Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.200 – Circuits & Electronics
Spring 2023

Midterm 1

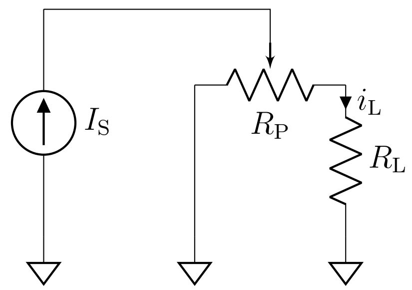
3/8/2023

Name: ______________________________

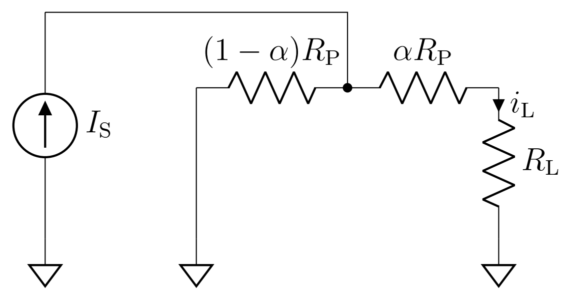
- There are 6 problems and 26 pages (including this cover page) in this exam.
- Do not remove any pages from this exam.
- Do your work for each question within the boundaries of that question. If you use the back of any pages for work, indicate this fact in text in the valid answer space. Enter your answer to each question in the corresponding answer box provided.
- You may refer to one 8.5”× 11” pages of notes, double-sided. Calculators, smartphones, and laptops are not permitted.
- Show your work. Unless you write out your thought process clearly, partial credit will not be awarded for incorrect solutions.

Problem 1, 16 pts: Hidden Potentiometer

Consider this design of a tunable current source where R_{L} is the load.

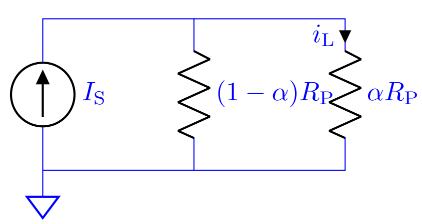


We can redraw the potentiometer as two resistors $(1-\alpha)R_{\mathrm{P}}$ and αR_{P} where α is the adjustable parameter.



(1A) (4 pt) Assume R_{L} is 0. Determine the current i_{L}.

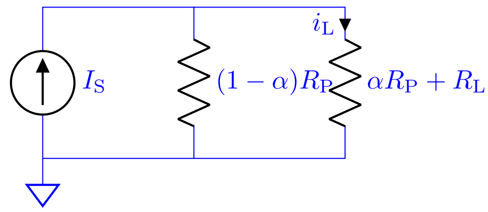
The circuit is a simple current divider, which we can redraw as:



From this picture, we can see that the current divider relation applies:

$$i_{\mathrm{L}} = I_{\mathrm{S}} \frac{(1-\alpha)R_{\mathrm{P}}}{(1-\alpha)R_{\mathrm{P}} + \alpha R_{\mathrm{P}}} = (1-\alpha) I_{\mathrm{S}}$$

(1B) (4 pt) Find the current i_L assuming R_L is nonzero.

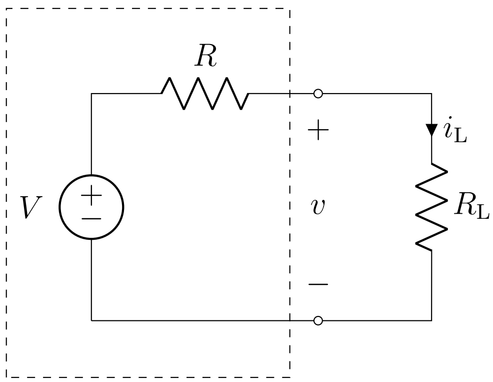


$$I_\mathrm{S}\frac{(1-\alpha)R_\mathrm{P}}{(1-\alpha)R_\mathrm{P}+\alpha R_\mathrm{P}+R_\mathrm{L}} = \frac{1-\alpha}{R_\mathrm{P}+R_\mathrm{L}}I_\mathrm{S}$$

$i_\mathrm{L} = \frac{1-\alpha}{R_\mathrm{P}+R_\mathrm{L}}I_\mathrm{S}$

An ideal current source should be able to provide a fixed current to a resistive load, completely independent of the load resistance. In reality, such a perfect circuit is impossible to realize, but it can be approximated.

(1C) (4 pt) A design for a (not tunable) non-ideal current source (providing current i_{L} to load R_{L}) is shown in the dashed box below.



Provide a specific value for V and R where the source supplies consistently approximately $10\,\mu\mathrm{A}$ with variation of less than $\pm 1\%$ (i.e. a minimum of $9.9\,\mu\mathrm{A}$ and a maximum of $10.1\,\mu\mathrm{A}$) even though the load R_{L} varies from 0 to $10\,\Omega$. V must be less than or equal to $10\,\mathrm{V}$, and R must be between $100\,\mathrm{m}\Omega$ and $100\,\mathrm{M}\Omega$).

It's perfectly acceptable for the answer to just say "well I'll need R to be big" and pick a really big number and then say $V = 10\,\mu\mathrm{A} \cdot R$.

