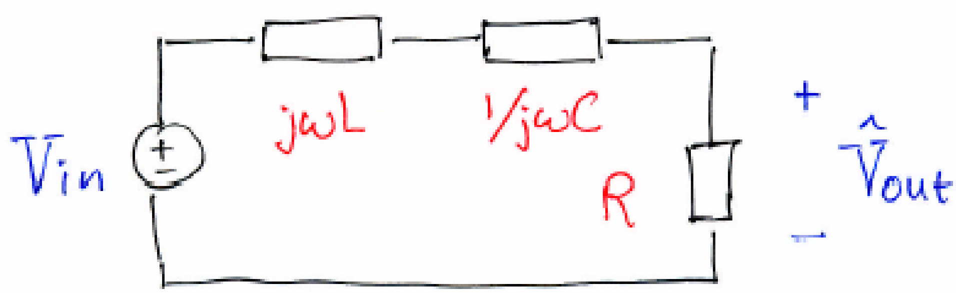


BSF:



Theory: Series-LC BPF



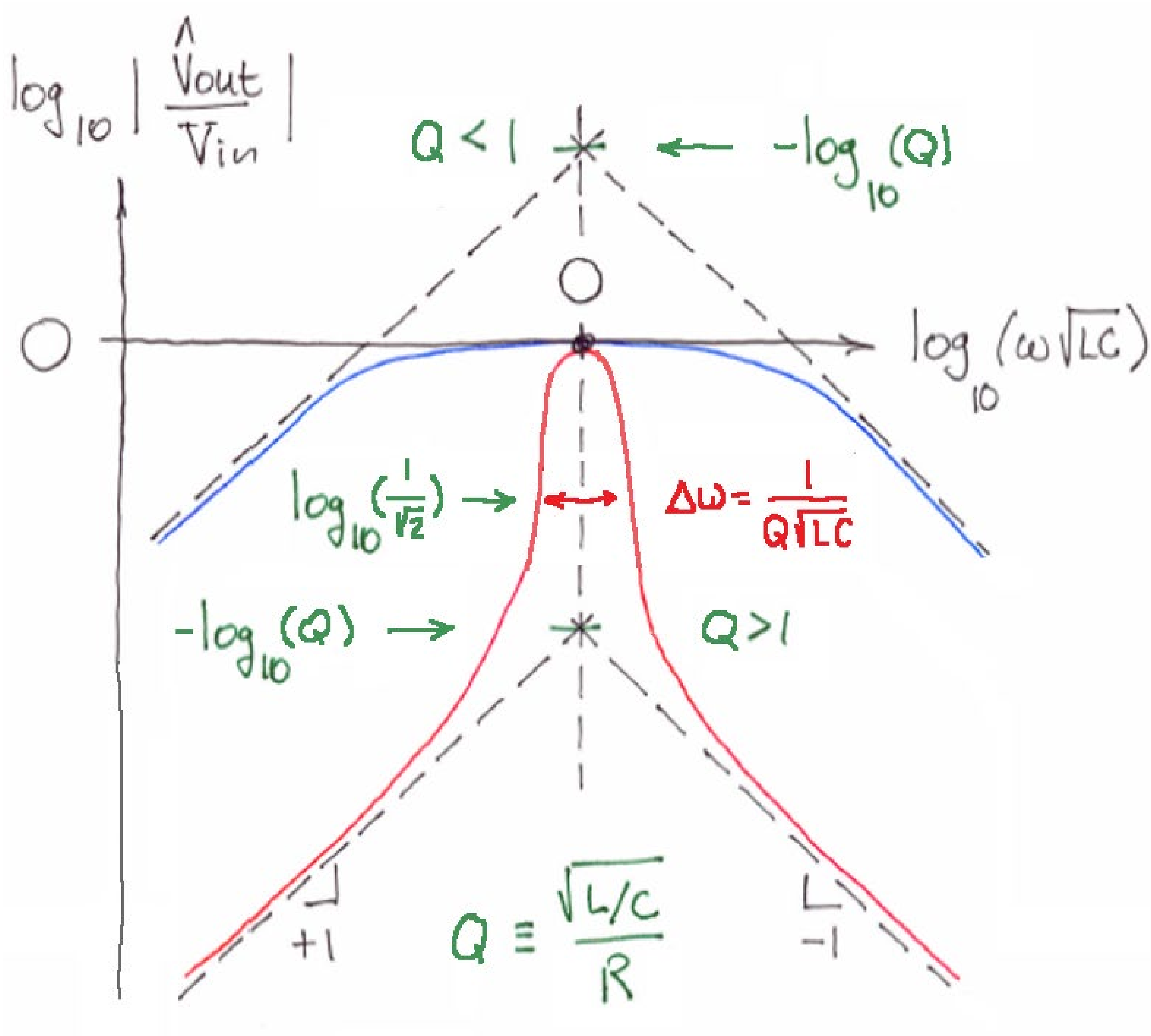


$$\hat{V}_{out} = \frac{R}{R + j\omega L + 1/j\omega C}\,\hat{V}_{in} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}\,V_{in}$$

$$V_{out} = \left|\hat{V}_{out}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}\,V_{in}$$

$$\phi = \measuredangle \hat{V}_{out} = \tan^{-1}\left(\frac{1 - \omega^2 LC}{\omega RC}\right)$$

