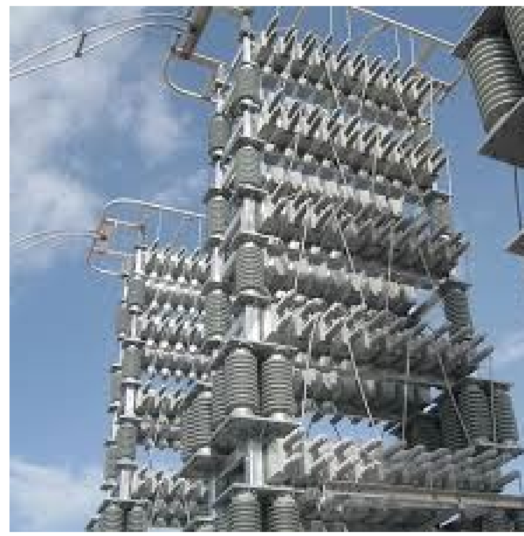
6.200 - Lecture 12B

Active “LC” Filters

- Successful LC Filters
- Active Filter Motivation
- Sallen-Key Topologies
- Examples

Successful LC Filters

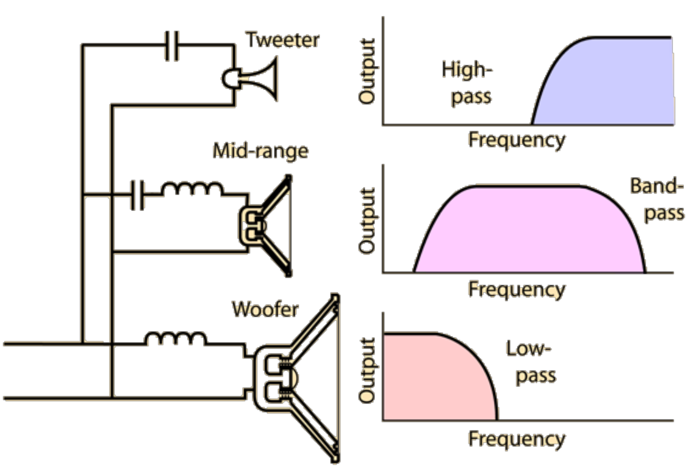
Power System Inductor & Capacitor Banks

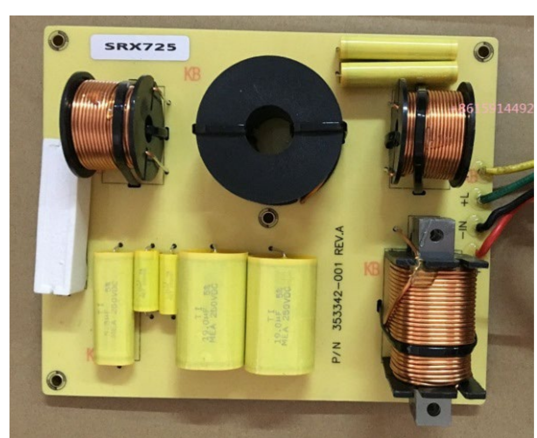


eaton.com

https://electrical-engineering-portal.com

Audio Crossover Networks

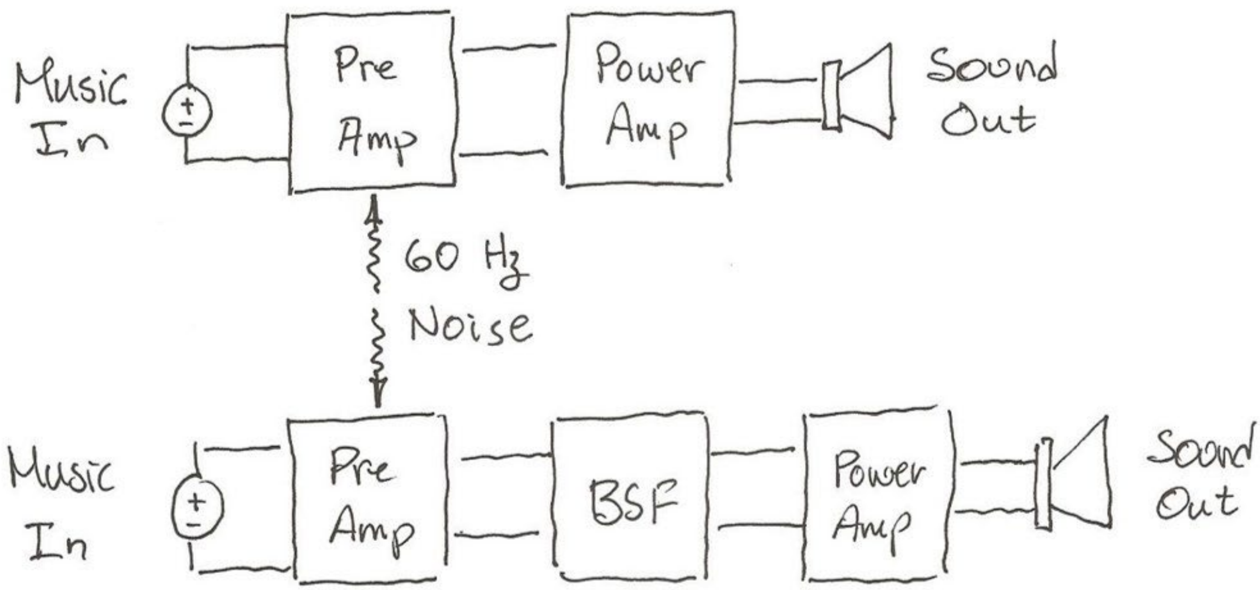




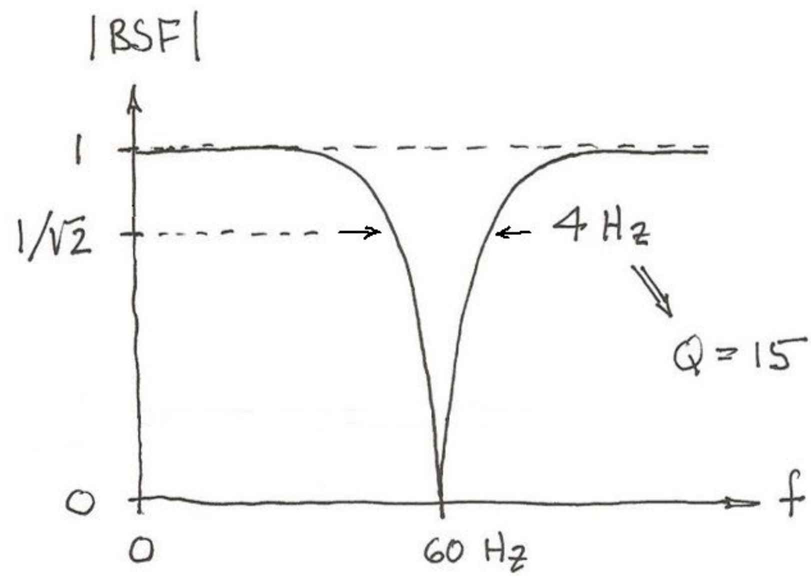