A more quantitative approach (but not necessarily preferred) is to realize that the larger current ($i_{\mathrm{L}} = 10.1\,\mu\mathrm{A}$) will correspond to the $0\,\Omega$ load, while the lower current ($i_{\mathrm{L}} = 9.9\,\mu\mathrm{A}$) will correspond to the $10\,\Omega$ load. From this we get that

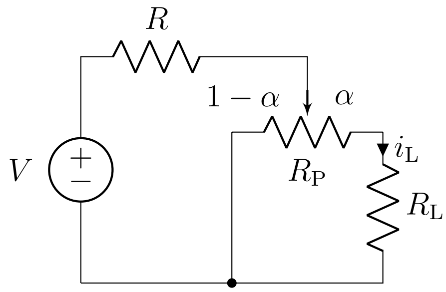
$$V = 10.1\,\mu\mathrm{A} \cdot R \qquad\qquad V = 9.9\,\mu\mathrm{A} \cdot (R + 10\,\Omega) \tag{1}$$

$$\Rightarrow 10.1\,\mu\mathrm{A} \cdot R = 9.9\,\mu\mathrm{A} \cdot (R + 10\,\Omega)$$

After a bit of algebra, this can be solved to yield $R = 495\,\Omega$. Any R larger than this value should work. Using eq 1 above, we can solve for V for this case ($4.9995\,\mathrm{mV}$) and in general, we want $V = 10.1\,\mu\mathrm{A} \cdot R$.

$V =$	$R =$

(1D) (4 pt) Finally, we will take the circuit presented in A, using the circuit from part C as the current source. Provide a symbolic expression (i.e. not using the values you chose in part C) for i_{L}. Note, you may use the notation $R_1//R_2$ to represent resistances in parallel in your answer without penalty.



The total current emerging from the positive terminal of the voltage supply i_{TOT} is V/R_{eff} where R_{eff} is the effective resistance of the entire resistor network when viewed from the voltage supply. $R_{\mathrm{eff}} = R + (1-\alpha)R_{\mathrm{P}}//(\alpha R_{\mathrm{P}} + R_{\mathrm{L}})$.

i_{L} can then be calculated by using the current divider relation, $i_{\mathrm{L}} = i_{\mathrm{TOT}} \frac{(1-\alpha)R_{\mathrm{P}}}{\alpha R_{\mathrm{P}} + R_{\mathrm{L}}}$. Substituting in our value above for i_{TOT} we find:

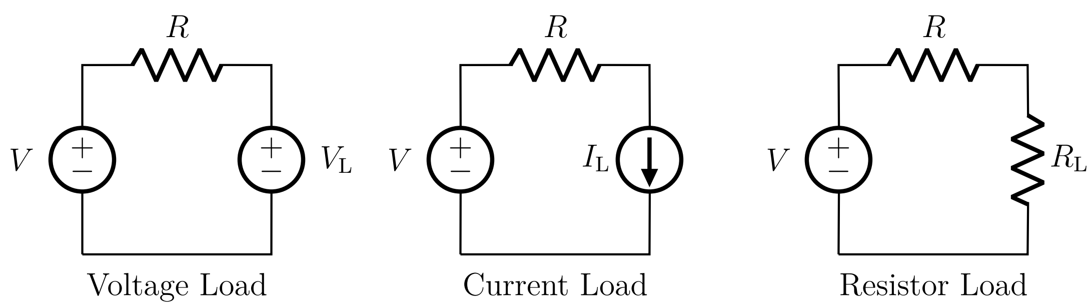
$$
\begin{aligned}
i_{\mathrm{L}} &= \frac{V}{R + (1-\alpha)R_{\mathrm{P}}//(\alpha R_{\mathrm{P}} + R_{\mathrm{L}})} \cdot \frac{(1-\alpha)R_{\mathrm{P}}}{\alpha R_{\mathrm{P}} + R_{\mathrm{L}}} \\
&= \frac{V}{R + \frac{(1-\alpha)R_{\mathrm{P}}(\alpha R_{\mathrm{P}} + R_{\mathrm{L}})}{R_{\mathrm{P}} + R_{\mathrm{L}}}} \cdot \frac{(1-\alpha)R_{\mathrm{P}}}{\alpha R_{\mathrm{P}} + R_{\mathrm{L}}} \\
&= \frac{V(R_{\mathrm{P}} + R_{\mathrm{L}})}{R(R_{\mathrm{P}} + R_{\mathrm{L}}) + (1-\alpha)R_{\mathrm{P}}(\alpha R_{\mathrm{P}} + R_{\mathrm{L}})} \cdot \frac{(1-\alpha)R_{\mathrm{P}}}{\alpha R_{\mathrm{P}} + R_{\mathrm{L}}}
\end{aligned}
$$

$$i_{\mathrm{L}} = \frac{V}{R + (1-\alpha)R_{\mathrm{P}}//(\alpha R_{\mathrm{P}} + R_{\mathrm{L}})} \cdot \frac{(1-\alpha)R_{\mathrm{P}}}{\alpha R_{\mathrm{P}} + R_{\mathrm{L}}}$$

Problem 2, 12 pts: Power

This problem is concerned with determining the value of a load that maximizes the power delivered to that load.

(2A) (8 pt) A Thevenin equivalent network having voltage V and resistance R is separately loaded with a voltage source, a current source, and a resistor, as shown below. For each load case, determine the value of the load (V_L, I_L or R_L) in terms of V and R that maximizes the power delivered to the load. Additionally, determine the maximized power delivered to the load.