Theory: Series-LC BPF



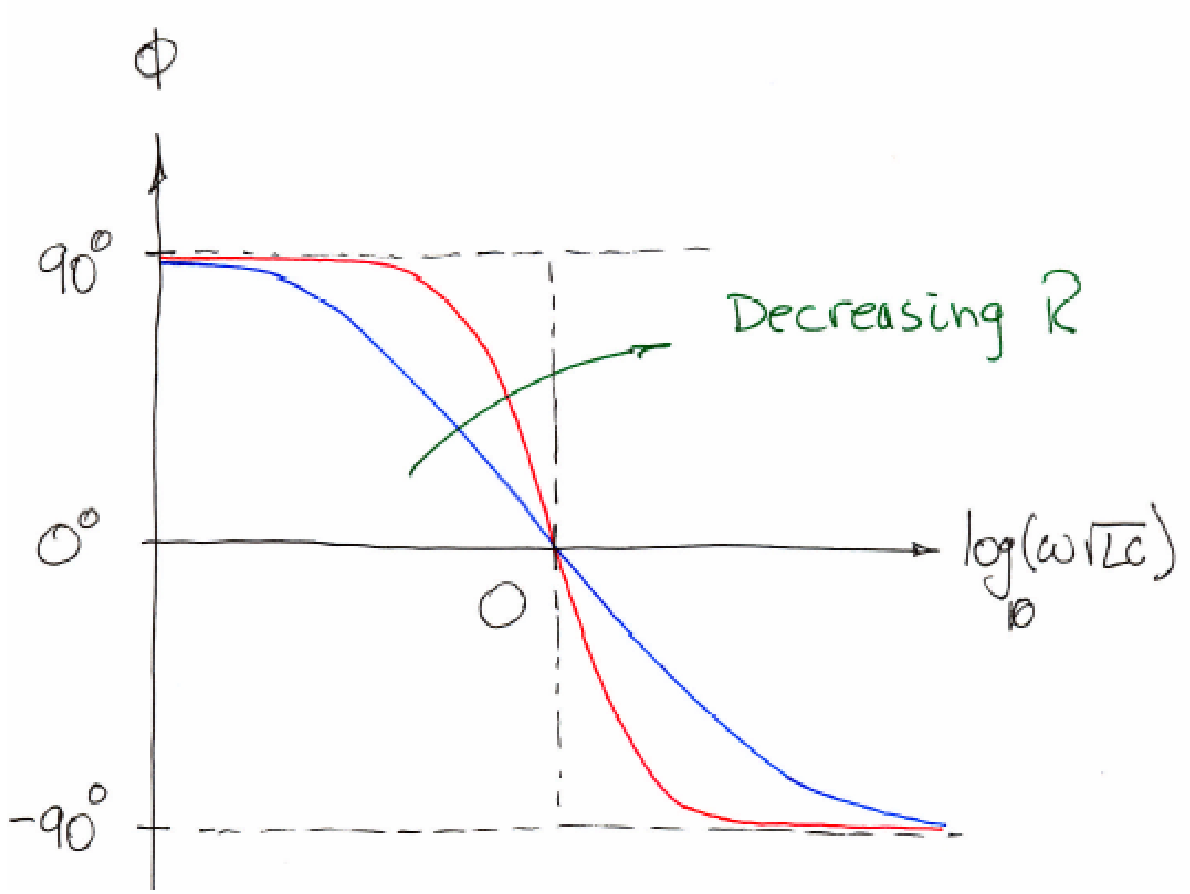
Low Frequency

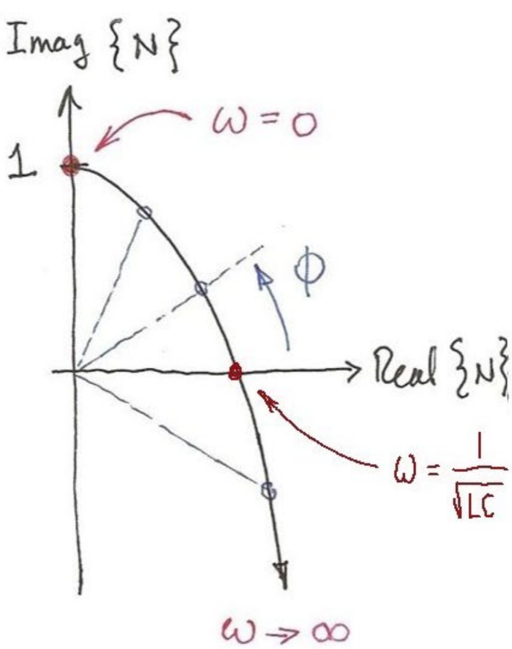
$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \omega RC = \frac{\omega\sqrt{LC}}{Q}$$

High Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \frac{R}{\omega L} = \frac{1}{\omega\sqrt{LC}\,Q}$$

Theory: Series-LC BPF





$$\phi \equiv \measuredangle \left| \frac{\hat{V}_{out}}{V_{in}} \right|$$

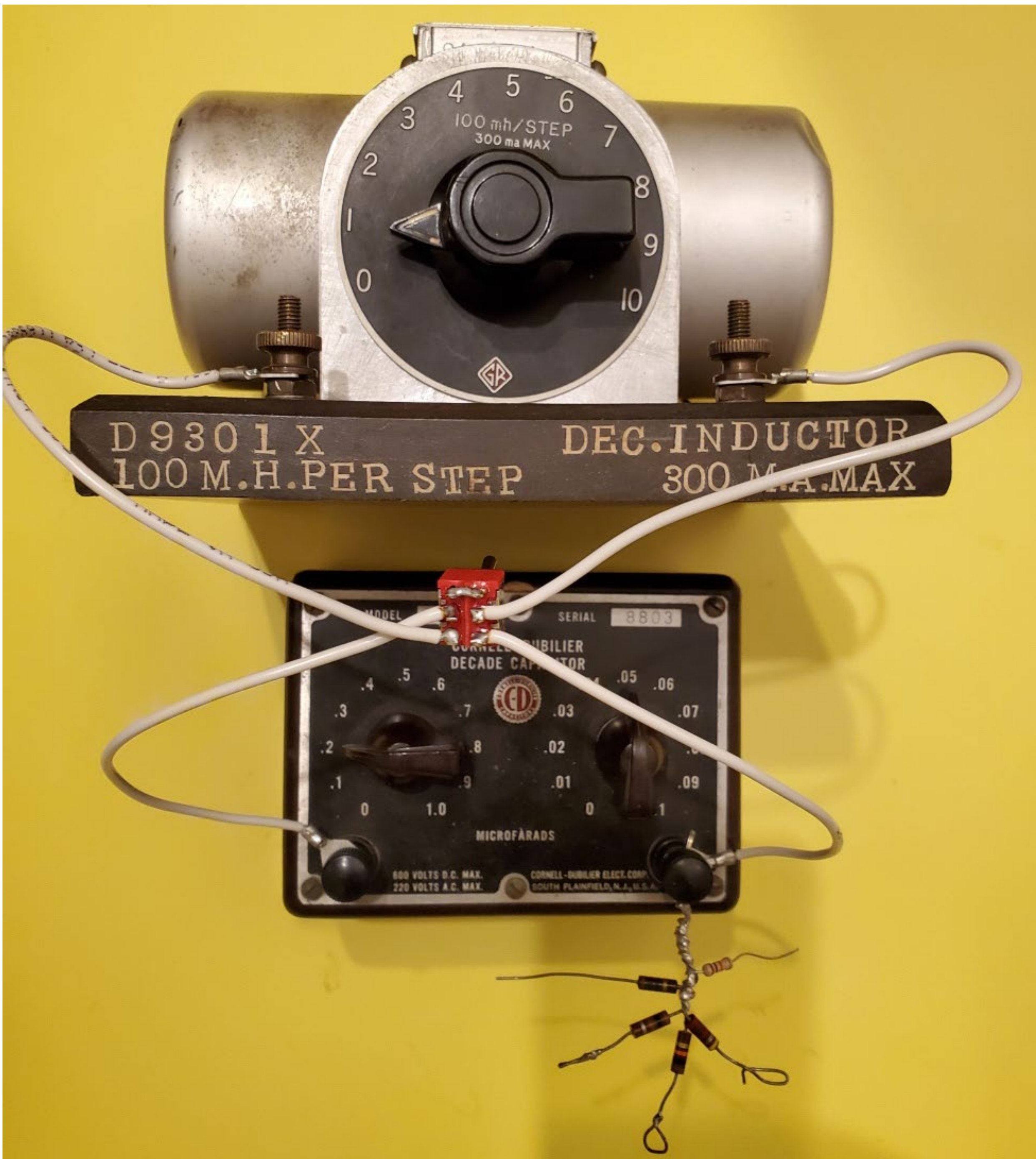
$$= \measuredangle \frac{j\omega RC}{1-\omega^2 LC + j\omega RC}$$

$$= \measuredangle \frac{j\omega RC\,(1-\omega^2 LC - j\omega RC)}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