http://hyperphysics.phy-astr.gsu.edu

guangshou shengda audio

Motivating Example

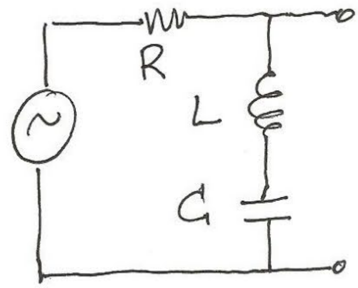


(Best is to eliminate the noise itself.
Next best is to employ a bandstop filter.)



Passive BSF

Design



$\omega_0 = 1/\sqrt{LC}$

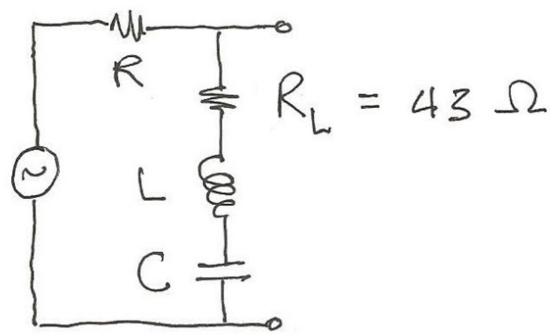
$Z_0 = \sqrt{L/C}$

$Q = Z_0/R$

$L = 1\,H$ $\quad C = 7\,\mu F$ $\quad R = 25\,\Omega$

$\omega_0 = 377\ \frac{rad}{s} = 60\ Hz$ $\quad Z_0 = 377\,\Omega$ $\quad Q = 15$