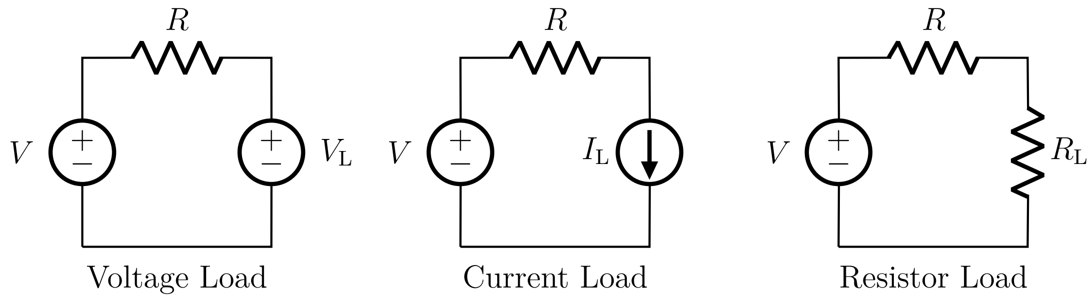


In all three cases, the power P delivered to the load is the current through it multiplied by the voltage across it, where the passive sign convention is used to define polarities. For the voltage load case, $P = V_\mathrm{L}\frac{V-V_\mathrm{L}}{R}$. For the current load case, $P = (V - RI_\mathrm{L})I_\mathrm{L}$. For the resistor load case, $P = V\frac{R_\mathrm{L}}{R+R_\mathrm{L}}V\frac{1}{R+R_\mathrm{L}}$. To maximize P for the three cases, differentiate P with respect to V_L, I_L, and R_L, respectively, set the result equal to zero, and solve for V_L, I_L, and R_L, respectively. This yields $V_\mathrm{L} = V/2$, $I_\mathrm{L} = V/(2R)$, and $R_\mathrm{L} = R$. Thus, in all three cases, the voltage across the load is $V/2$, the current through it is $V/(2R)$, and the power delivered to the load is $V^2/(4R)$.

The key learning here is that for maximum-power-transfer loading, the load operates just as would the Thevenin resistance. This is referred to as "matched" loading.

Please put your answers on the following page.

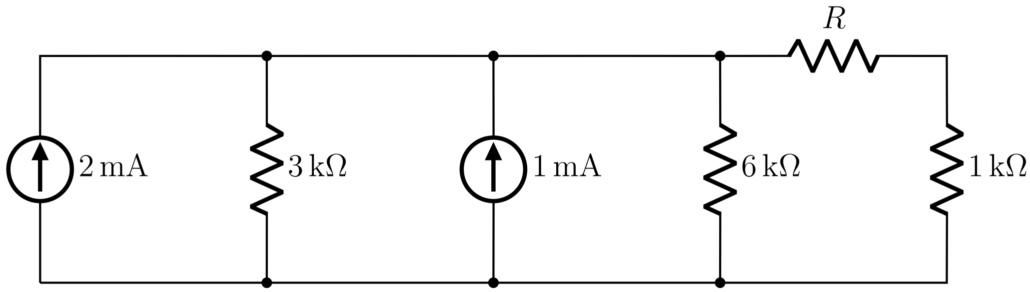
(2A Continued) Circuits repeated for your convenience



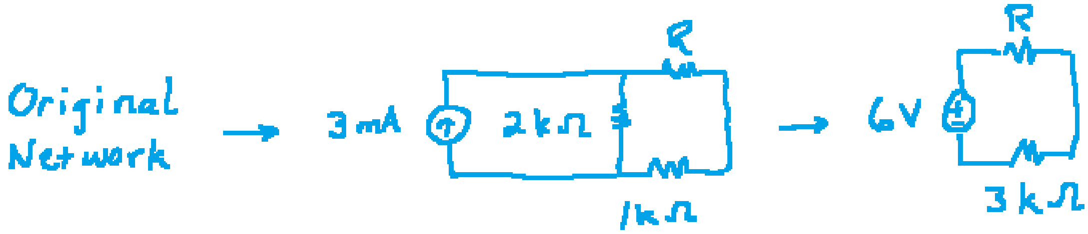
	V_L Load	I_L Load	R_L Load
Load Value	V/2	V/(2R)	R
Maximized Power	$V^2/(4R)$	$V^2/(4R)$	$V^2/(4R)$

(2B) (4 pt) The network shown below contains a resistor having an unknown resistance R. Determine the resistance R that maximizes the power dissipated in the corresponding resistor. Additionally, determine the maximized power. *Hint: How are this part and Part A related?*

Numerical values with proper units are expected.



Following the key learning from the Part (2A) above, reduce the majority of the network to its Thevenin equivalent as follows. First, combine the two parallel current sources into a single current source, and the two parallel resistors (3 kΩ and 6 kΩ) into a single resistor (2 kΩ). Then, combine all but the unknown resistor in a Thevenin equivalent. This is as shown below.

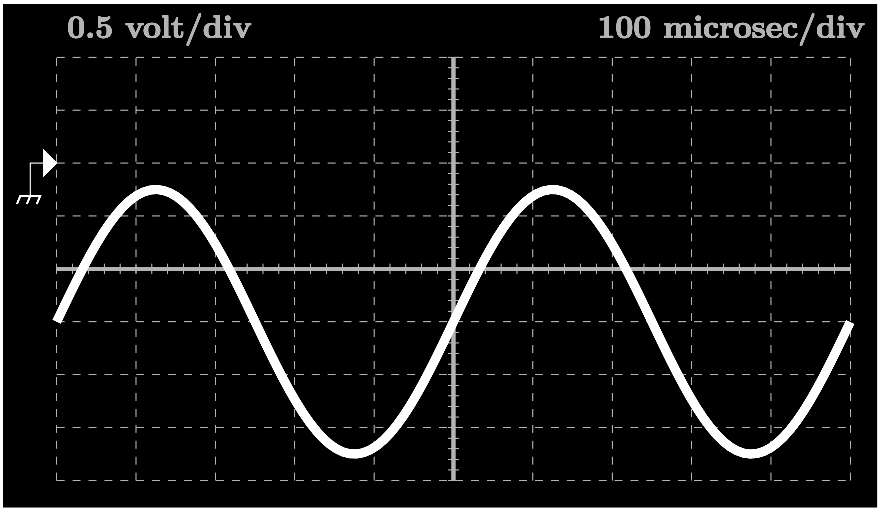


Following the results of Part (2A), R should be 3 kΩ, and the resulting power will be 3 mW.