$$= \measuredangle\, j\,(1-\omega^2 LC) + \omega RC$$

$$\equiv \measuredangle\, N$$

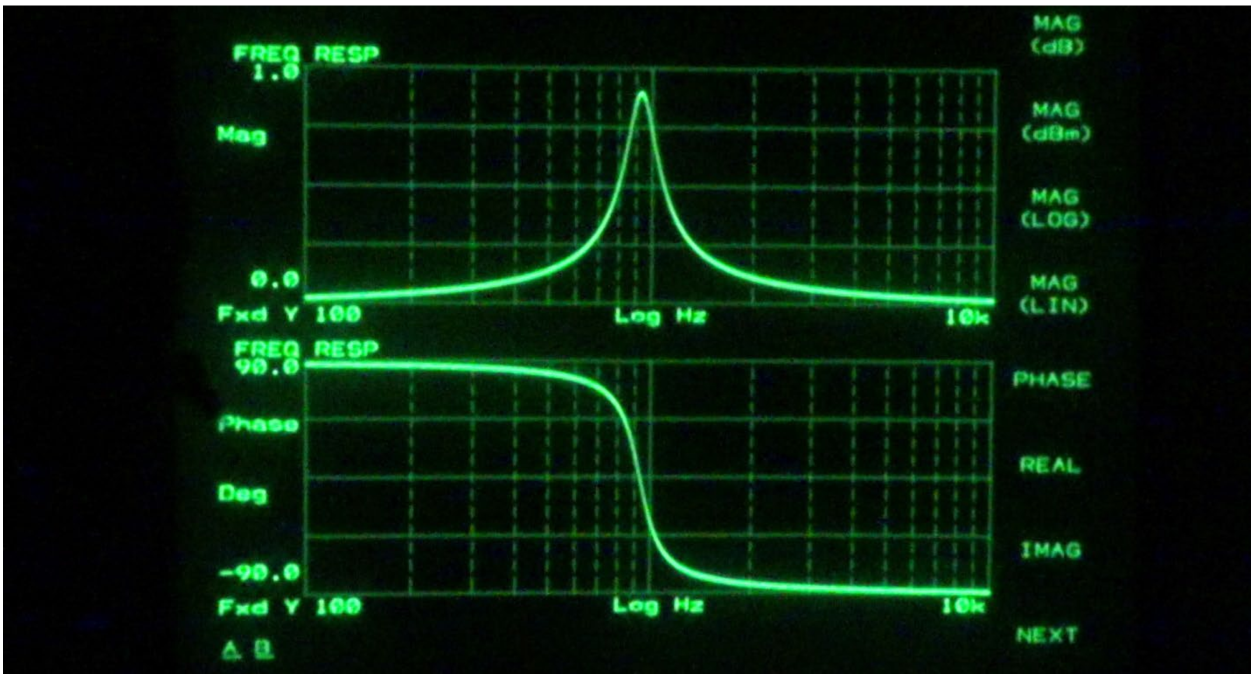
RLC Demos

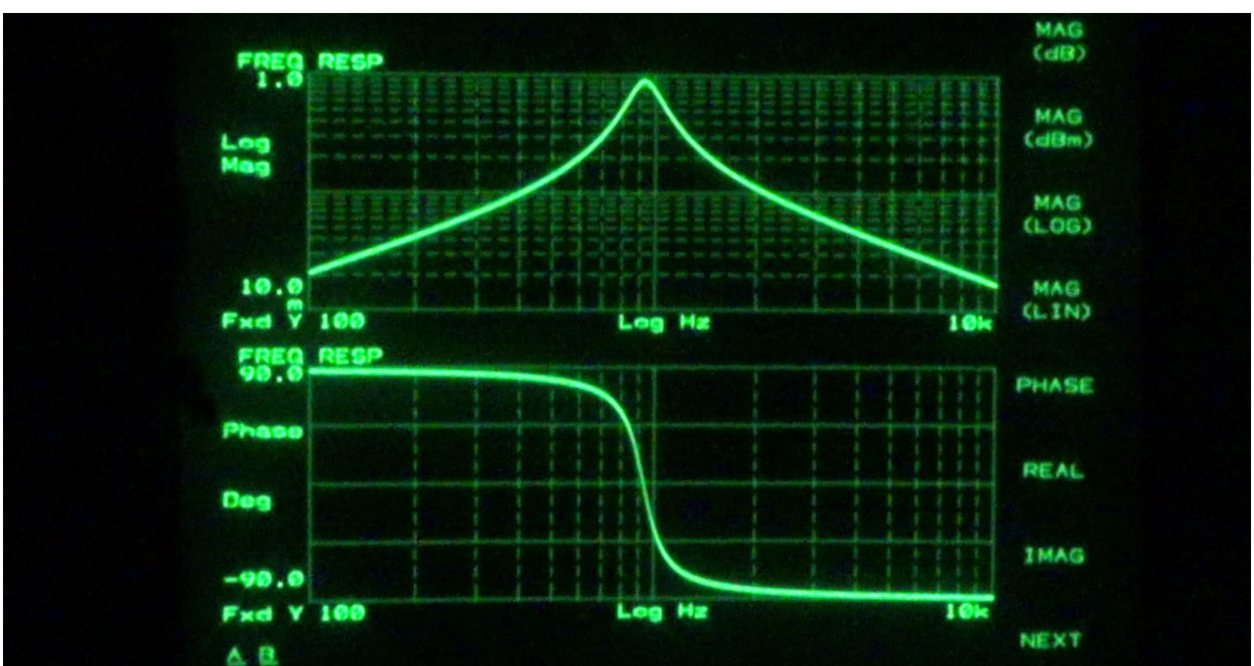

100 mh/STEP
300 ma MAX
D9301X
DEC.INDUCTOR
100 M.H.PER STEP
SERIAL
8803
MICROFARADS
600 VOLTS D.C. MAX.
220 VOLTS A.C. MAX.

Demo: Series-LC BPF

L = 0.1 H ; C = 0.25 μF ; R = 100 Ω

$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ = 632 Ω

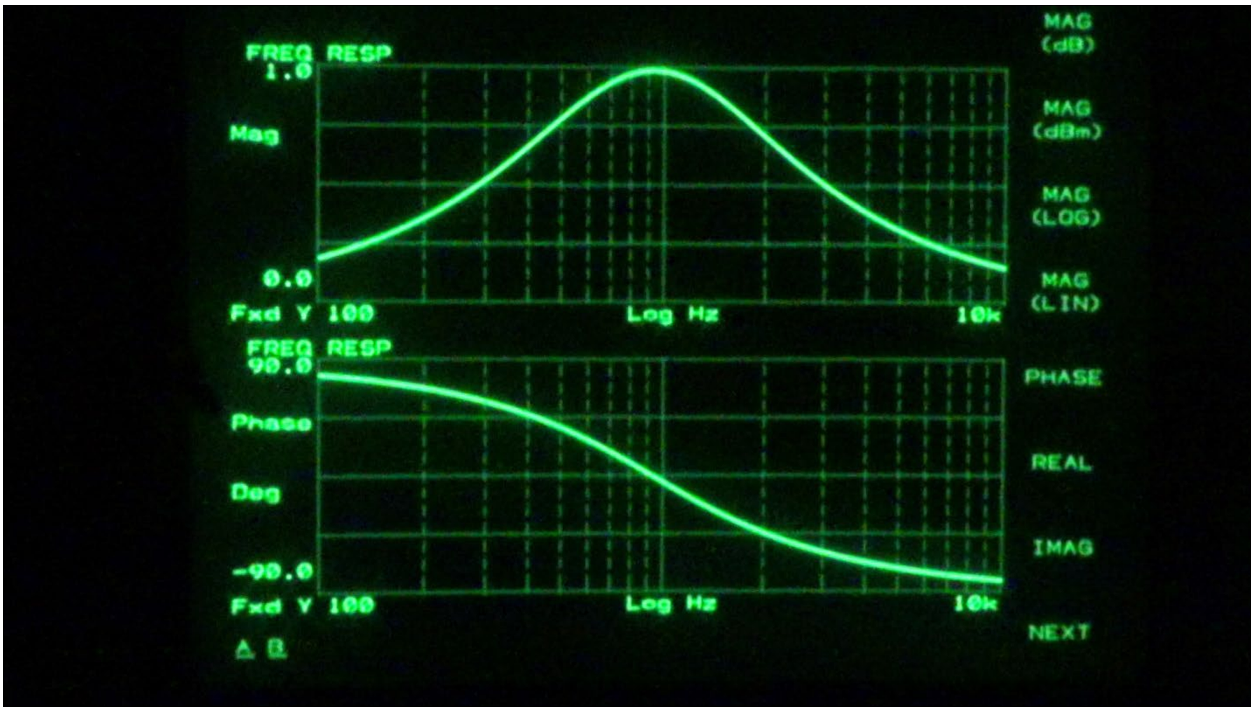




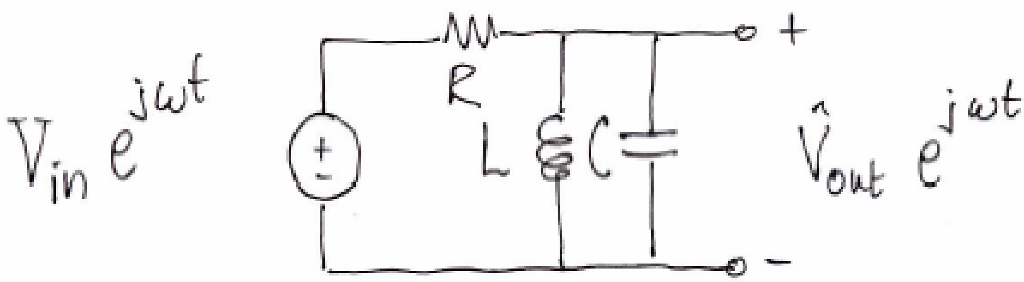
Demo: Series-LC BPF

L = 0.1 H ; C = 0.25 μF ; R = 1 kΩ

$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ = 632 Ω

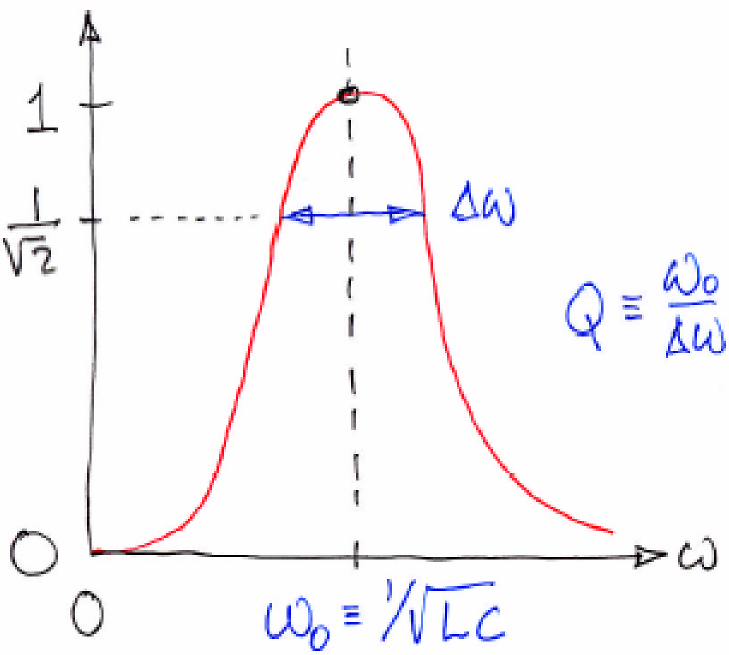


Theory: Parallel-LC BPF



$$\frac{\hat{V}_{out}}{V_{in}} = \frac{1/R}{1/R + 1/j\omega L + j\omega C} = \frac{j\omega L/R}{1-\omega^2 LC + j\omega L/R}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{(\omega L/R)}{\sqrt{(1-\omega^2 LC)^2 + (\omega L/R)^2}}$$



$$Q \equiv \frac{\omega_0}{\Delta\omega} = \frac{R}{\sqrt{L/C}}$$

Numbers:

