

Electrical Engineering and Computer Science

6.200 – Circuits & Electronics  
Spring 2023

Midterm 2

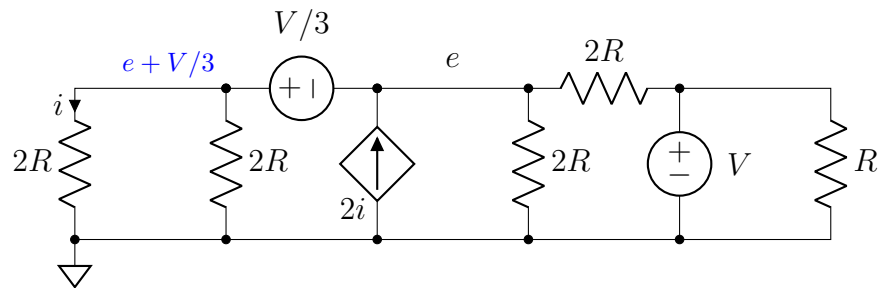
4/19/2023

Name: \_\_\_\_\_

- There are 7 problems and 23 pages (including this cover page) in this exam.
- There are 90 possible points you can earn on this exam.
- Do not remove any pages from this exam.
- Do your work for each question within the boundaries of that question. If you use the back of any pages for work, indicate this fact in text in the valid answer space. Enter your answer to each question in the corresponding answer box provided.
- You may refer to one 8.5" × 11" pages of notes, double-sided. Calculators, smart-phones, and laptops are not permitted.
- Show your work. Unless you write out your thought process clearly, partial credit will not be awarded for incorrect solutions.

**Problem 1, 6 pts: Node Your Typical Circuit**

Consider this circuit:



- (1A) (6pts) Use the node method to solve for the node potential  $e$ . Note:  $i$  may not appear in your solution.

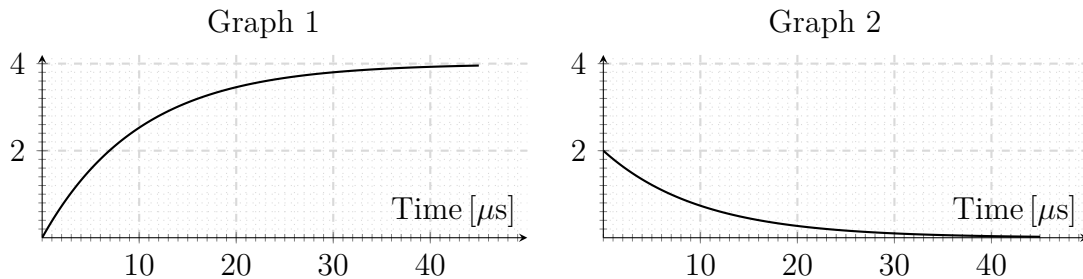
Use KCL on supernode

$$\begin{aligned} \frac{(e + \frac{V}{3})}{2R} + \frac{(e + \frac{V}{3})}{2R} - 2i + \frac{e}{2R} + \frac{e - V}{2R} &= 0 \quad \text{where } (e + V/3)/2R = i \\ \Rightarrow \frac{(e + \frac{V}{3})}{R} - \frac{2(e + \frac{V}{3})}{2R} + \frac{e}{2R} + \frac{e}{2R} - \frac{V}{2R} &= 0 \\ \Rightarrow 2e &= V \\ \Rightarrow e &= \frac{V}{2} \end{aligned}$$

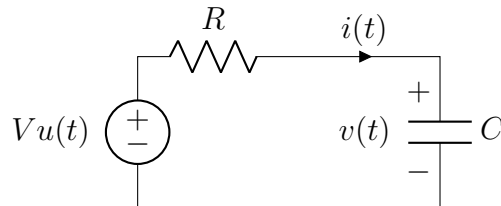
$$e = V/2$$

**Problem 2, 14 pts: Riding the Graphy Train**

The two graphs below show electrical variables as functions of time. One graph shows voltage in volts while the other shows current in milliamps. Which graph shows which variable is to be determined.