Reality



$$\text{Minimum Gain} = \frac{43}{43+25} = 0.63$$

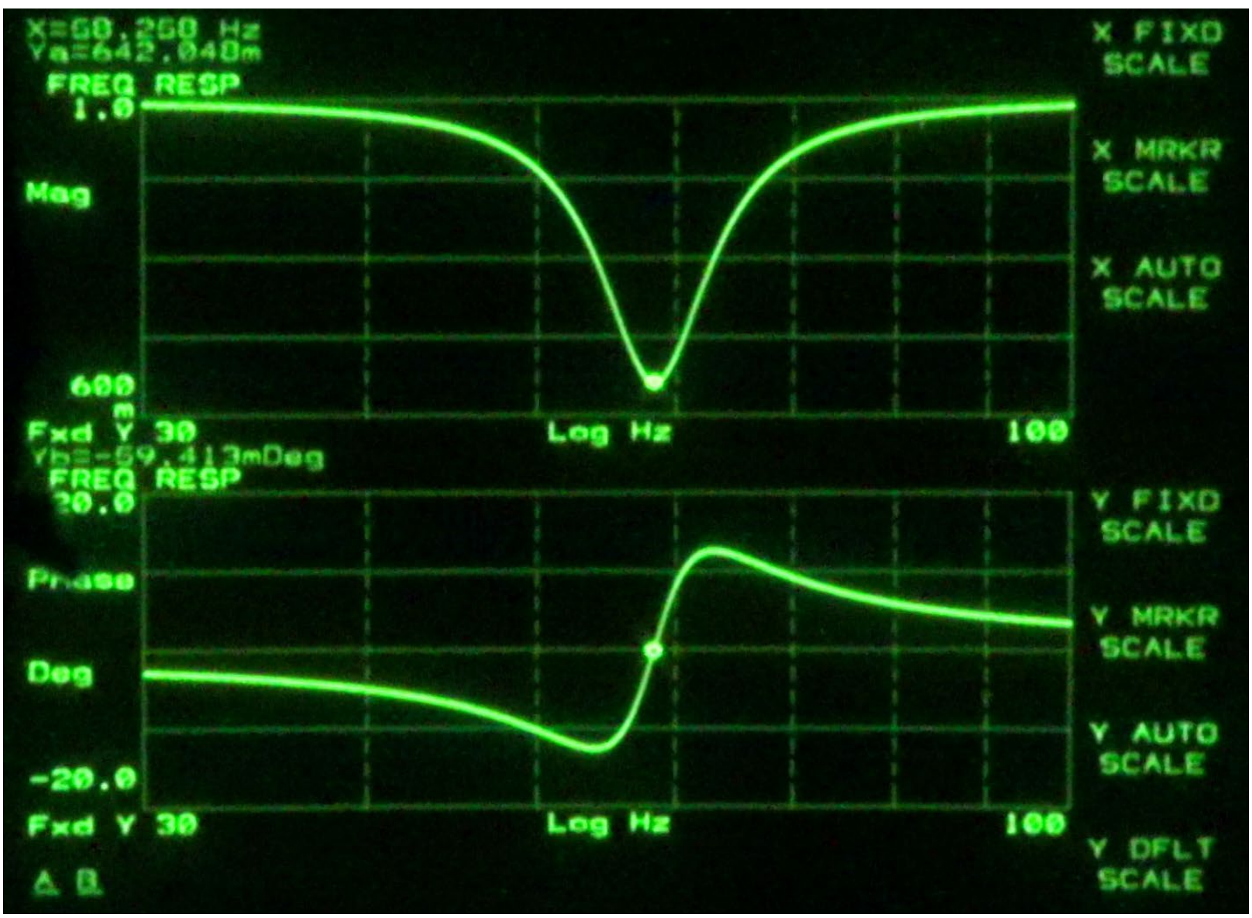
$$Q = Z_0/(R + R_L) = 5.5$$

Demo

L = 1 H ; C = 7 μF ; R = 25 Ω (Hopefully)

$(2\pi\sqrt{LC})^{-1}$ = 60 Hz ; $\sqrt{L/C}$ = 377 Ω

Q = 15 (Hopefully)

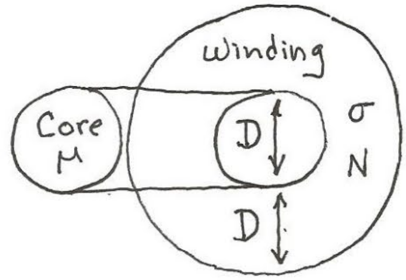


The DC resistance of the inductor is 43 Ω, so the stop band will drop only to 0.63!