$L = 0.1$ H

$C = 0.25\ \mu F$

$R = 10\ k\Omega$

$\frac{1}{2\pi\sqrt{LC}} = 1\ kHz$

$\sqrt{L/C} = 632\ \Omega$

$Q = 16$

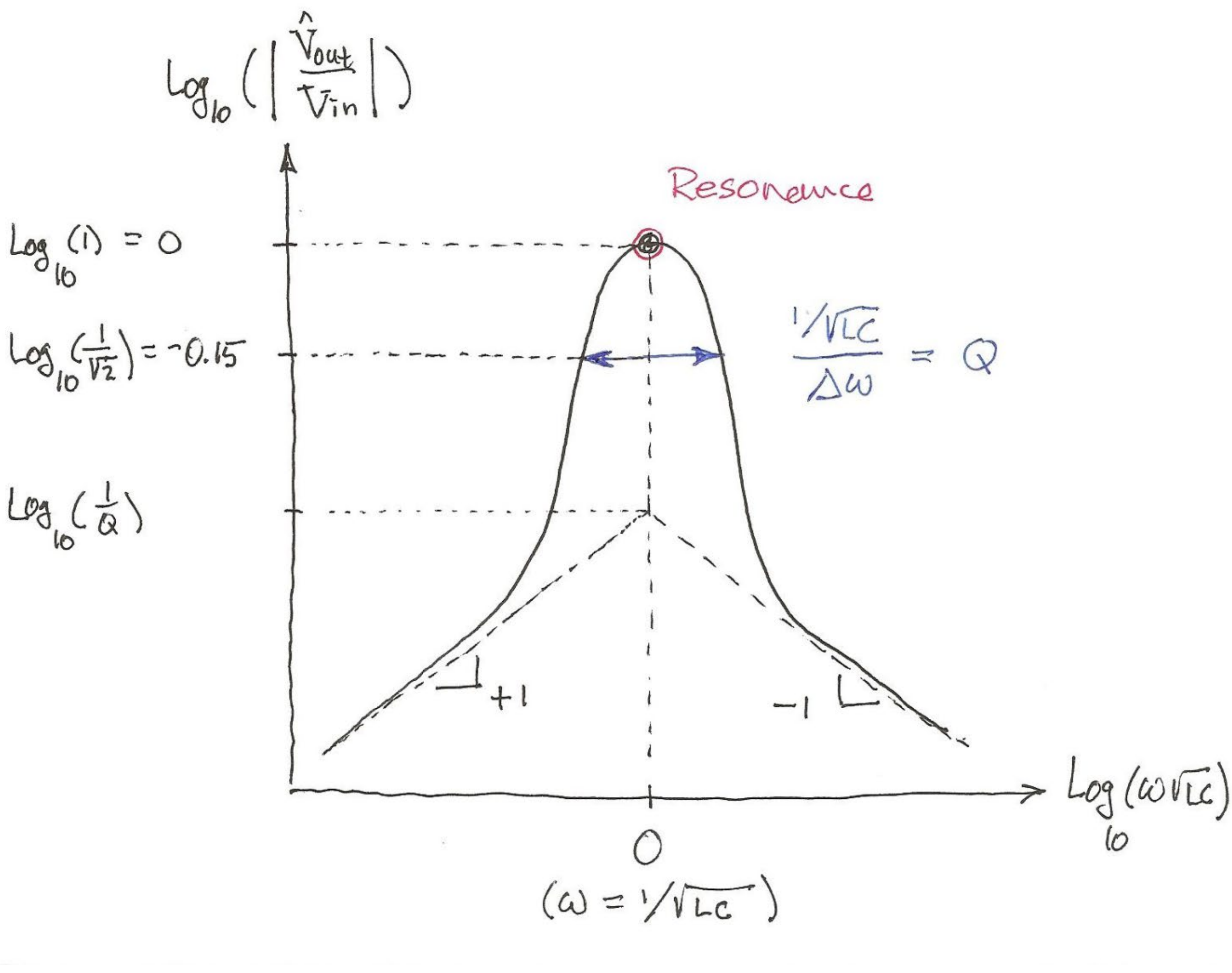
Theory: Parallel-LC BPF

Low ω: $\left|\frac{\hat{V}_{out}}{\hat{V}_{in}}\right| \approx \frac{\omega L}{R} = \frac{\omega\sqrt{LC}}{Q}$

High ω: $\left|\frac{\hat{V}_{out}}{\hat{V}_{in}}\right| \approx \frac{1}{\omega RC} = \frac{1}{\omega\sqrt{LC}\,Q}$

$\Bigg\}$ Assymptotes

Assymptotes cross at $\omega\sqrt{LC} = 1$ with value $1/Q$.



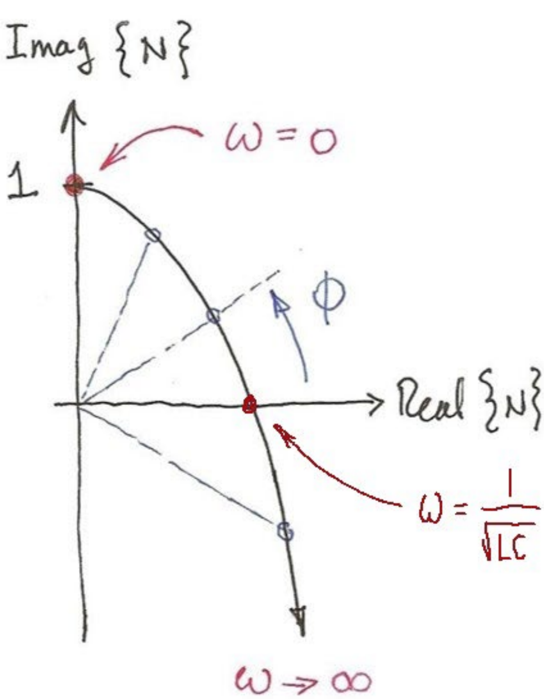
Theory: Parallel-LC BPF

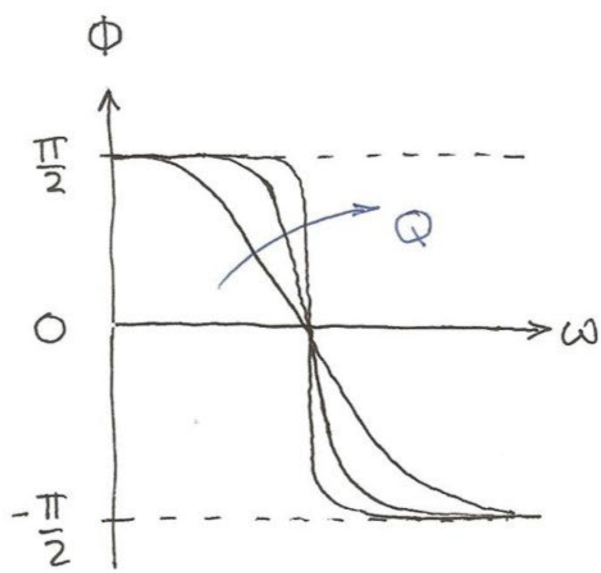
$$\measuredangle \left| \frac{\hat{V}_{out}}{V_{in}} \right| \equiv \phi = \measuredangle \frac{j\omega L/R}{1-\omega^2 LC + j\omega L/R}$$

$$= \measuredangle \frac{j\omega L/R\,(1-\omega^2 LC - j\omega L/R)}{(1-\omega^2 LC)^2 + (\omega L/R)^2}$$