(2A) (4pts) The network shown below comprises a voltage source, a resistor and a capacitor. Prior to  $t = 0$ , the capacitor has no charge and the source voltage is zero. At  $t = 0$ , the voltage source steps to  $V$ . Determine which graph above shows  $i(t)$  and which graph shows  $v(t)$ . **To discourage guessing, 1 pt of partial credit will be provided for each answer left blank.**



where

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0. \end{cases} \quad (1)$$

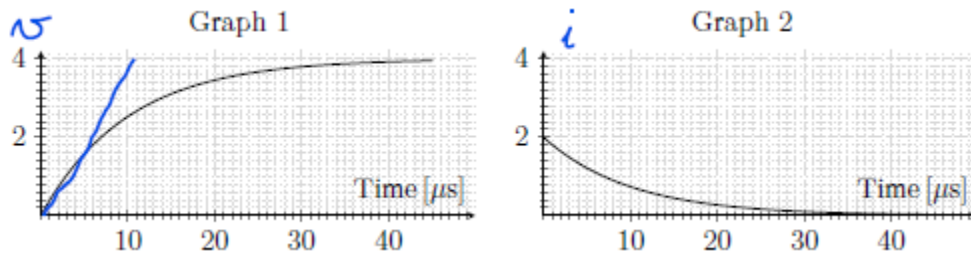
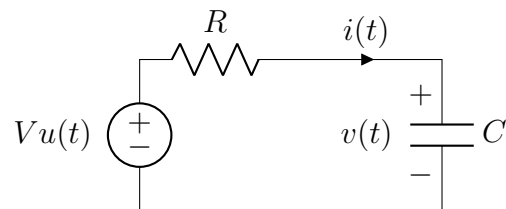
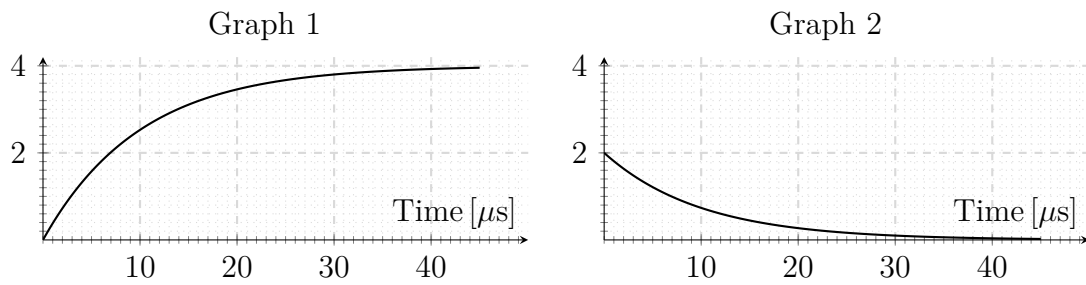
At  $t = 0_-$ ,  $v = 0 \Rightarrow t = 0_+ v = 0$  by continuity of charge  $\Rightarrow$  graph 1  $\equiv v$ .

At  $t = 0_+$   $C \rightarrow$  voltage source with strength 0  $\Rightarrow i = V/R \neq 0 \Rightarrow$  graph 2.

Circle the correct graphs:

$i(t) =$	<input type="radio"/> Graph1	<input type="radio"/> Graph2	$v(t) =$	<input type="radio"/> Graph1	<input type="radio"/> Graph2
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(2B) (3pts) Following Part (A), determine  $R$ ,  $C$  and  $V$  from the graphical information. The graphs and circuit schematic are repeated below for your convenience. Remember to include units in your answer.