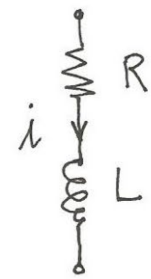
Magnetics Scaling

Inductor

$\Rightarrow$



$\Rightarrow$



$$L = \frac{N^2 \mu \text{ Area}}{\text{Length}} = \frac{N^2 \mu \, \pi D^2/4}{\pi D} = \frac{N^2 \mu D}{4}$$

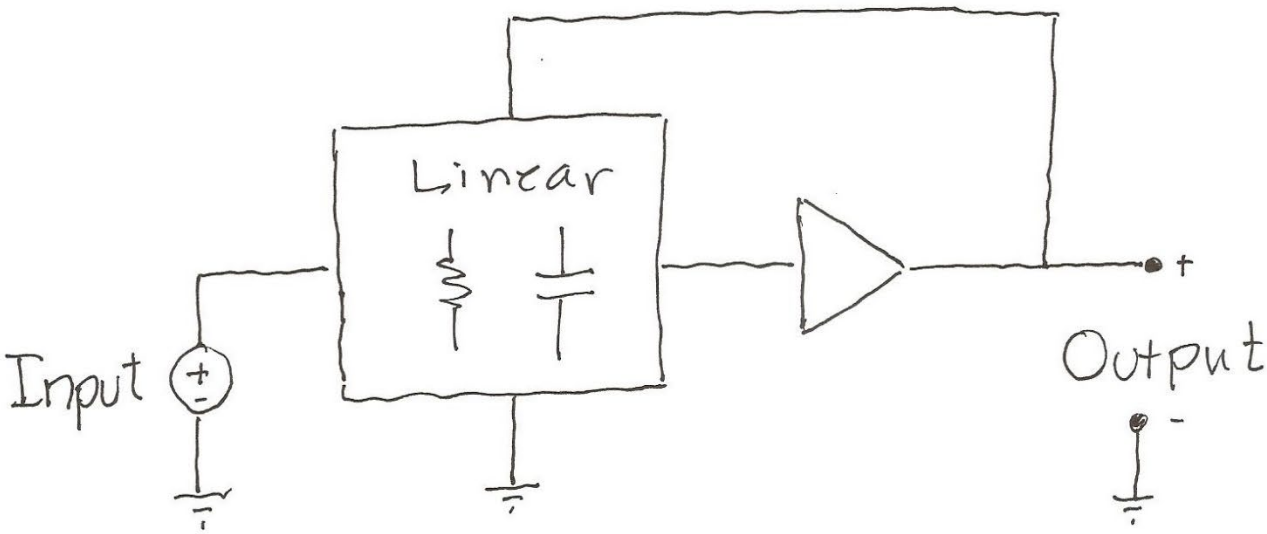
$$R = \frac{N^2 \text{ Length}}{\sigma \text{ Area}} = \frac{N^2 \pi D}{\sigma \pi D^2/4} = \frac{4N^2}{\sigma \pi D}$$

Sinusoidal Steady State $\Rightarrow$ $i = I \cos(\omega t)$

$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy Lost Per Cycle}} = 2\pi \frac{L I^2/2}{\frac{1}{2} I^2 R \frac{2\pi}{\omega}} = \frac{\omega \mu \sigma D^2}{64}$$

$$Q \sim D^2 \Rightarrow \begin{cases} \text{Bigger is better} \\ \text{Smaller is worse} \end{cases}$$

Single-Amplifier Topology



Amplifier:

- Linear
- Finite ± Gain gain
- Infinite bandwidth
- Infinite input impedance
- Zero output impedance