R	Power
3 kOhm	3 mW

Problem 3, 8 pts: Oscilloscope

Provide the requested information for the voltage-time oscilloscope trace shown below:



(3A) (2 pt) What is the approximate period T of the signal, with engineering units (i.e. mega, kilo, milli, micro, etc.)?

The period is the shortest time T in a repeating signal for which $f(t) = f(t+T)$ for all t, i.e. the time between repeating cycles of a signal. In this case, that is 5 divisions, thus $500\,\mu\,\mathrm{sec} = 5 \times 10^{-4}$ seconds in scientific notation note, engineering units were requested, not scientific notation.

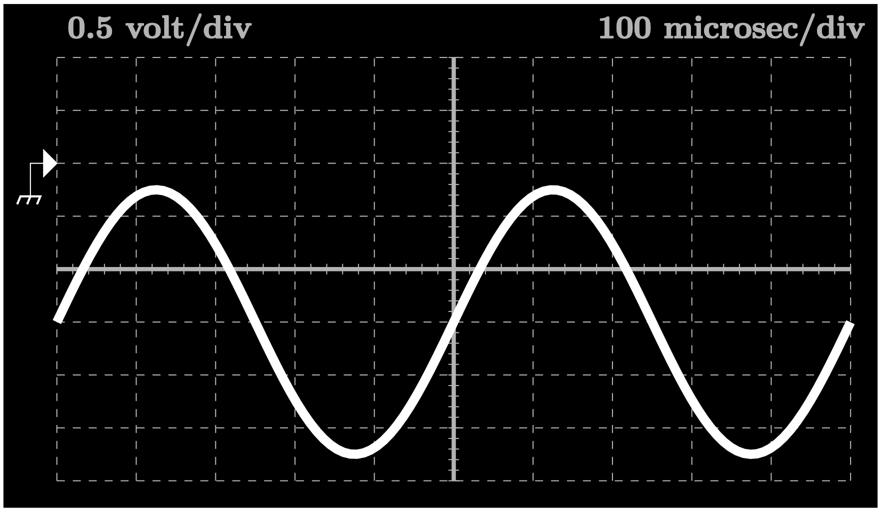
$T = 500\,\mu\,\mathrm{sec}$

(3B) (2 pt) What is the approximate frequency of the signal, with engineering units (i.e. mega, kilo, milli, micro, etc.)?

Frequency $f = 1/T = 1/(500\,\mu\,\mathrm{sec} = 1/(500 \times 10^{-6}) = (1/500) \times 10^{6} = 0.002 \times 10^{6}\mathrm{Hz} = 2 \times 10^{3}\mathrm{Hz} = 2\,\mathrm{kHz}$

frequency $= 2\,\mathrm{kHz}$

(3) Continued: Oscilloscope image repeated for your convenience.



(3C) (2 pt) What is the approximate value of V_{pp} the peak-to-peak voltage of the signal, with appropriate units?

The peak-to-peak voltage is the voltage difference between the maximum and minimum of a periodic signal. Counting carefully, that's approximately 5 divisions, which corresponds to $5\text{div} \cdot 0.5\text{V/div} = 2.5$ V.

$V_{pp} = 2.5$ V

(3D) (2 pt) What is the approximate offset of the signal, with appropriate units?

The offset is indicated by comparing the average signal level to the ground level. The ground in this case is indicated with the triangle with the spade on the left hand margin of the scope, 2 divisions above the x axis. The signal average is 1 division below the x axis (the symmetric point between the min and max signal). Thus the signal is 3 divisions (1.5 V) below the ground.

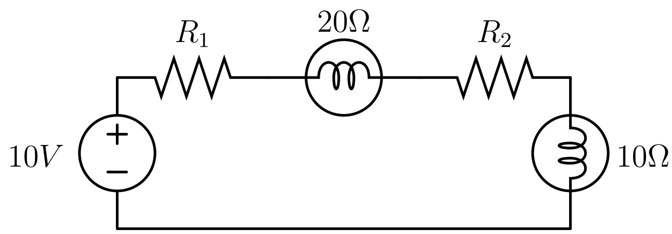
offset = -1.5 V

Problem 4, 12 pts: Let There Be Light

In each of the following circuits, is it possible to adjust the positive, finite, non-zero resistances R_1 and R_2 such that the light bulbs have the same voltage drop across them? If so, write *any single* example of a valid combination of R_1 and R_2 in the boxes. If not, write `None` in each box.

We can model each light bulb as a resistor. In each circuit, there are two different bulbs (one with a 10Ω resistance, and one with a 20Ω resistance).

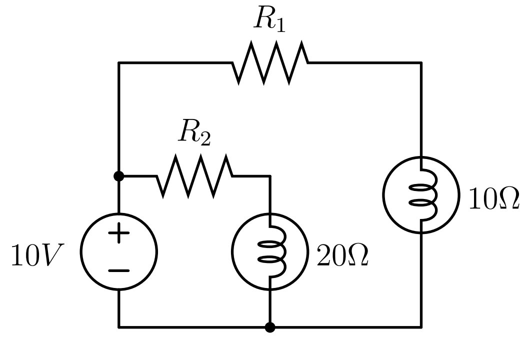
(4A) (4 pt)



Here, we must have the same current flowing through the two light bulbs. Since they have the same current and different resistances, they cannot have the same voltage drop.

R_1: None	R_2: None

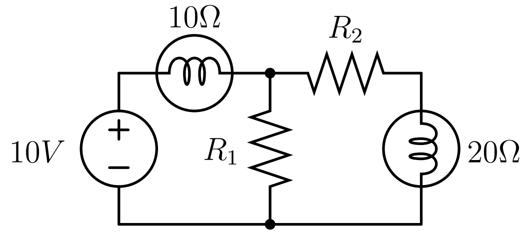
(4B) (4pt)



The 20Ω bulb forms a voltage divider with R_2, and the 10Ω bulb forms a voltage divider with R_1. So in order for the two to have the same voltage drops, we need $\frac{20\Omega}{R_2} = \frac{10\Omega}{R_1}$.