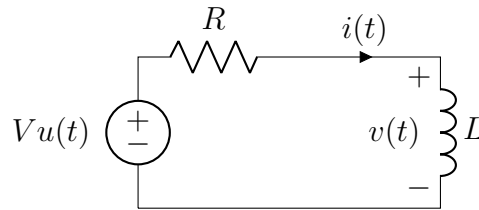
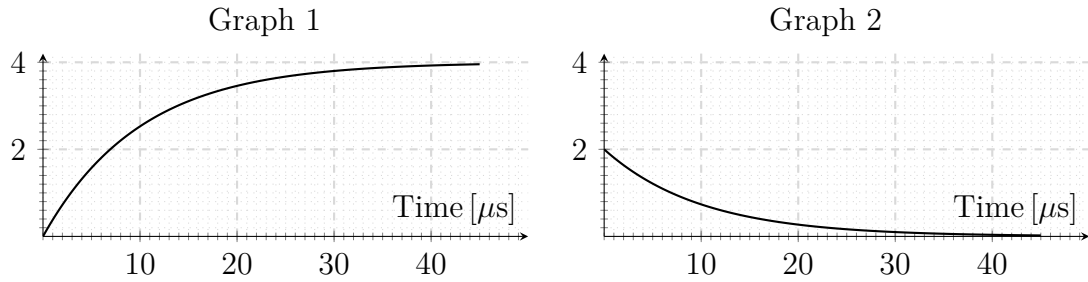
From graph 2  $i(0_+) = \frac{V}{R} = 2A$   
 From intercept with asymptote  $\tau \approx 10\mu s$   
 $\Rightarrow RC = 10\mu s$   
 From  $v(\infty) = 4V \Rightarrow V = 4V$   
 $\Rightarrow R = 2\Omega$   
 $\Rightarrow C = 5\mu F$

$$R = 2\Omega$$

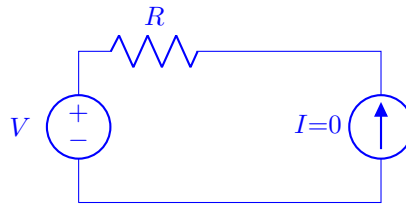
$$C = 5\mu F$$

$$V = 4V$$

(2C) (4pts) The network shown below comprises a voltage source, a resistor and an inductor. Prior to  $t = 0$ , the inductor is at rest and the source voltage is zero. At  $t = 0$ , the voltage source steps to  $V$ . Determine which graph above shows  $i(t)$  and which graph shows  $v(t)$ . The graphs and circuit schematic are repeated below for your convenience. **To discourage guessing, 1 pt of partial credit will be provided for leaving the answer blank.**



At  $t = 0_-$   $i = 0$   
 from continuity  $i(0_+) = i(0_-) = 0$   
 $\Rightarrow i$  is graph 1  
 Doing KVL at  $t = 0_+$

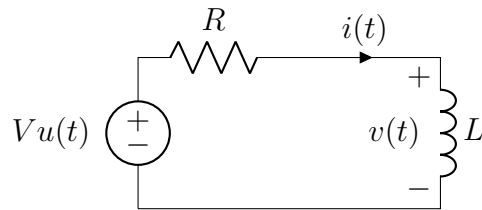
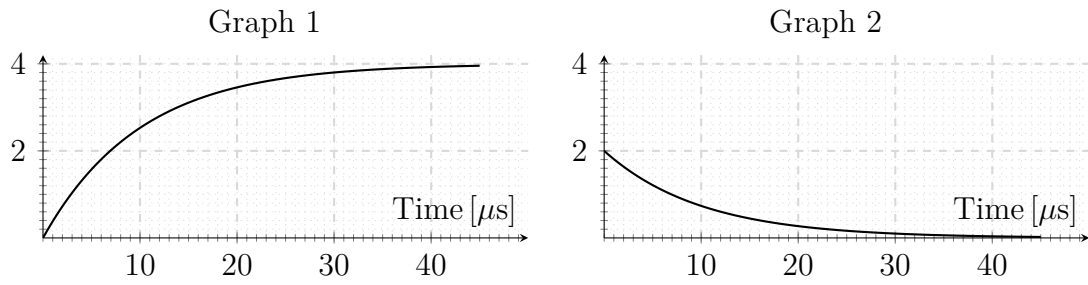