Performance Issues

- Gain versus frequency
- Phase versus frequency
- Sensitivity to component values
- Sensitivity to amplifier behavior
- Simplicity and cost

Unity-Gain Second-Order Filters

$$s \equiv j\omega \quad \longleftrightarrow \quad d/dt$$

LPF $$\frac{V_{out}}{V_{in}} = \frac{\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$

HPF $$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + s\omega_0/Q + \omega_0^2}$$

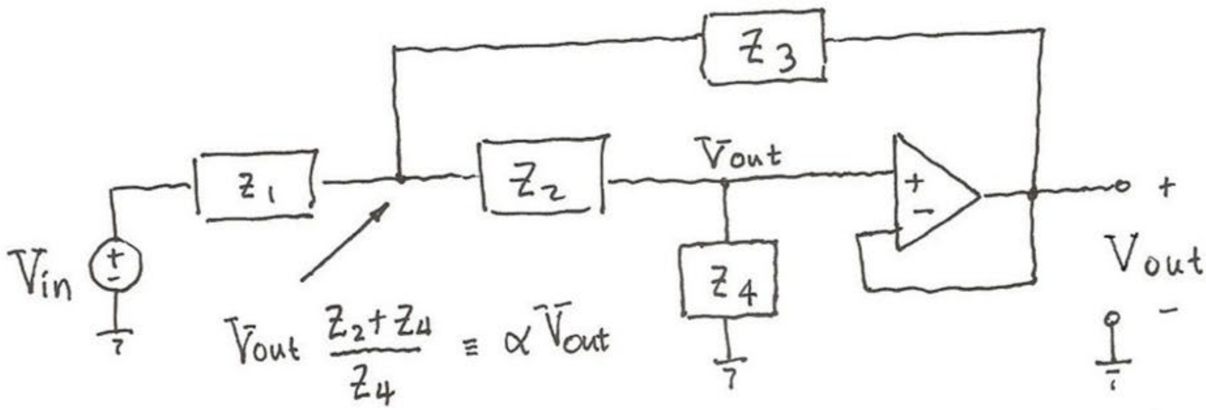
BPF $$\frac{V_{out}}{V_{in}} = \frac{s\omega_0/Q}{s^2 + s\omega_0/Q + \omega_0^2}$$

BSF $$\frac{V_{out}}{V_{in}} = \frac{s^2 + \omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$

$$\ddot{V}_{OUT} + \frac{\omega_0}{Q}\dot{V}_{OUT} + \omega_0^2 V_{OUT} = \underbrace{\ddot{V}_{IN} + \frac{\omega_0}{Q}\dot{V}_{IN} + \omega_0^2 V_{IN}}_{\text{Selected by filtering type}}$$

Simple Sallen-Key Filter

R. P. Sallen and E. L. Key, “A Practical Method of Designing RC Active Filters”, IRE Circuit Theory, 2:1, 74-85, March 1955



$$V_{out}\,\frac{Z_2+Z_4}{Z_4} \equiv \alpha V_{out}$$

$$\text{KCL} \Rightarrow \frac{\alpha V_{out} - V_{in}}{Z_1} + \frac{V_{out}}{Z_4} + \frac{\alpha V_{out} - V_{out}}{Z_3} = 0$$

$$V_{in}\left[\frac{1}{Z_1}\right] = V_{out}\left[\frac{\alpha}{Z_1} + \frac{1}{Z_4} + \frac{Z_2}{Z_3 Z_4}\right]$$

$$V_{in} = V_{out}\left[\frac{Z_2+Z_4}{Z_4} + \frac{Z_1}{Z_4} + \frac{Z_1 Z_2}{Z_3 Z_4}\right]$$