R_1: any $\quad$ R_2: $2R_1$

(4C) (4pt)

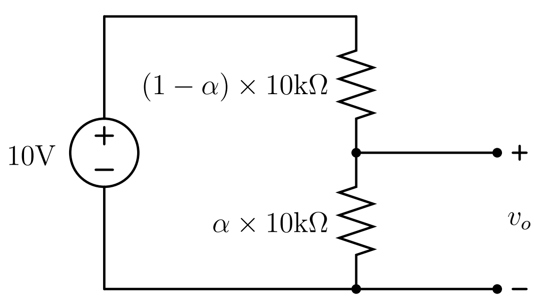


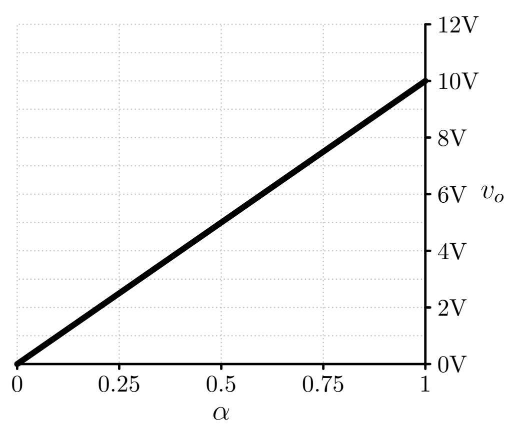
The important thing to notice here is that the current flowing through the 10ω resistor will split at the junction in the top middle of the circuit, with some flowing through R_1 and the rest flowing through R_2 and the 20Ω bulb. In order for the voltage drops to be the same, the 20Ω bulb needs to have half the current of the 10Ω bulb. R_1 and $(R_2 + 20\Omega)$ form a current divider, so if we want half of the current to flow through the 20Ω bulb, we need $R_1 = R_2 + 20\Omega$.

R_1: any $R_1 > 20\Omega$	R_2: $R_1 - 20\Omega$

Problem 5, 16 pts: The Turning Po(in)t

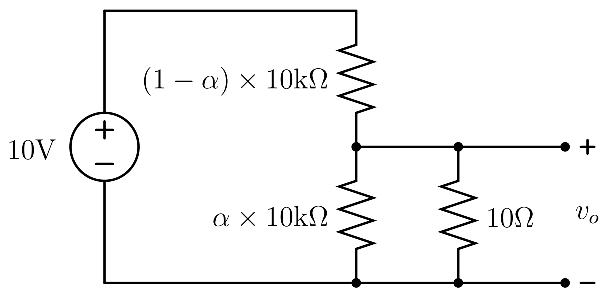
In Lab 2, we connected a potentiometer as a voltage divider configuration like the one shown below. In this configuration, v_o and α are related as shown in the graph on the right.



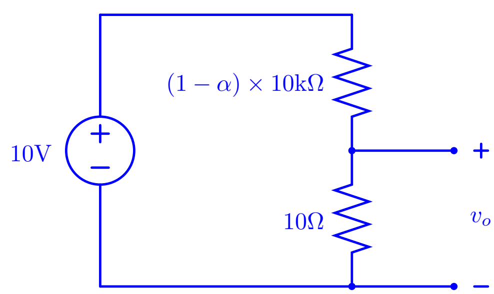


For each of the configurations on the following pages, sketch the relationship between v_o and α in that circuit. The original curve has been reproduced in grey on each graph and can be used as a reference. Note that this pot's total internal resistance is 10kΩ.

(5A) (4pt)



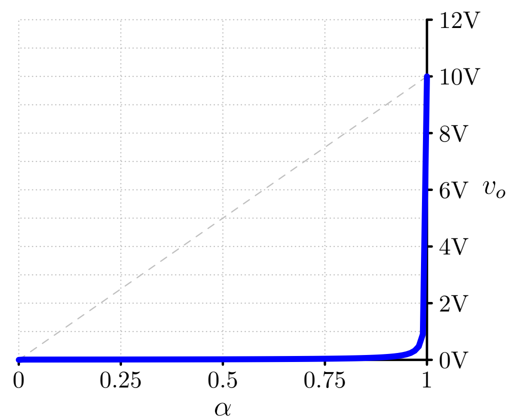
For more of the range of α (except when α is really small), the circuit will be roughly equivalent to the following (the bottom resistor is actually $10\Omega || \alpha \times 10\text{k}\Omega$, but when $\alpha \times 10\text{k}\Omega >> 10\Omega$, that parellel combination will be approximately 10Ω):



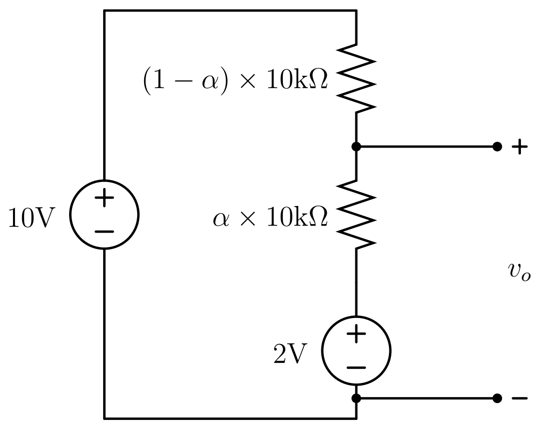
This is a voltage divider, and for most of the range of α, the bottom resistance is tiny compared to the top resistance, so only a tiny fraction of the voltage will drop there. Thus, for most of the range of α, we sit very close to $v_o = 0\text{V}$.

