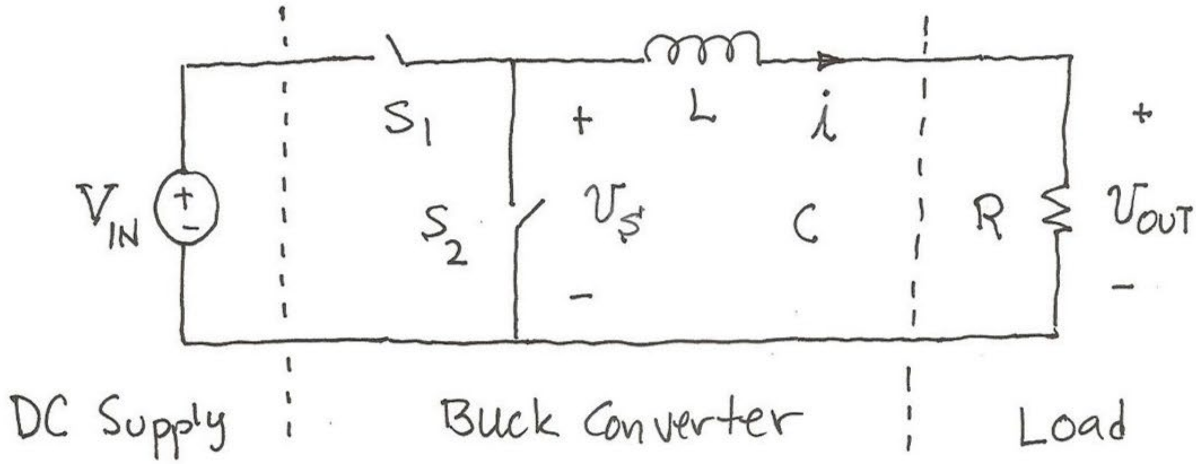
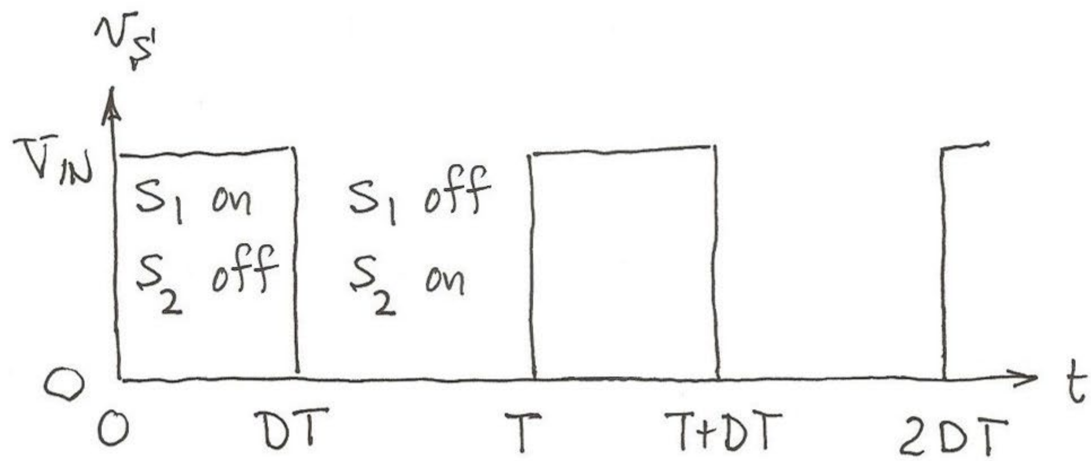
6.200 - Lecture 12A

Buck Converter

- Principles of operation
- Performance improvement
- Analysis via averaging
- Analysis via filtering
- Natural response