$\Rightarrow v = V$   
 $\Rightarrow v$  is graph 2

Circle the correct graphs:

$i(t) =$      Graph1     Graph2     $v(t) =$      Graph1     Graph2

(2D) (3pts) Following Part (C), determine  $R$ ,  $L$  and  $V$  from the graphical information. The graphs and circuit schematic are repeated below for your convenience. Remember to include units in your answer.



From part B  $\tau = 10\mu s = \frac{L}{R}$

From  $i(\infty) = \frac{V}{R} = 4A$

At  $0_+$   $v = V = 2V$

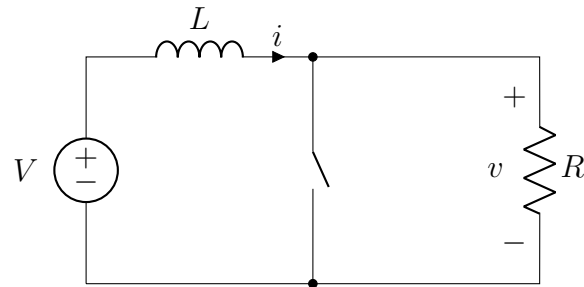
$\Rightarrow R = 0.5\Omega$

$\Rightarrow L = 5\mu H$

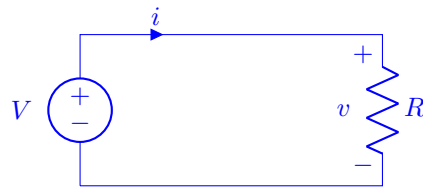
$R = 0.5\Omega$                        $L = 5\mu H$                        $V = 2V$

### Problem 3, 14 pts: A Fowl In-Duck-tor

Consider the circuit below which comprises a voltage source, a resistor, an inductor and an ideal switch. Assume that the switch has been open for a very long time prior to  $t = 0$  so that the circuit operates in steady state at that time.



(3A) (2pts) Determine  $i(t)$  and  $v(t)$  for the time just before  $t = 0$ , i.e. at the time  $t = 0_-$ .

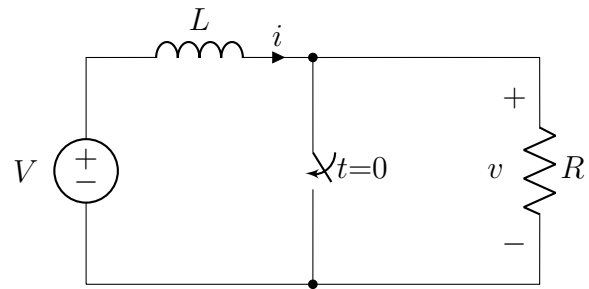


Inductor becomes a short

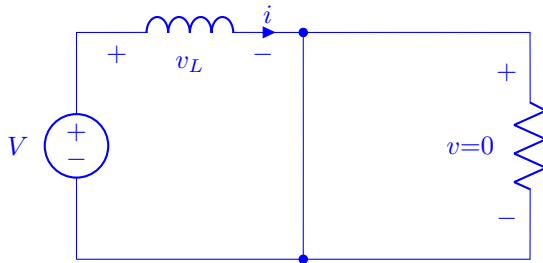
$$i(0_-) = V/R$$

$$v(0_-) = V$$

Now, as shown below, at  $t = 0$  assume the switch is suddenly closed.