It's only when $\alpha \to 1$ that this starts to change. When $\alpha = 1$, we have $v_0 = 10\text{V}$; and we cross over $v_o = 5\text{V}$ when the two resistors are equal, i.e. when $(1-\alpha) \times 10\text{k}\Omega \approx 10\Omega$, or at around $\alpha = 0.999$. So the change in v_0 happens very rapidly near the $\alpha = 1$ edge of the graph.

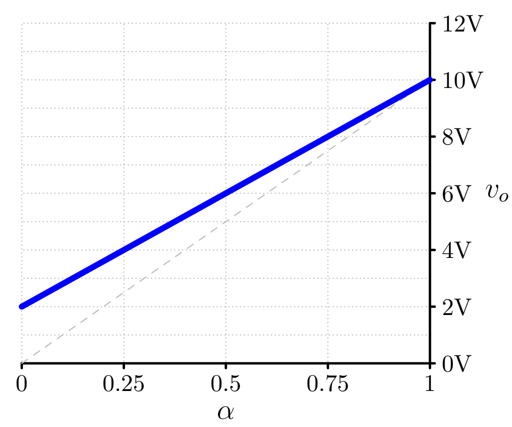
Putting that all together, the graph should look something like this:



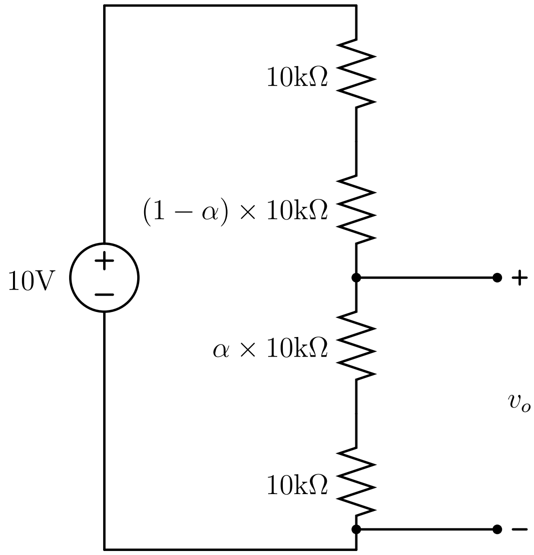
(5B) (4pt)



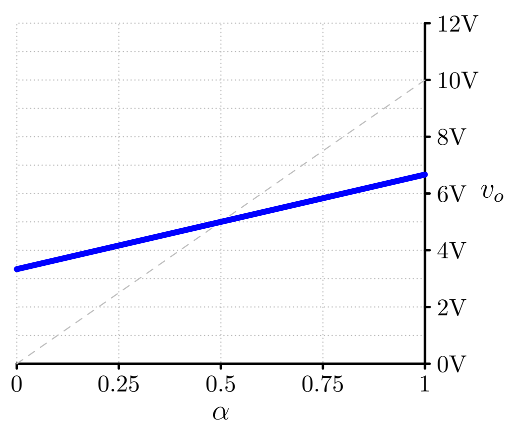
$$v_o = 2\text{V} + 8\text{V} * \frac{\alpha \times 10\text{k}\Omega}{10\text{k}\Omega}$$
$$= 2\text{V} + \alpha \times 8\text{V}$$



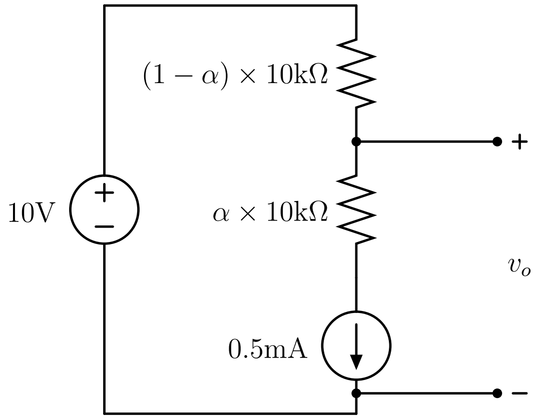
(5C) (4pt)



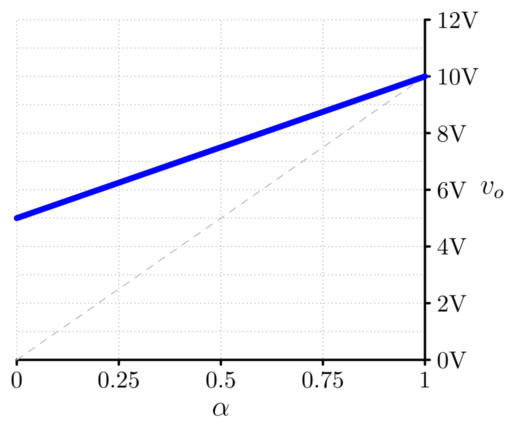
$$v_o = 10\text{V} \times \frac{10\text{k}\Omega(1+\alpha)}{30k\Omega}$$
$$= \left(\frac{10}{3} + \frac{10}{3}\alpha\right)\text{V}$$



(5D) (4pt)



$$
\begin{aligned}
v_o &= 10\text{V} - (1-\alpha)\times 10\text{k}\Omega \times 0.5\text{mA} \\
&= 10\text{V} - (5-5\alpha)\text{V} \\
&= (5+5\alpha)\text{V}
\end{aligned}
$$



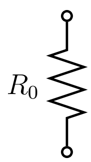
Problem 6, 14 pts: The I-V League

For each of the circuits and current-voltage characteristics shown below and on the following pages (6 in total), specify a single component that could be added in the box to cause the current i and the voltage v to have the pictured relationship. Valid components are limited to:

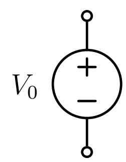
- resistors with finite positive resistance,
- voltage sources with finite nonzero voltage,
- and current sources with finite nonzero current.