Buck Converter



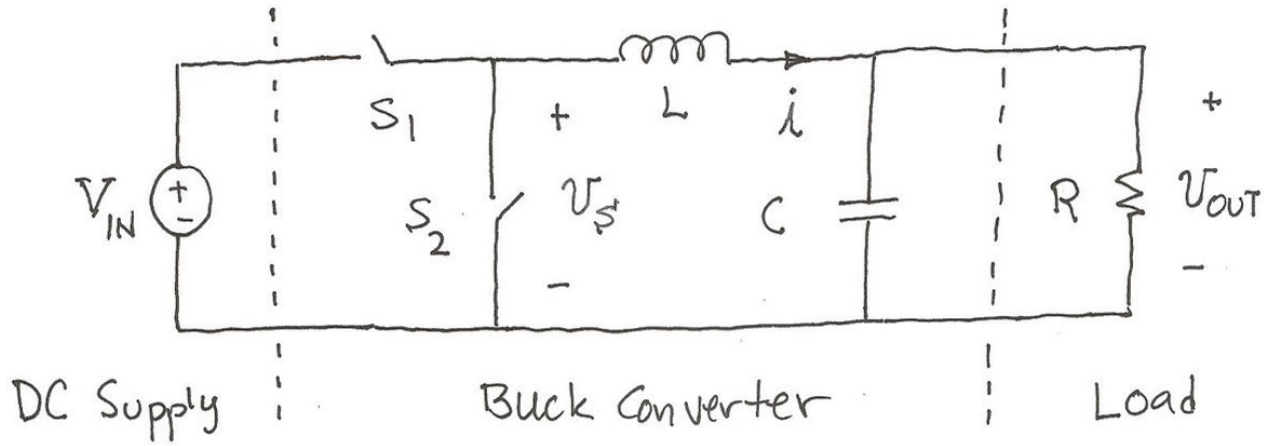


* Digital electronics wish to operate at low voltage (V_{OUT}) to reduce losses and heat.

* Power is cheaper to deliver at high voltages due to wiring costs at high current.

* Buck converters bridge the high → low voltage gap.

Improved Buck Converter



* The original buck converter exhibited a significant ripple current through, and hence a significant ripple voltage across, the load.

* The added capacitor offers a low-impedance path for the ripple current and hence filters out the load ripple current and voltage.

* But, the converter natural response may now be oscillatory.

Average Analysis

* Assume cyclic continuous ($i>0$) operation.

* Define an average: $\langle x(t) \rangle \equiv \frac{1}{T} \int_0^T x(s)\,ds.$

Inductor $\Rightarrow$ $v_s(t) - v_{OUT}(t) = L \frac{di}{dt}$

$$\langle v_s(t) \rangle - \langle v_{OUT}(t) \rangle = \frac{L}{T}\left(i(T) - i(0)\right) \equiv 0$$

$$\langle v_{OUT}(t) \rangle = D\,V_{IN}$$

Capacitor $\Rightarrow$ $i(t) - \frac{v_{OUT}(t)}{R} = C \frac{dv_{OUT}}{dt}$

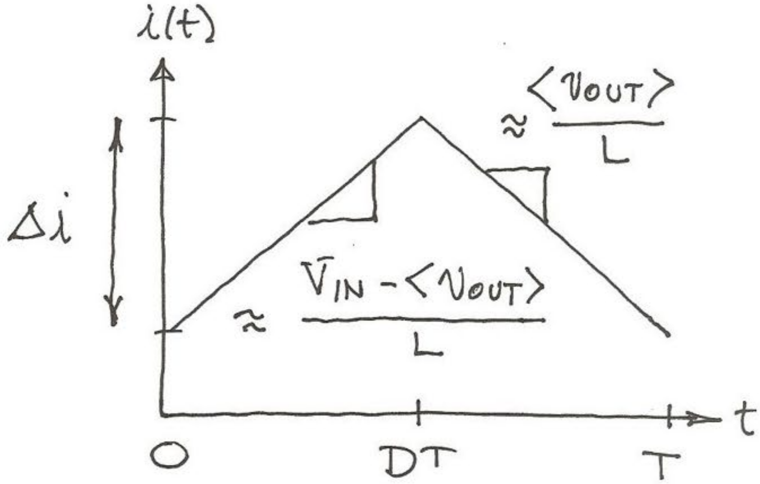
$$\langle i(t) \rangle - \frac{\langle v_{OUT}(t) \rangle}{R} = \frac{C}{T}\left(v_{OUT}(T) - v_{OUT}(0)\right) \equiv 0$$

$$\langle i(t) \rangle = \frac{\langle v_{OUT}(t) \rangle}{R} = \frac{D\,V_{IN}}{R}$$

Duty cycle D is used to control v_{OUT}.