$$V_{out} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4}\,V_{in}$$

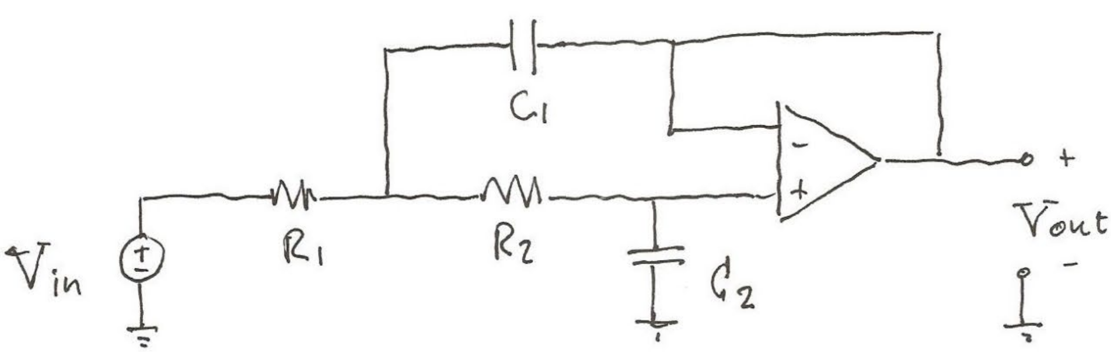
$$= \frac{Y_1 Y_2}{Y_1 Y_2 + Y_1 Y_4 + Y_2 Y_4 + Y_3 Y_4}\,V_{in}$$

Example: LPF Design

Resistor: $Z = R$; $Y = 1/R$

Capacitor: $Z = 1/sC$; $Y = sC$

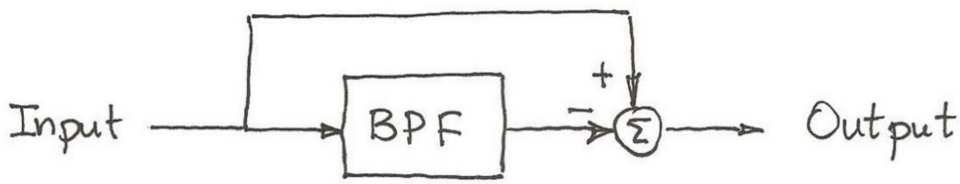
Constant LPF transfer function numerator $\Rightarrow Y_1 = 1/R_1$ and $Y_2 = 1/R_2$. Second-order system $\Rightarrow Y_3 = sC_1$ and $Y_4 = sC_2$.

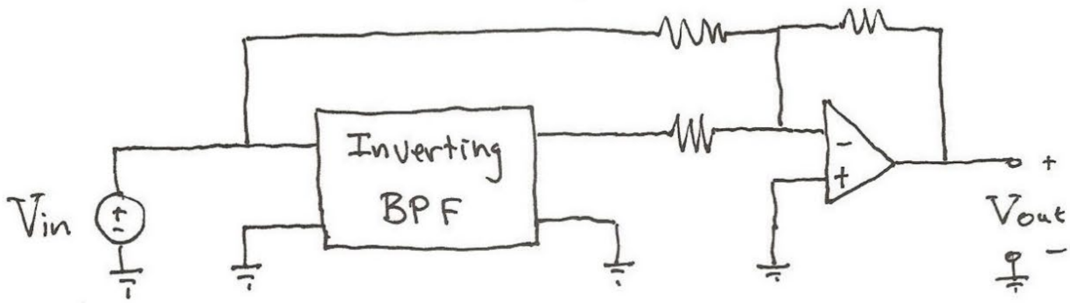


$$V_{out} = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}} V_{in}$$

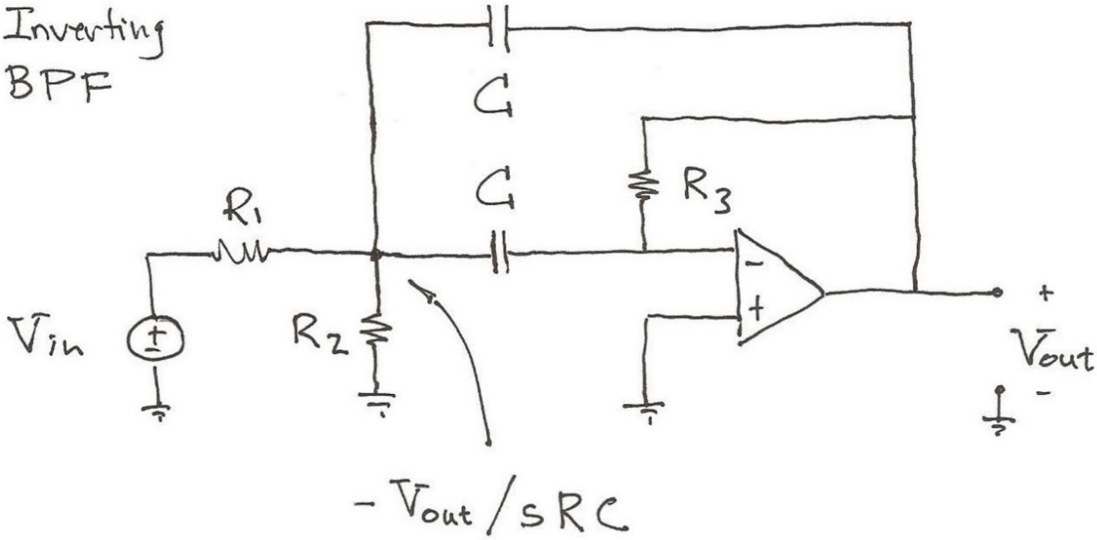
$$\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2} \qquad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2) C_2}$$