Specify the appropriate component by circling its type and specifying its value (be careful of directionality and specify units!). **If no single component would work, circle "None" and leave the value blank.**

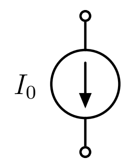
Valid circuit elements:



Type: Resistor
Value: R_0

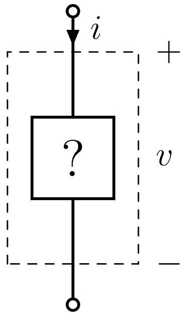


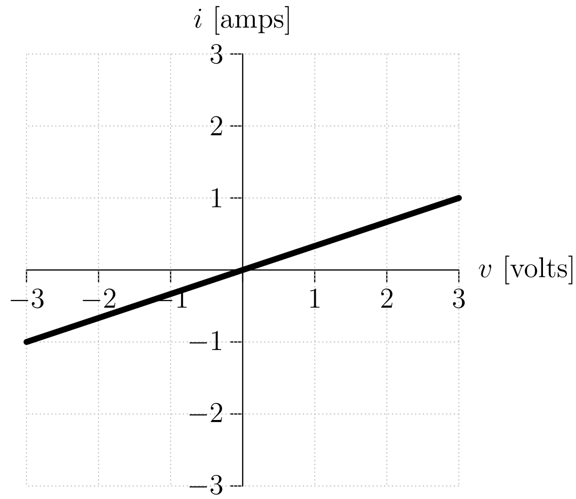
Type: Voltage Source
Value: V_0



Type: Current Source
Value: I_0

(6A) (2pt)



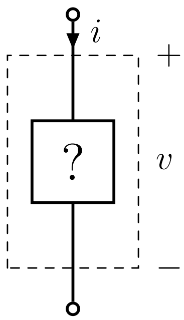


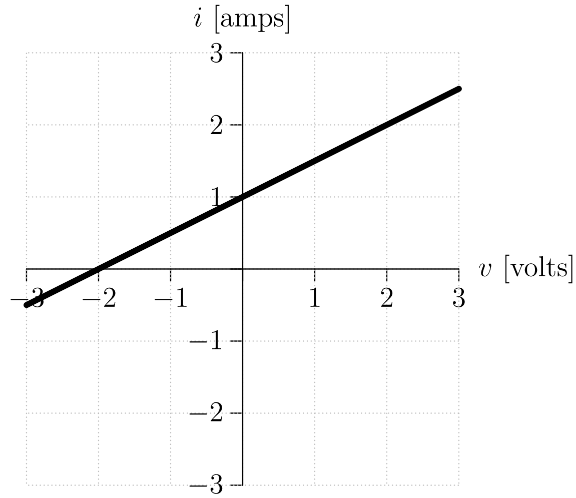
Since it passes through the origin, this graph must represent a pure resistance. Since its slope is $\frac{1}{3}$ Amperes per Volt, the resistance must be 3 Volts per Ampere.

Type (Circle One): $\boxed{\text{Resistor}}$ Voltage Source Current Source None

Value and Units: 3Ω

(6B) (2pt)

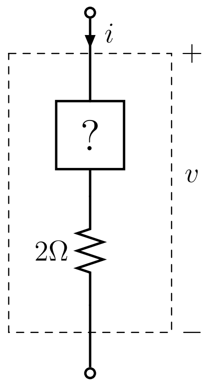


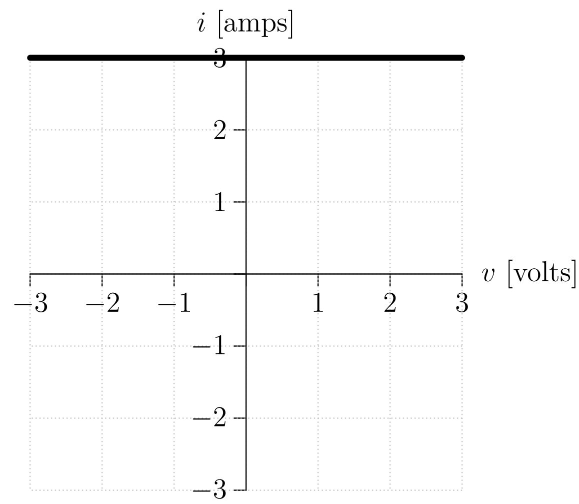


No single component can account for both the finite, nonzero slope of the graph and the fact that this graph does not pass through the origin.

Type (Circle One):	Resistor	Voltage Source	Current Source	$\boxed{\text{None}}$
Value and Units: None				

(6C) (2pt)



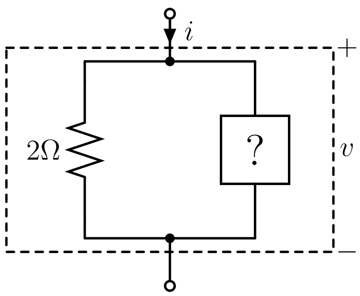


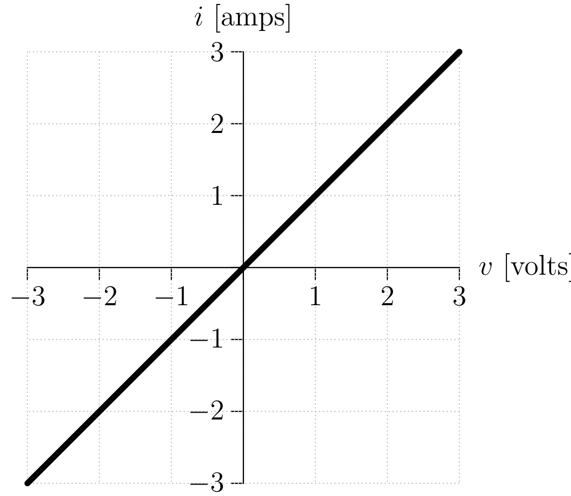
Regardless of the voltage drop across this combination, the current flowing through it is always 3 Amperes. The only way to do that is with a 3A current source.