Ripple Analysis

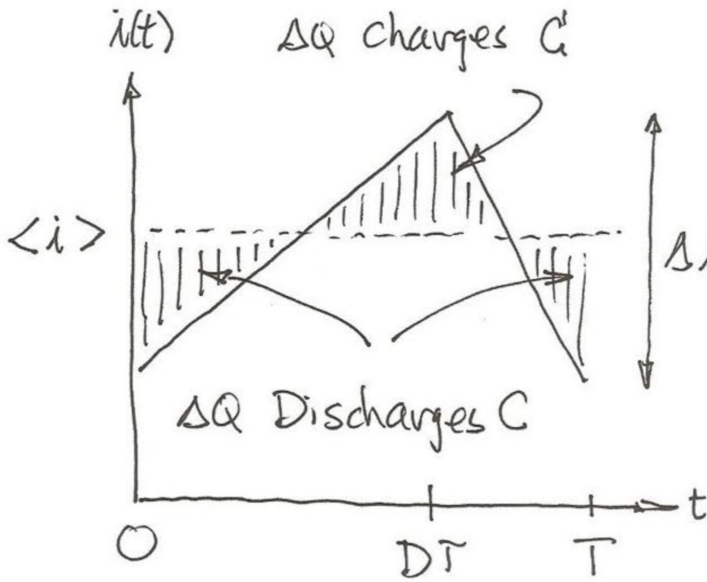
Ideal inductor $\Rightarrow$ inductor current is approximately piecewise linear.



$$\Delta i \approx \frac{\langle v_{OUT}\rangle}{L}(1-D)$$

$$= \frac{D(1-D)\,T\,\hat{V}_{IN}}{L}$$

High-frequency inductor current passes largely through capacitor because $\frac{1}{\omega C} \ll R$.



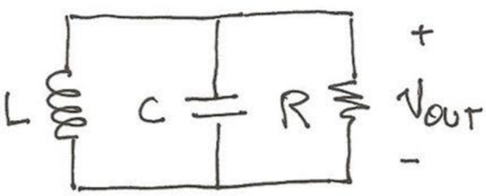
$$\Delta Q \approx \frac{1}{2}\,\frac{\Delta i}{2}\,\frac{T}{2}$$

$$= \frac{D(1-D)\,T^2\,\hat{V}_{IN}}{8L}$$

$$\Delta v_{OUT} = \frac{\Delta Q}{C}$$

$$= \frac{D(1-D)\,T^2\,\hat{V}_{IN}}{8LC}$$

Natural Response



$$C\frac{dv_{OUT}}{dt} + \frac{v_{OUT}}{R} + \frac{1}{L}\int_{-\infty}^{t} v_{OUT}\,dt = 0$$

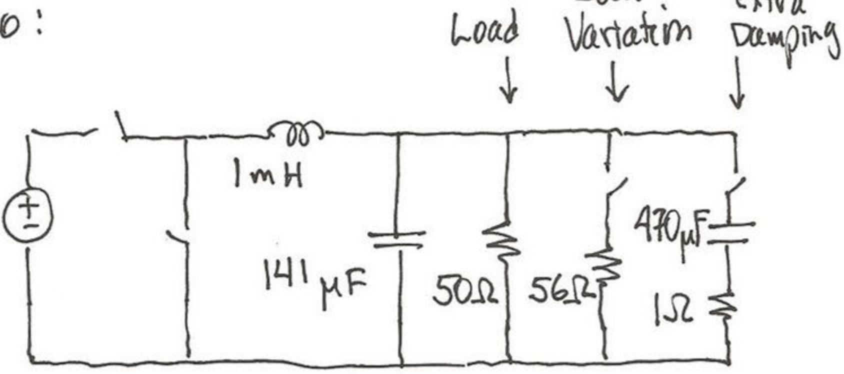
$$\frac{d^2 v_{OUT}}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha}\frac{dv_{OUT}}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} v_{OUT} = 0$$

$$e^{st} \Rightarrow s = -\alpha \pm j\sqrt{\omega_0^2-\alpha^2} \underset{\text{Lightly Damped}}{\approx} -\alpha + j\omega_0$$

Natural response will appear for:

* change in input voltage V_{IN};
* change in duty cycle D;
* change in load resistance R.