$$= \measuredangle\ j(1-\omega^2 LC) + \omega L/R$$

$$\equiv \measuredangle\ N$$

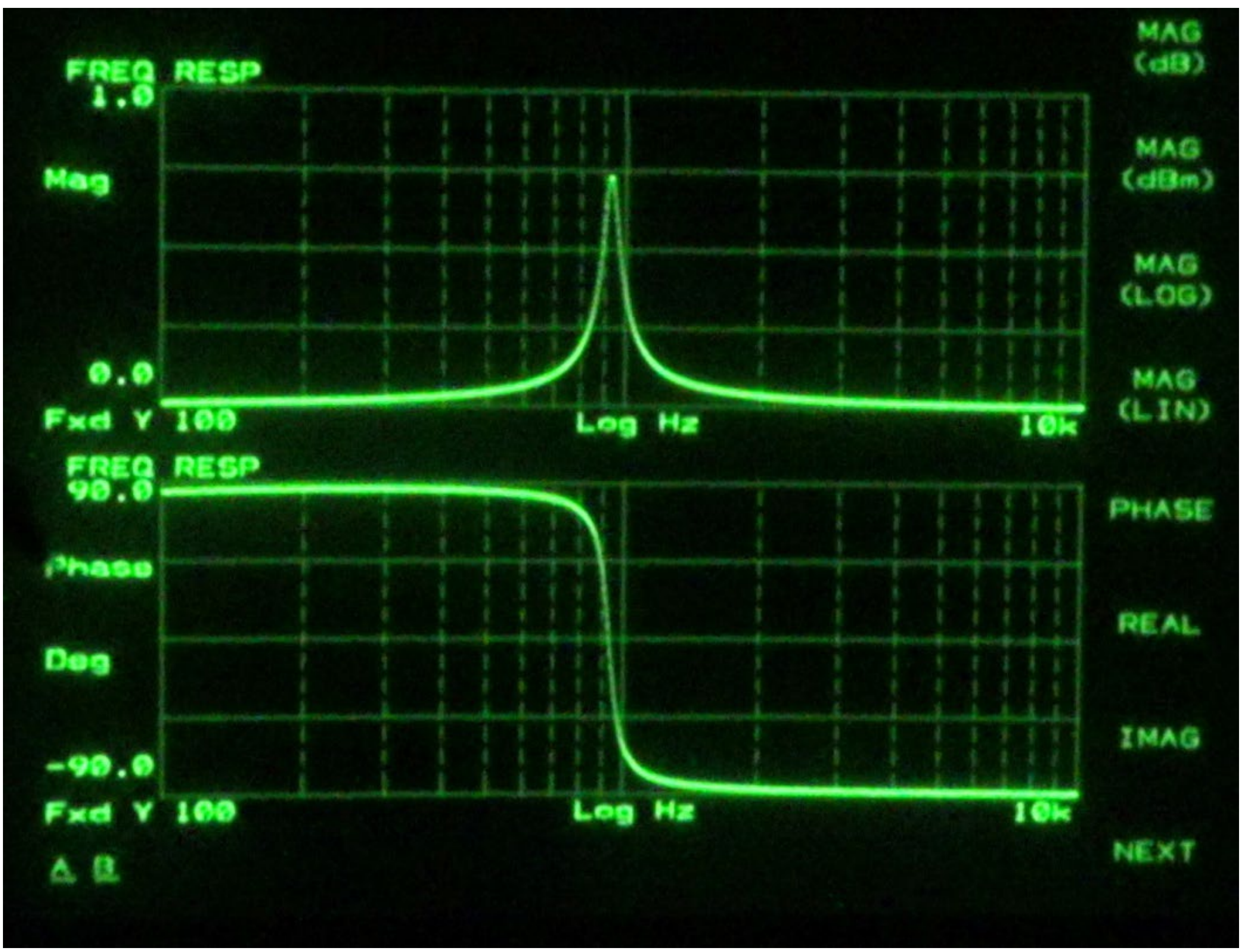




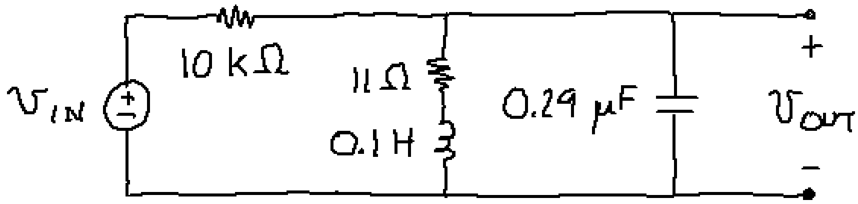
Demo: Parallel-LC BPF

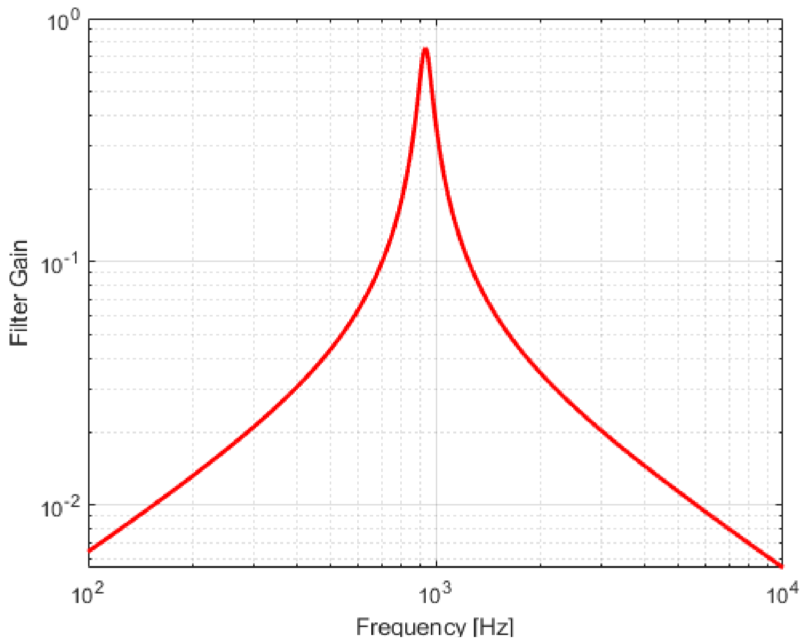
L = 0.1 H ; C = 0.25 μF ; R = 10 kΩ

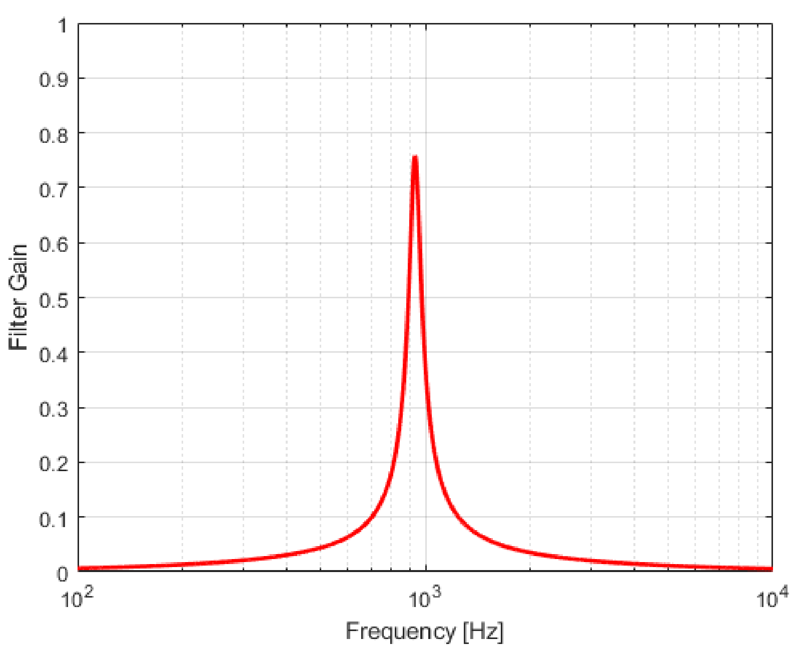
$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $R/\sqrt{L/C}$ = 16