Type (Circle One):	Resistor	Voltage Source	$\boxed{\text{Current Source}}$	None

Value and Units: 3A

(6D) (2pt)



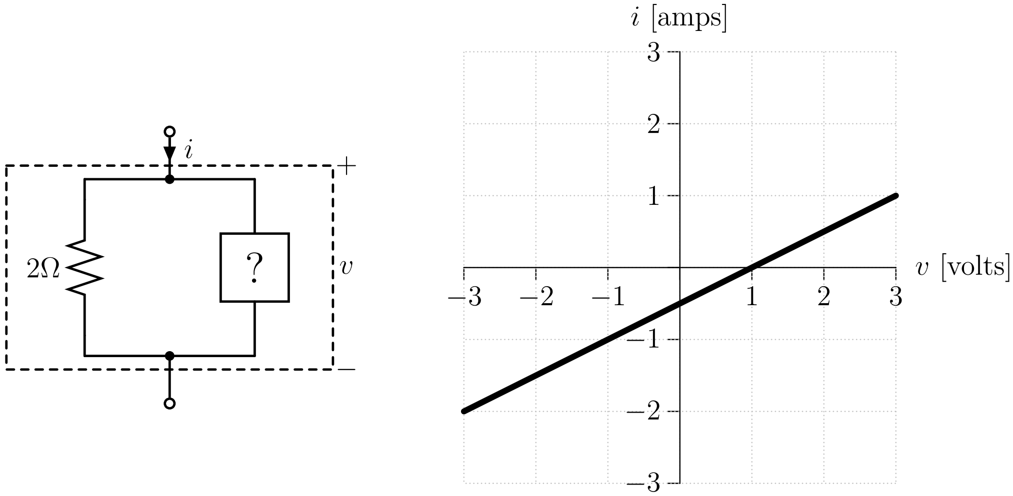


Since it passes through the origin, this graph represents a pure resistance (no sources), so the missing component must be a resistor in parallel with the given 2Ω resistor. Since the equivalent resistance is 1Ω, the missing component must be a 2Ω resistor.

Type (Circle One): $\boxed{\text{Resistor}}$ Voltage Source Current Source None

Value and Units: 2Ω

(6E) (2pt)

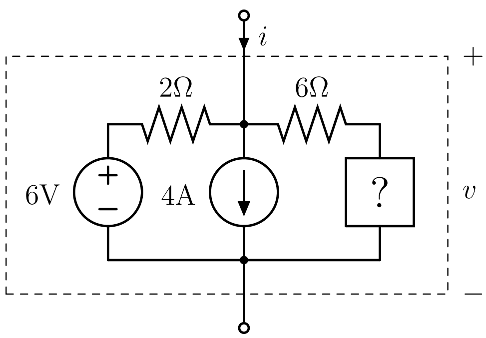


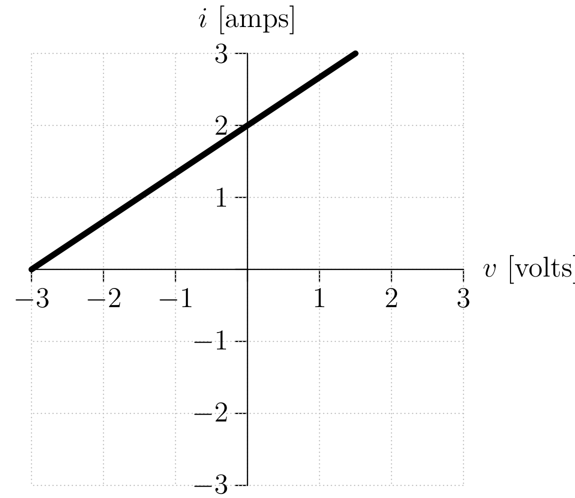
The missing component can't be a voltage source (the line would be vertical) or a resistor (it would pass through the origin), so the missing component must be a current source. Putting a current source in means that i_{sc} (the value of i when $v = 0$) would be precisely the value of that current source, so the missing component must be a $-\frac{1}{2}$A current source (or, equivalently, a $\frac{1}{2}$A current source pointing upwards).

Type (Circle One): Resistor Voltage Source $\boxed{\text{Current Source}}$ None

Value and Units: $-\frac{1}{2}$A

(6F) (4pt)





We know that if we were to find this circuit's Thévenin equivalent at the indicated port, its Thévenin voltage would be -3V and its Thévenin resistance would be $\frac{3}{2}\Omega$.

So let's start by finding the Thévenin resistance. We can do this by trying to put all of the possible components in there. If it were a current source, the equivalent resistance would be 2Ω, so that can't be it. If it were a resistor, it would need to have a resistance of 0Ω, which is not allowed per the problem specification. But if it's a voltage source, the resistance matches. So the only possibility is a voltage source.

Let's check to make sure that works, though.

From the graph, we know that the open-circuit voltage must be -3V. And we can set up a system of equations that ultimately ends up getting us to something like the following (KCL at the top node after substituting some things in, where V_M is the voltage of the missing voltage source):

$$\frac{9\text{V}}{2\Omega} = 4\text{A} + \frac{-3\text{V} - V_M}{6\Omega}$$

Solving, we find that V_M must be -6V.

Type (Circle One): Resistor $\boxed{\text{Voltage Source}}$ Current Source None

Value and Units: -6V

Worksheet (intentionally blank)