Demo:



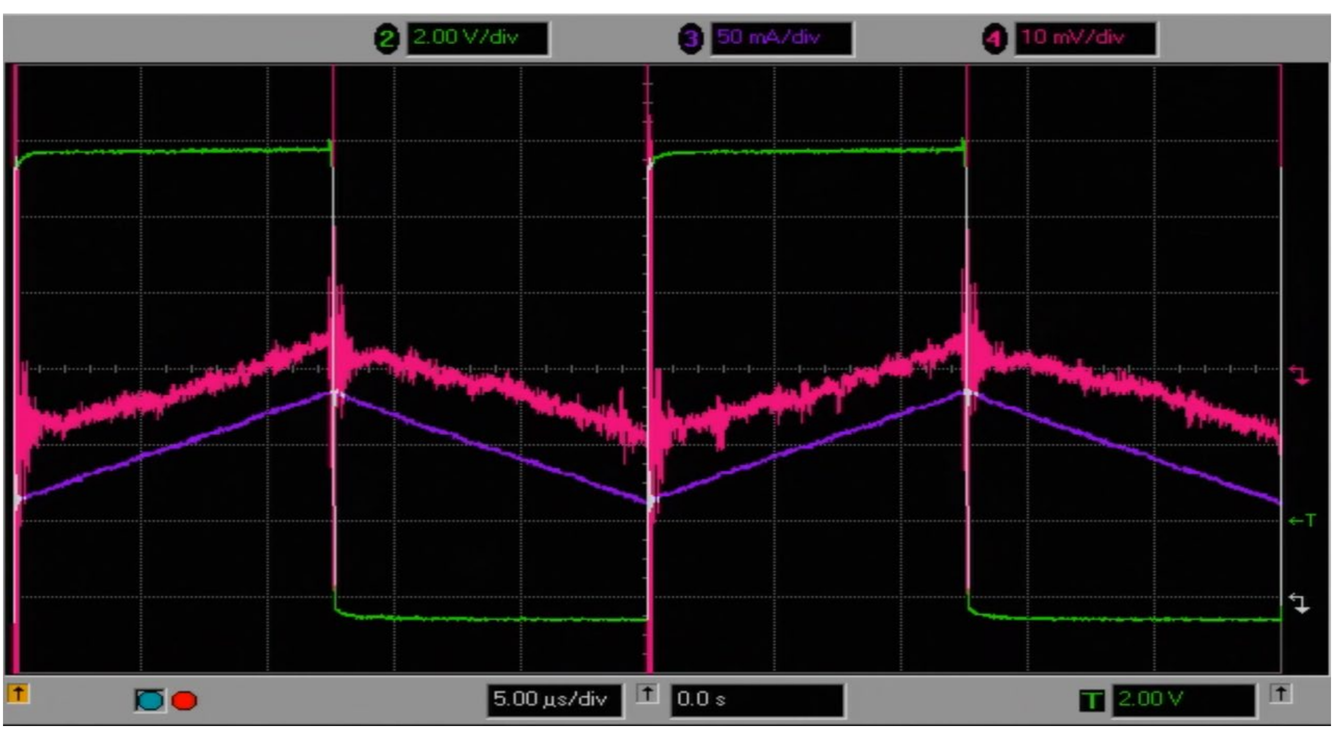
Demo - Ripple Analysis

$L = 1\,mH$ $\quad D = 0.5$ $\quad V_{IN} = 12\,V$

$C = 141\,\mu F$ $\quad T = 25\,\mu s$

$$\Delta i = \frac{D(1-D)\,T\,V_{IN}}{L} = 75\ mA$$

$$\Delta V_{OUT} = \frac{\Delta i\,T}{C} = 13\ mV$$



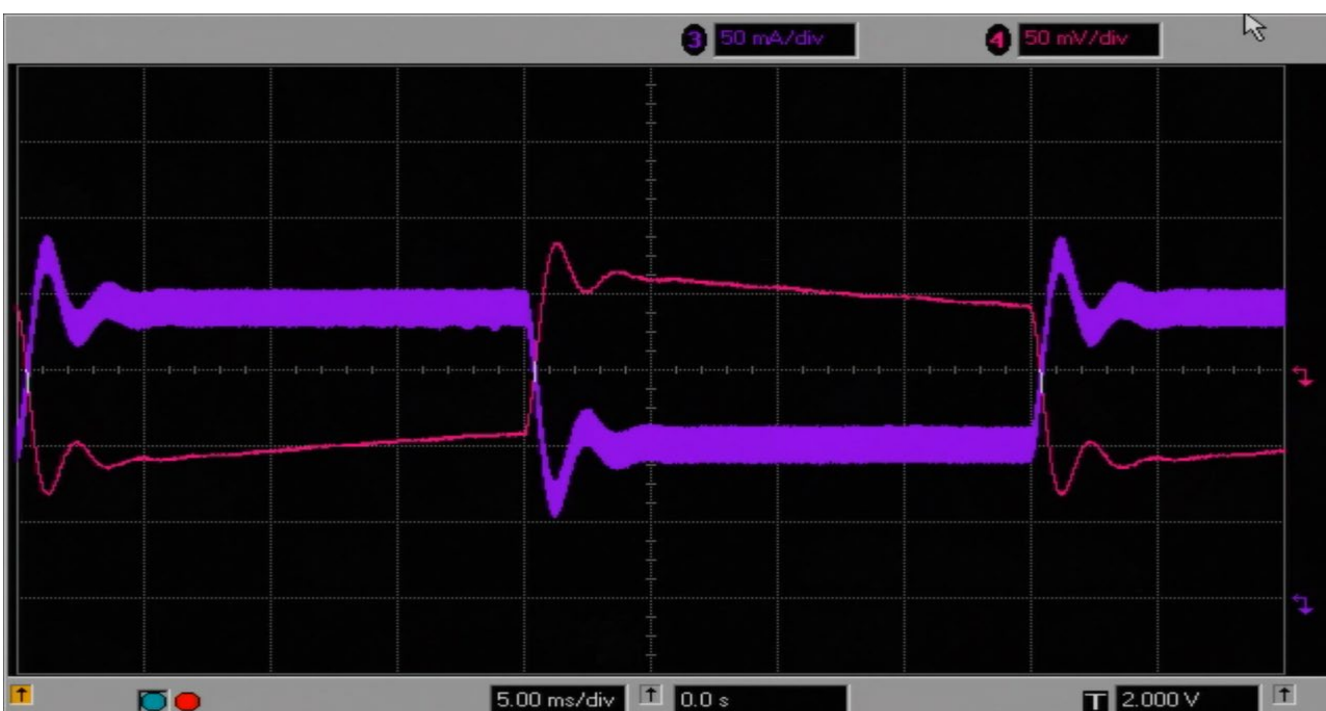
Demo - Natural Response

$L = 1\ mH$ $\quad$ $D = 0.5$ $\quad$ $V_{IN} = 12\ V$

$C = 141\ \mu F$ $\quad$ $T = 25\ \mu s$ $\quad$ $R = 50\ \Omega$

$$\text{Period} = 2\pi\sqrt{LC} = 2.4\ ms \leftrightarrow 424\ Hz$$

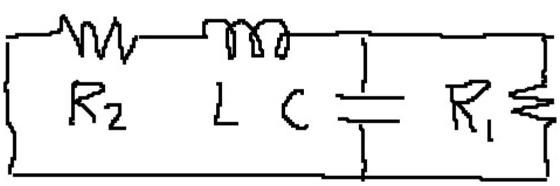
$$Q = \frac{R}{\sqrt{L/C}} = 19 \text{ Without Inductor Loss}$$