BSF Design





Inverting
BPF



Analysis

$$\text{KCL} \Rightarrow \quad \frac{V_{in} + \frac{V_{out}}{SR_3C}}{R_1} + \frac{\frac{V_{out}}{SR_3C}}{R_2} + \frac{V_{out}}{SR_3C}SC + \left[V_{out} + \frac{V_{out}}{SR_3C}\right]SC = 0$$

$$V_{in}\left[\frac{S}{R_1C}\right] + V_{out}\left[S^2 + \frac{S}{R_3C} + \frac{S}{R_3C} + \frac{1}{R_1R_3C^2} + \frac{1}{R_2R_3C^2}\right] = 0$$

$$V_{out}\left[S^2 + \underbrace{\frac{2}{R_3C}}_{\omega_o/Q} S + \underbrace{\frac{R_1+R_2}{R_1R_2R_3C}}_{\omega_o^2}\right] = -V_{in}\left[\underbrace{\frac{1}{R_1C}}_{G\,\omega_o/Q} S\right]$$

$$\omega_o^2 = \frac{R_1+R_2}{R_1R_2R_3C^2} \qquad\qquad R_1 = \frac{Q}{G\,\omega_o C}$$

$$Q = \frac{1}{2}\sqrt{\frac{(R_1+R_2)R_3}{R_1R_2}} \qquad\qquad R_2 = \frac{Q}{[2Q^2 - Q]\,\omega_o C}$$