(3B) (6pts) Write an expression for  $i(t)$  and  $v(t)$  for all times  $t > 0$ .



$$i = \frac{1}{L} \int_{0+}^t v_L dt + i(0_+) \quad (2)$$

$$= \frac{1}{L} \int_{0+}^t V dt + \frac{V}{R} \quad (3)$$

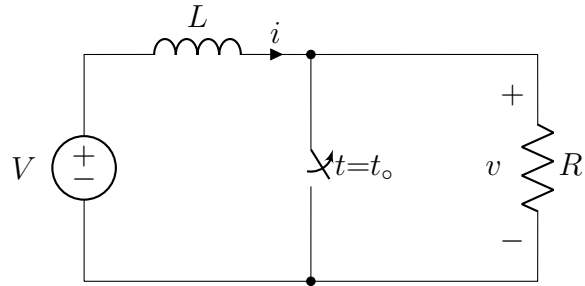
$$= \frac{V}{L} t + \frac{V}{R} \quad (4)$$

$$i(t) = \frac{V}{L} t + \frac{V}{R}$$

$$v(t) = 0$$



At some time  $t_o$ , where  $t_o > 0$ , assume the switch opens again, as shown below.



(3C) (6pts) Determine  $i(t)$  and  $v(t)$  for  $t > t_o$ . To avoid propagating possible prior errors, assume that immediately before  $t_o$ ,  $i(t_{o-}) = i_o$  and  $v(t_{o-}) = v_o$ . Find  $i(t)$  and  $v(t)$  for  $t > t_o$ .

$$v(t_{o+}) = v_o \qquad v(\infty) = V \text{ because } L \rightarrow \text{short} \qquad (5)$$

$$v(t) = (v(t_o) - v(\infty))e^{-(t-t_o)/(\frac{L}{R})} + v(\infty) \qquad (6)$$

$$= (v_o - v)e^{-(t-t_o)(\frac{R}{L})} + V \qquad (7)$$

$$i(t_{o+}) = i \qquad i(\infty) = V/R \qquad (8)$$

$$\therefore \text{ same expression} \qquad (9)$$

$$i(t) = (i_o - \frac{V}{R})e^{-(t-t_o)(\frac{R}{L})} + V/R \qquad (10)$$

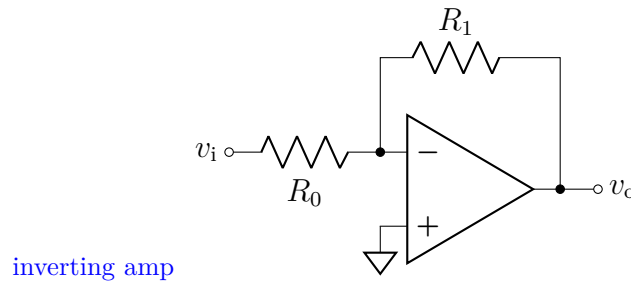
$$i(t > t_o) = (i_o - \frac{V}{R})e^{-(t-t_o)(\frac{R}{L})} + \frac{V}{R} \qquad v(t > t_o) = (v_o - V)e^{-(t-t_o)(\frac{R}{L})} + V$$

### Problem 4, 18 pts: Ohm Sweet Ohm

For each of the following circuits, determine the gain  $G = \frac{v_o}{v_i}$ . Make the ideal op-amp assumption, assume all the op-amps are stable, and ignore the output limitations of the op-amps. All labeled voltages are measured relative to the same ground node.

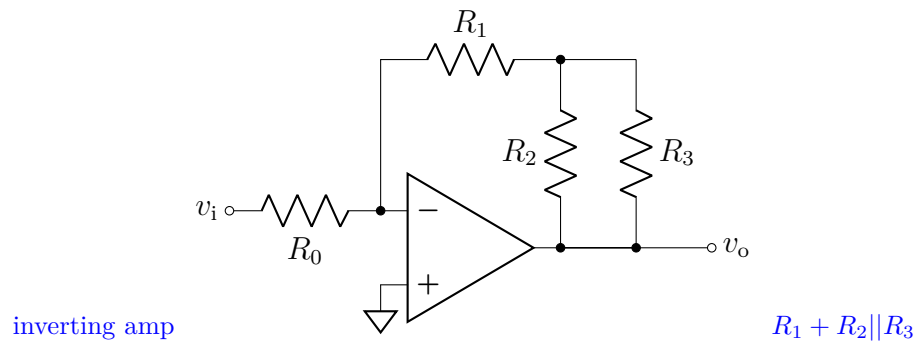
Feel free to use the symbols  $+$  and  $||$  to represent series and parallel combinations of resistors, respectively, rather than solving completely.