Q

$$Q = 2\pi \frac{\text{Stored Energy}}{\text{Energy Lost Per Cycle}}$$

$$\frac{1}{Q} = \frac{1}{2\pi} \frac{1}{\text{Stored Energy}} \sum_i \text{Cycle Loss}_i$$



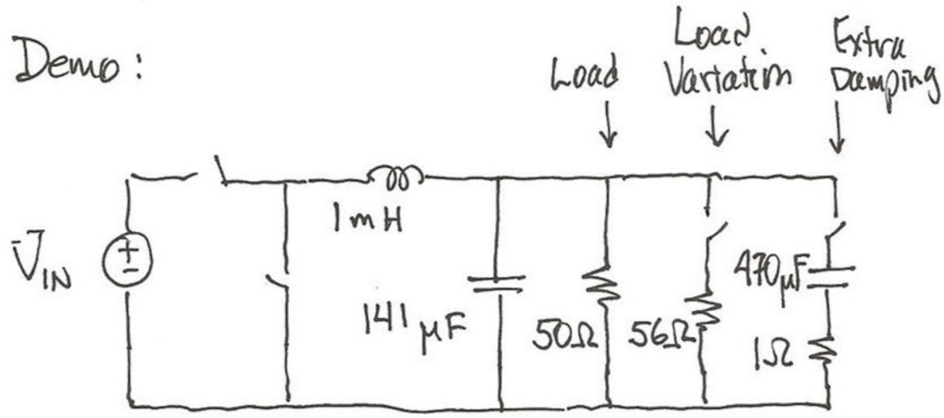
$$Q_1 = \frac{R_1}{\sqrt{L/C}} \quad Q_2 = \frac{\sqrt{L/C}}{R_2}$$

$L = 1\ \mathrm{mH}$ $C = 141\ \mu\mathrm{H}$ $R_1 = 50\ \Omega$ $R_2 = 0.5\ \Omega$

$Q_1 = 19$ $Q_2 = 5.3$ $Q = 4.2$

Damping

Demo:



Objective is to reduce the oscillatory load-step response.

* Does not affect average (DC) behavior because of series capacitor.

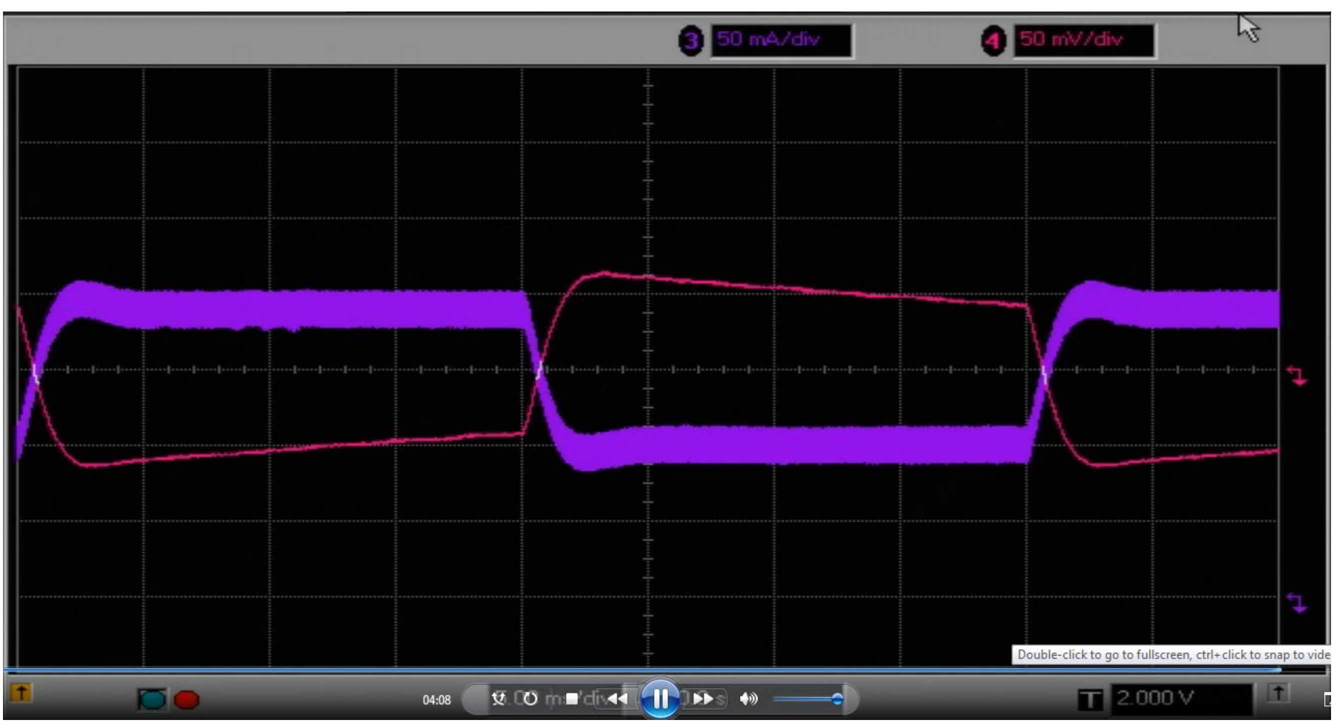
* Does not carry much ripple current (get hot) because $1\Omega \gg 1/(\omega_s \cdot 141\,\mu F)$.