$$G = \frac{R_3}{2R_1} \qquad\qquad R_3 = \frac{2Q}{\omega_o C}$$

Demo Design

$\omega_0 = 2\pi \times 60$ Hz

Q = 15

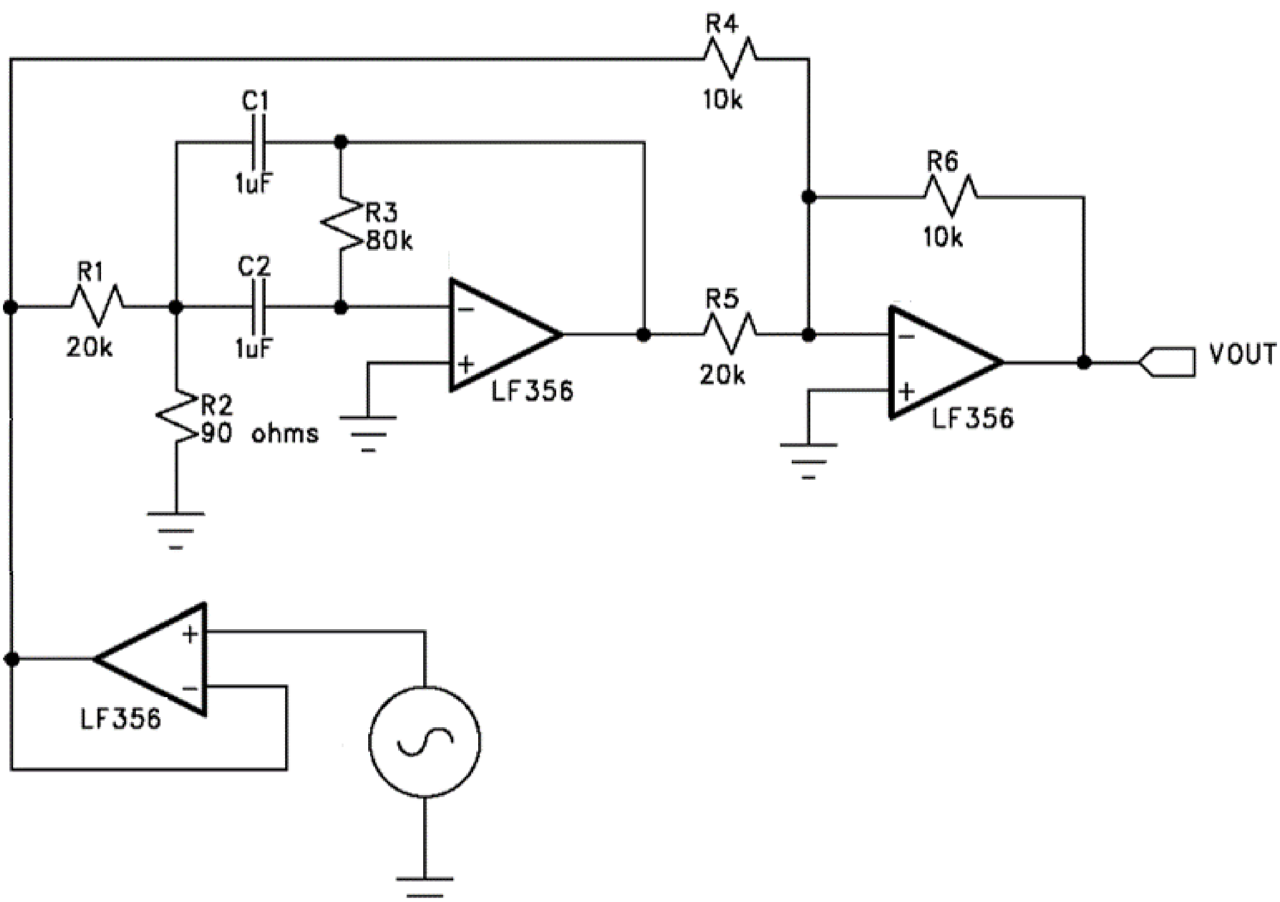
G = 2

C = 1 µF

R1 = 20 kΩ

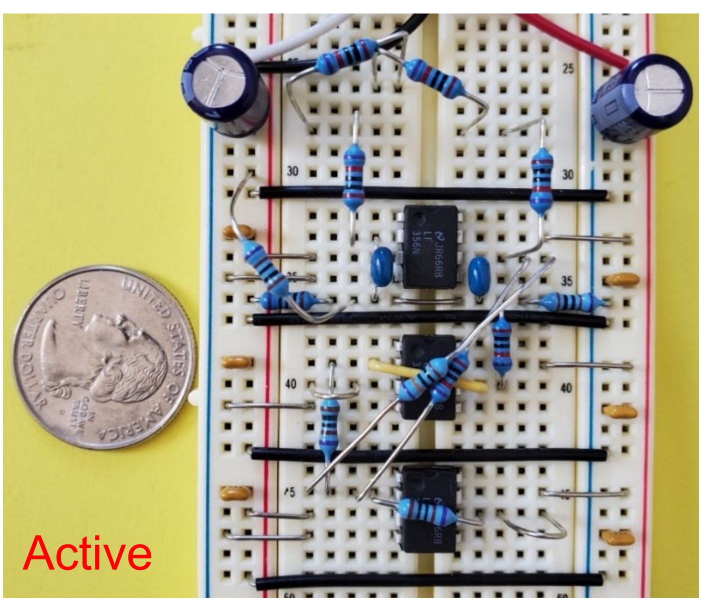
R2 = 90 Ω

R3 = 80 kΩ



Demo


X=56.1778 Hz
Ya=-25.361 dB
FREQ RESP
0.0
dB
-30.0
0.054
Log Hz
80
Fxd Y 40
Yb=157.672 Deg
FREQ RESP
260
Phase
Deg
100
Log Hz
80
Fxd Y 40
Measurement Complete
A B
X FIXD SCALE
X MRKR SCALE
X AUTO SCALE
Y FIXD SCALE
Y MRKR SCALE
Y AUTO SCALE
Y DFLT SCALE


Active


D9301 X
100 M.H.PER STEP
DEC.INDUCTOR
300 M.A.MAX
Passive