#### (4A) (2pts) Circuit 1



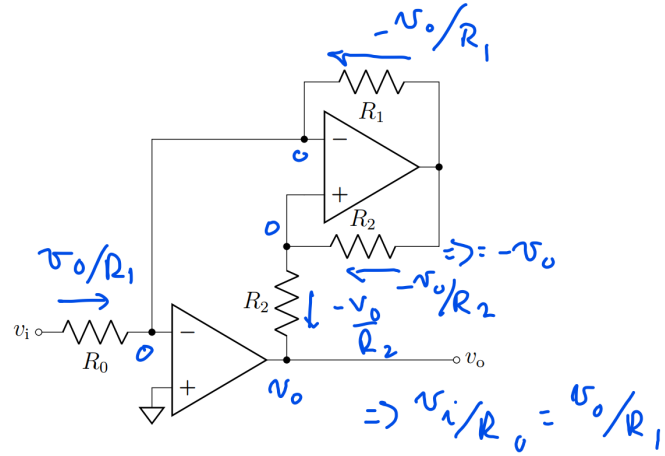
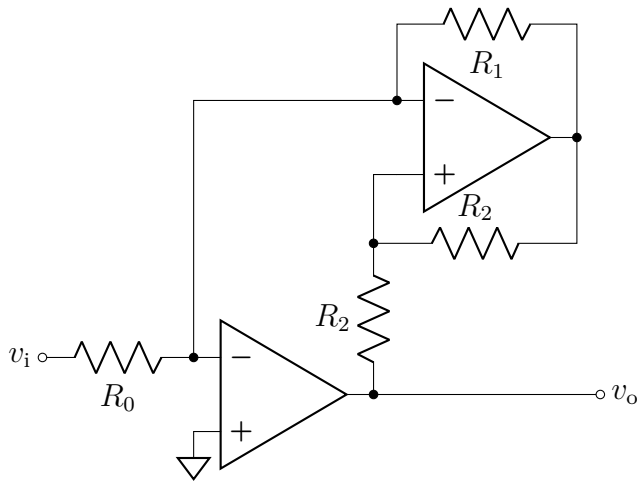
$$G = \frac{v_o}{v_i} = -\frac{R_1}{R_0}$$

#### (4B) (3pts) Circuit 2



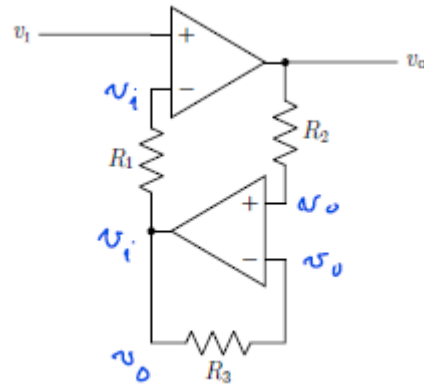
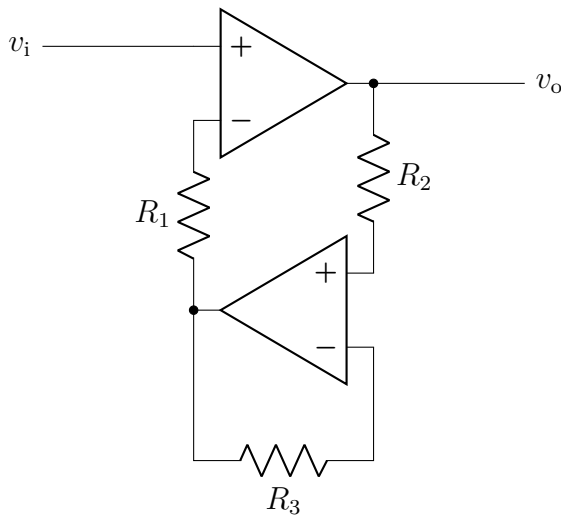
$$G = \frac{v_o}{v_i} = -\frac{(R_1 + (R_2 || R_3))}{R_0}$$