* Provides significant damping because 1Ω and $1/(\omega_N \cdot 470\,\mu F) \ll 50\,\Omega$.

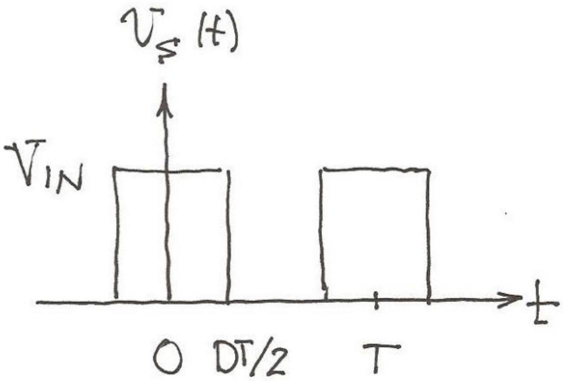
Damping

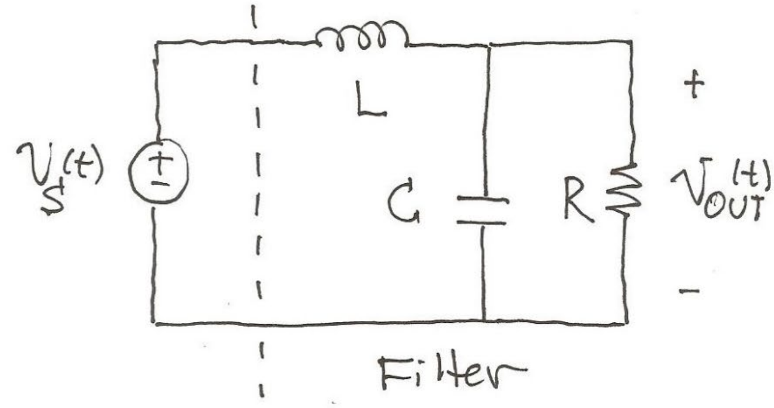
Damping network $\Rightarrow$ $R = 1\,\Omega$; $C = 470\,\mu F$

* $1/\omega_N C = 0.8\,\Omega$ at $\omega_N = 424$ Hz $\Rightarrow$ small but not strictly negligible. Ignore anyway.
* $Q = \dfrac{R}{Z_0} = 0.4$ $\Rightarrow$ Overdamped



Buck Converter ⇒ Low-Pass Filter





Fourier decomposition of $V_S(t)$:

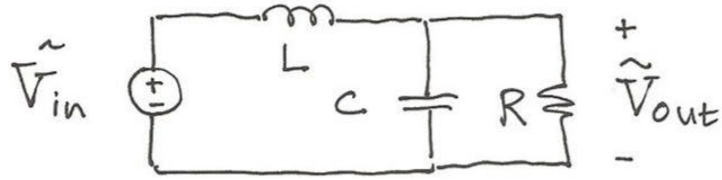
$$V_S(t) = D V_{IN} + \sum_{n=1}^{\infty} \frac{2 V_{IN} \sin(n \pi D)}{n\pi} \cos(n \omega_s t)$$

$$\omega_s T \equiv 2\pi$$

Filter each Fourier component separately and then recombine results to get complete response.