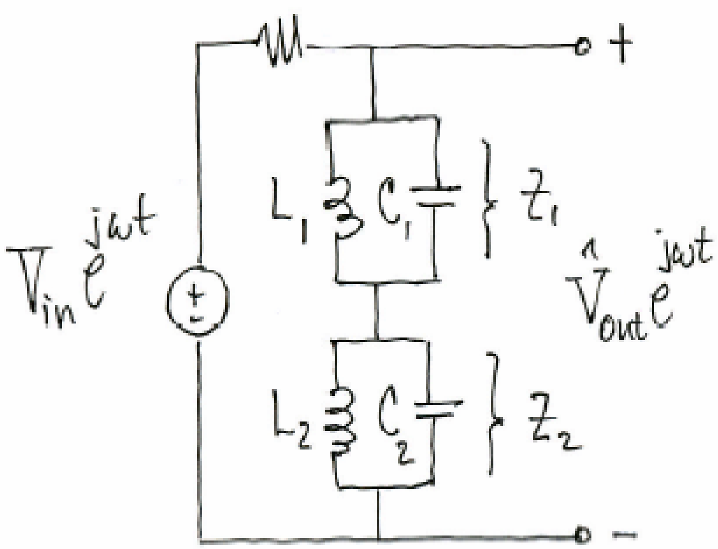
Numerical: Parallel-LC BPF







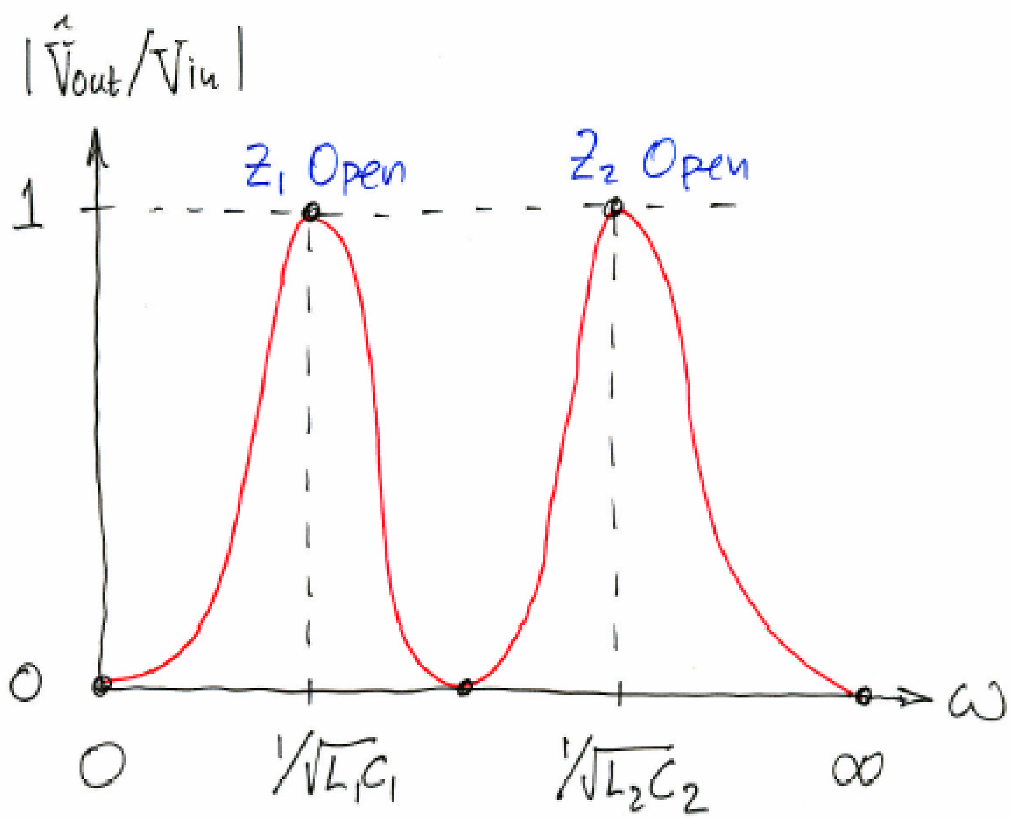
LC Comb Filter



Z_1 = Open at $\omega = \frac{1}{\sqrt{L_1 C_1}}$

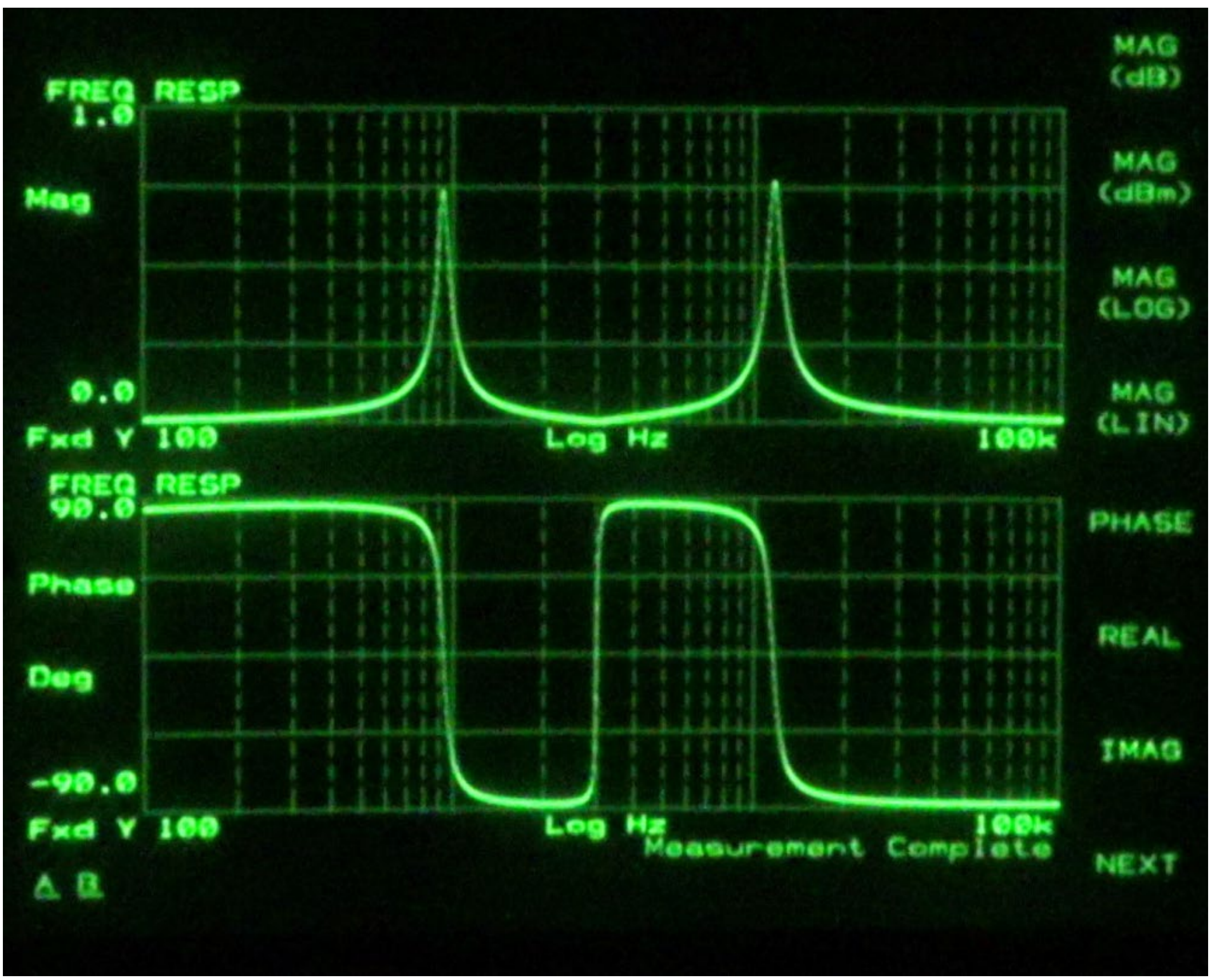
Z_2 = Open at $\omega = \frac{1}{\sqrt{L_2 C_2}}$

$Z_1 + Z_2$ = Short at $\omega = 0, \infty$ and between the two opens

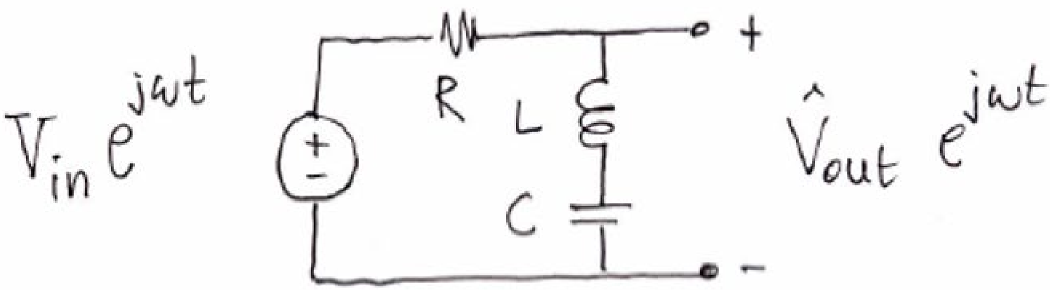


Demo: LC Comb Filter

$L_1 = 0.1$ H ; $C_1 = 0.25$ μF ; $f_1 = 1$ kHz
$L_2 = 0.01$ H ; $C_2 = 0.022$ μF ; $f_2 = 10$ kHz
R = 10 kΩ ; Q1 = Q2 = 16