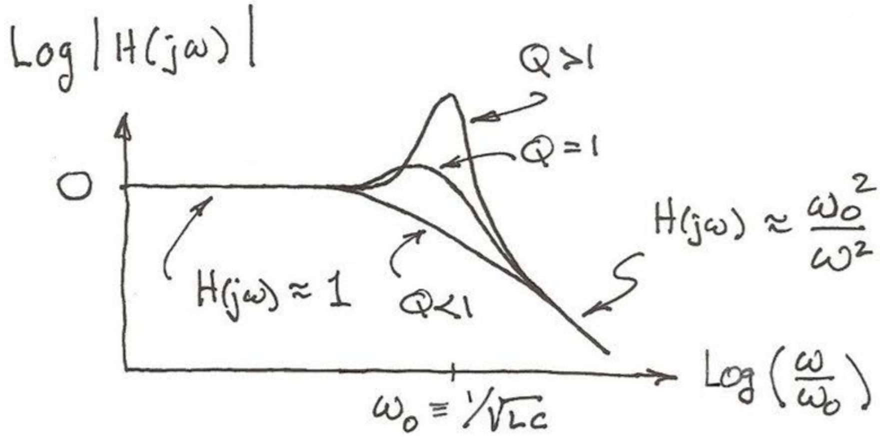
Can do this for all signals: $V_{OUT}(t)$, $i(t)$, ...

Filter Analysis (Driven Response)



$$\tilde{V}_{out} = \frac{\left(R \parallel \frac{1}{j\omega C}\right)\tilde{V}_{in}}{j\omega L + \left(R \parallel \frac{1}{j\omega C}\right)} = \frac{\tilde{V}_{in}}{1 - \omega^2 LC + \frac{j\omega L}{R}} \equiv H(j\omega)\,\tilde{V}_{in}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \left(\frac{\omega L}{R}\right)^2}} \qquad \measuredangle H(j\omega) = \mathrm{Tan}^{-1}\left(\frac{-\omega L/R}{1-\omega^2 LC}\right)$$



$$\omega_0 \equiv \frac{1}{\sqrt{LC}}$$

$$Z_0 \equiv \sqrt{\frac{L}{C}}$$

$$Q \equiv \frac{R}{Z_0}$$

$$H(j\omega_0) = Q$$

Demo $\Rightarrow$ $L = 1\,\mathrm{mH}$; $C = 141\ \mu F$

$\Rightarrow$ $\omega_0 = 2.6\ \frac{\mathrm{krad}}{\mathrm{s}}$ (424 Hz)

Filtering Interpretation

- At $\omega = 0$, $H(j\omega) = 1 \Rightarrow \tilde{V}_{out} = D\bar{V}_{IN} = \langle v_s(t) \rangle$.

  Same result as before!

- $f_s = 40\ kHz \Rightarrow \omega_s = 80\pi\ \frac{krad}{s} \gg \omega_o = 2.6\ \frac{krad}{s}$.

  $\Rightarrow$ All switching harmonics are greatly attenuated

  $\Rightarrow$ Small ripple!

- Ripple fundamental $\Rightarrow \dfrac{2\bar{V}_{IN}\sin(\pi D)}{\pi}\dfrac{\omega_o^2}{\omega_s^2}$ amplitude.

  For $D = \frac{1}{2}$, amplitude $= \dfrac{2\bar{V}_{IN}}{\pi}\dfrac{T^2}{4\pi^2 LC}$

  $= \dfrac{T^2\bar{V}_{IN}}{2\pi^3 LC} \approx \dfrac{T^2\bar{V}_{IN}}{62\ LC}$

  Compare with $\dfrac{1}{2}\dfrac{T^2\bar{V}_{IN}}{32\ LC} = \dfrac{T^2\bar{V}_{IN}}{64\ LC}$ from ripple analysis!