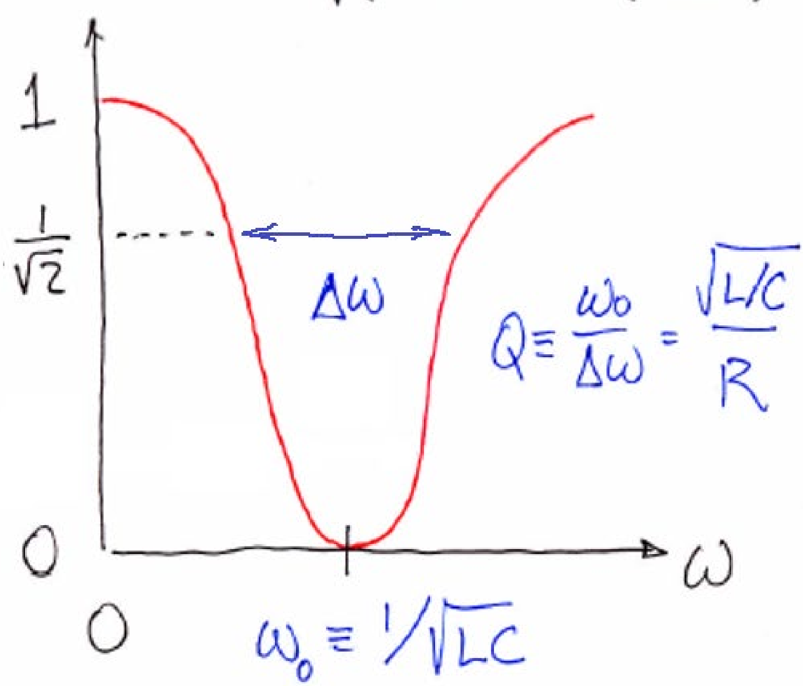


Theory: Series-LC BSF



$$\frac{\hat{V}_{out}}{V_{in}} = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1-\omega^2 LC}{1-\omega^2 LC + j\omega RC}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{1-\omega^2 LC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$



Numbers:

$L = 0.1\ \text{H}$

$C = 0.25\ \mu\text{F}$

$R = 100\ \Omega$

$\frac{1}{2\pi\sqrt{LC}} = 1\ \text{kHz}$

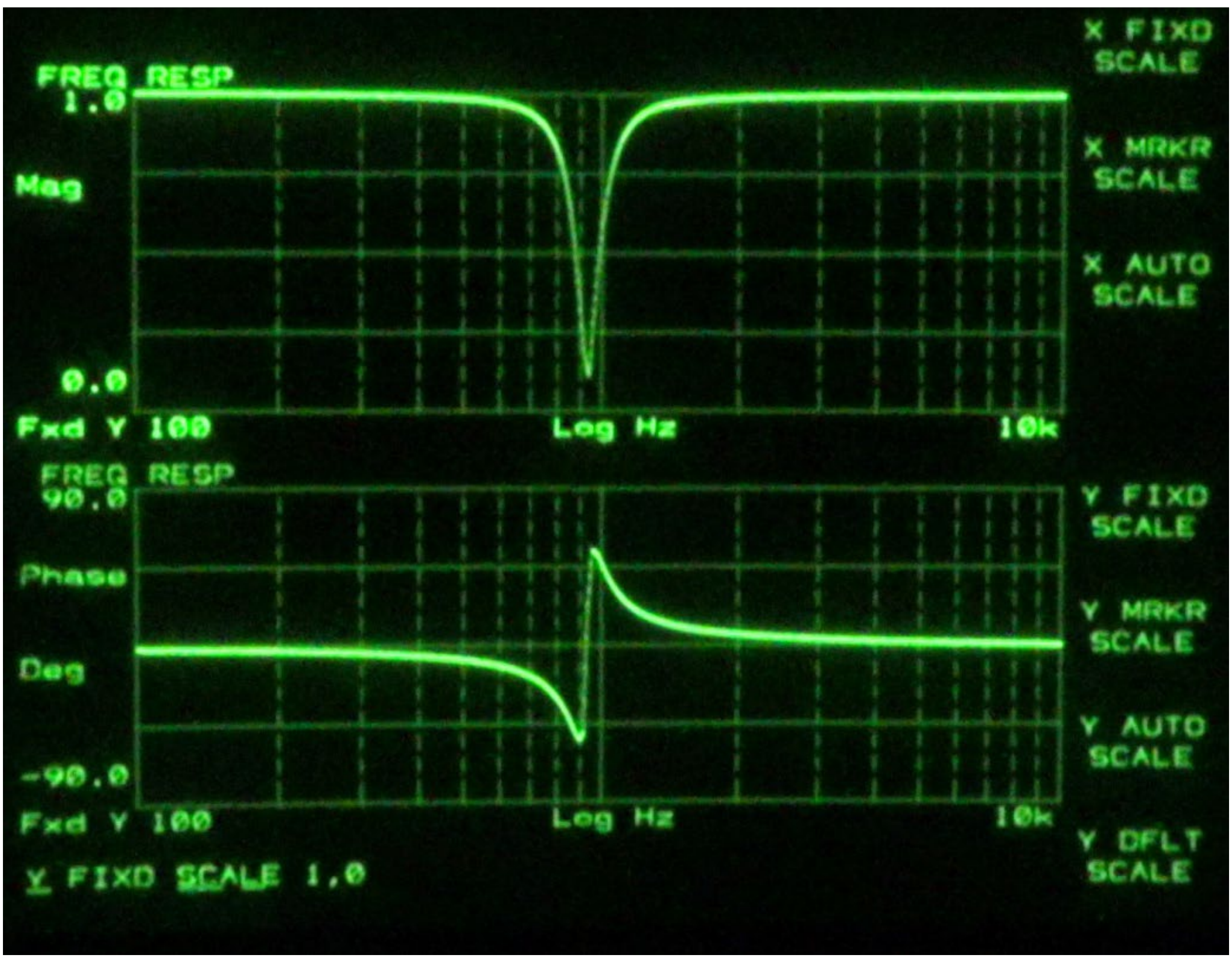
$\sqrt{L/C} = 632\ \Omega$

$Q = 6.3$

Demo: Series-LC BSF

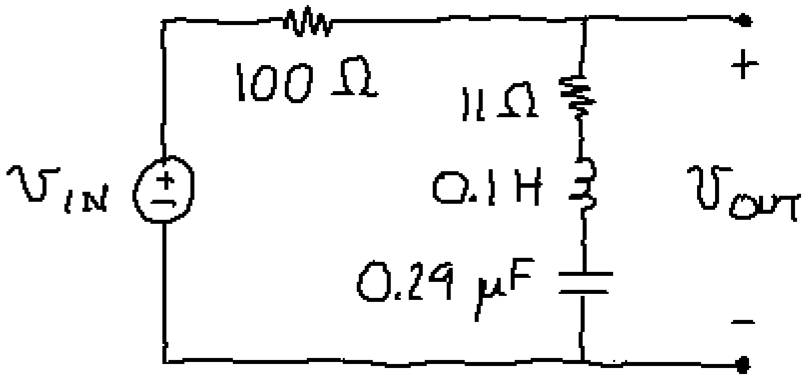
L = 0.1 H ; C = 0.25 μF ; R = 100 Ω

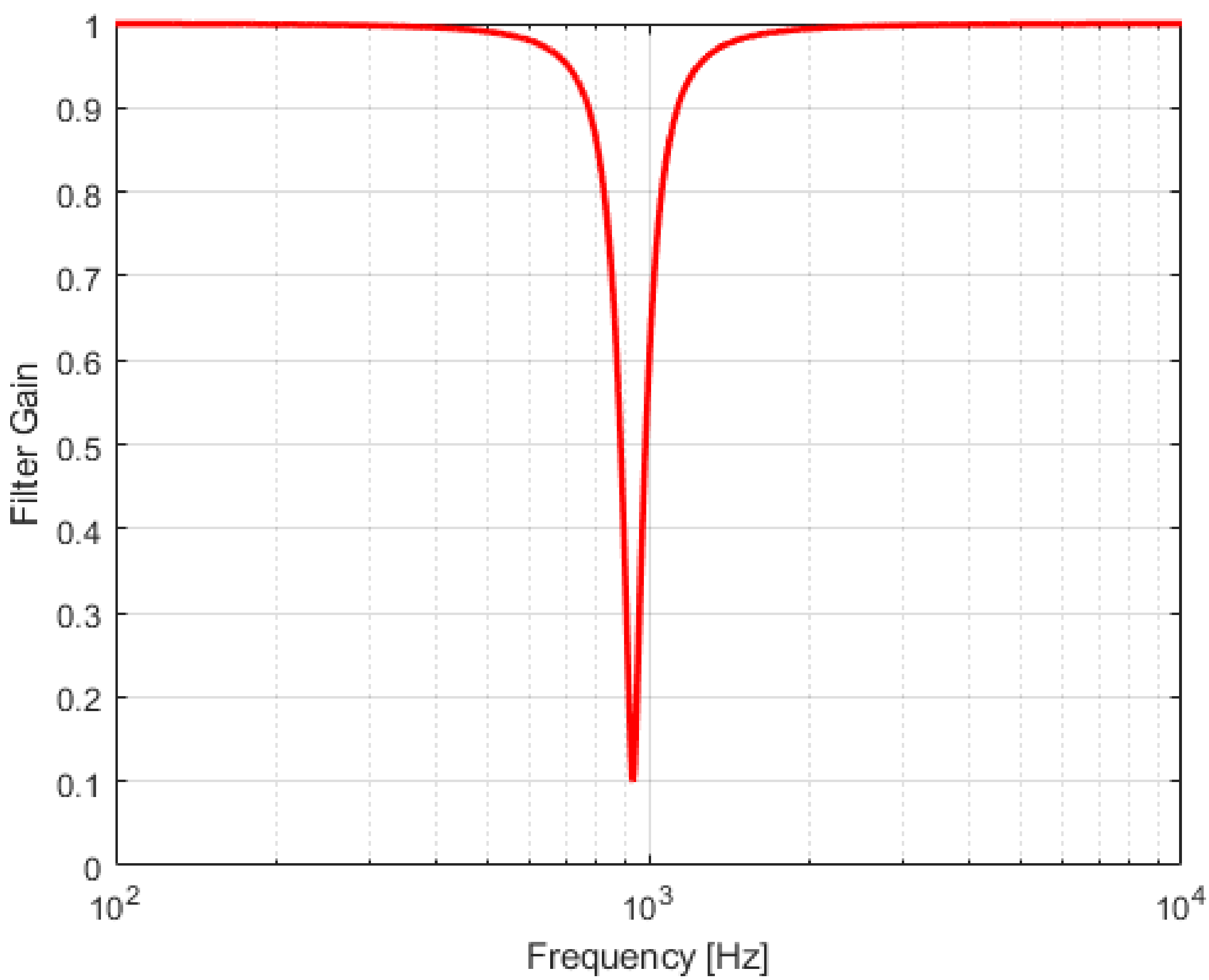
$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ / R = 6.3



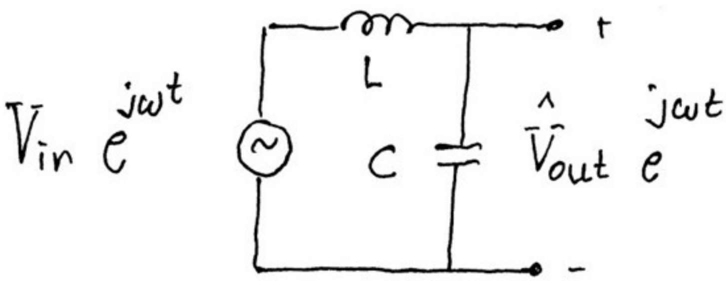
Note that the notch does not go to zero due to the parasitic series resistance of the inductor.

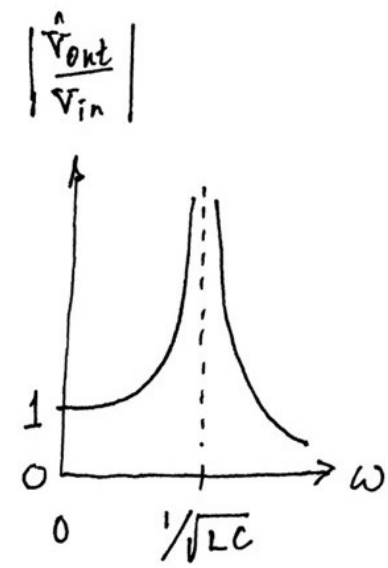
Numerical: Series-LC BSF





Resonators





$$\frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{1/j\omega C}{j\omega L + 1/j\omega C} = \frac{1}{1-\omega^2 LC}$$

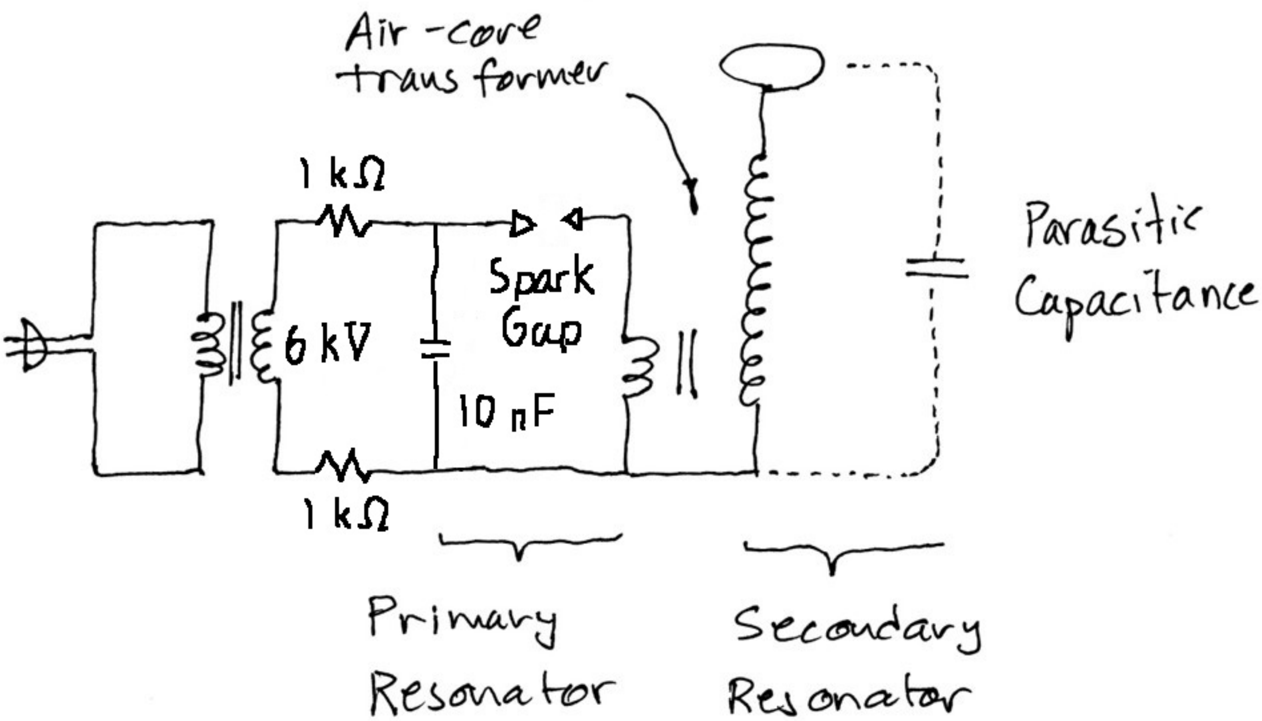
Peak voltage limited by parasitic resistances.

Applications include:

- capacitive shunts along inductive power-grid transmission lines;
- Tesla coil (dual tuned resonator).

Demo: Tesla Coil

The Tesla Coil employs two coupled resonators, with both resonators tuned to the same frequency.



The resistors protect the transformer. At 60 Hz, the transformer fill up the capacitor. When the capacitor voltage gets high enough, the spark gap sparks, and becomes a short allowing the LC resonator to oscillate, driving the rest of the Tesla coil.

Demo: Tesla Coil