(4C) (4pts) **Circuit 3**



$$G = \frac{v_o}{v_i} = \frac{R_1}{R_0}$$

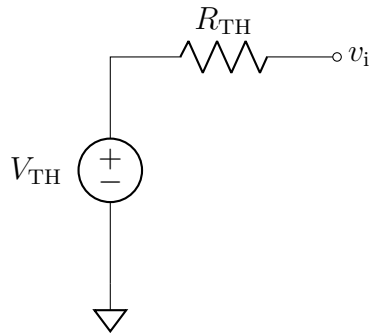
(4D) (4pts) **Circuit 4:**



$$G = \frac{v_o}{v_i} = 1$$

## Persistence

In the previous sections of this problem, we calculated the gain relative to the potential at the input,  $v_i$ . However, typically we are more interested in the gain relative to the source voltage of a network with a finite output resistance, as shown below:



where  $V_{TH}$  represents some earlier voltage (e.g. from a sensor, or a signal source) and  $R_{TH}$  represents the output resistance of that earlier element.

Now imagine connecting this divider to each of the circuits from the previous sections (as enumerated circuits 1-4) so that the node labeled  $v_i$  in this diagram is connected to the node labeled  $v_i$  in the diagrams above one at a time.

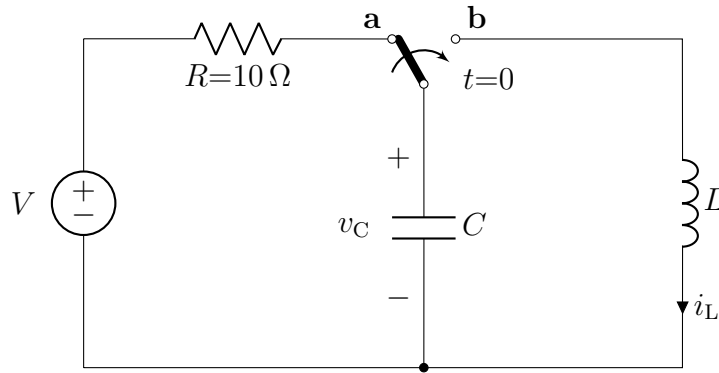
- (4E) (5pts) For which of these circuits, if any, will the new output voltage **be something other than**  $G \cdot V_{TH}$ ? Circle the numbers for each of those circuits in the box below, or enter **None** if none of the circuits have this property.

Circle all answers where  $v_o \neq G \cdot V_{TH}$  or None. **Earn one point for leaving blank.**

Circuit(s) with $v_o \neq G \cdot V_{TH}$ :	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4	<input type="radio"/> none
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**Problem 5, 18 pts: A Dastardly Plot**

Consider the circuit shown below. The switch is at position “a” for a long time before it switches to position “b” at  $t = 0$ .



(5A) (6pts) Write a symbolic expression for  $v_c(t)$  for  $t > 0$ .

$$v_C(0_-) = V \qquad \Rightarrow v_C(0_+) = 0 \qquad (11)$$

$$i_C(0_-) = 0 \qquad \Rightarrow i_L(0_+) = 0 \qquad (12)$$

$$(13)$$

$$V = A(\cos \omega_o t + \varphi) \qquad \omega_o = \frac{1}{\sqrt{LC}} \qquad (14)$$

$$\Rightarrow A \cos \varphi = 0 \qquad (15)$$

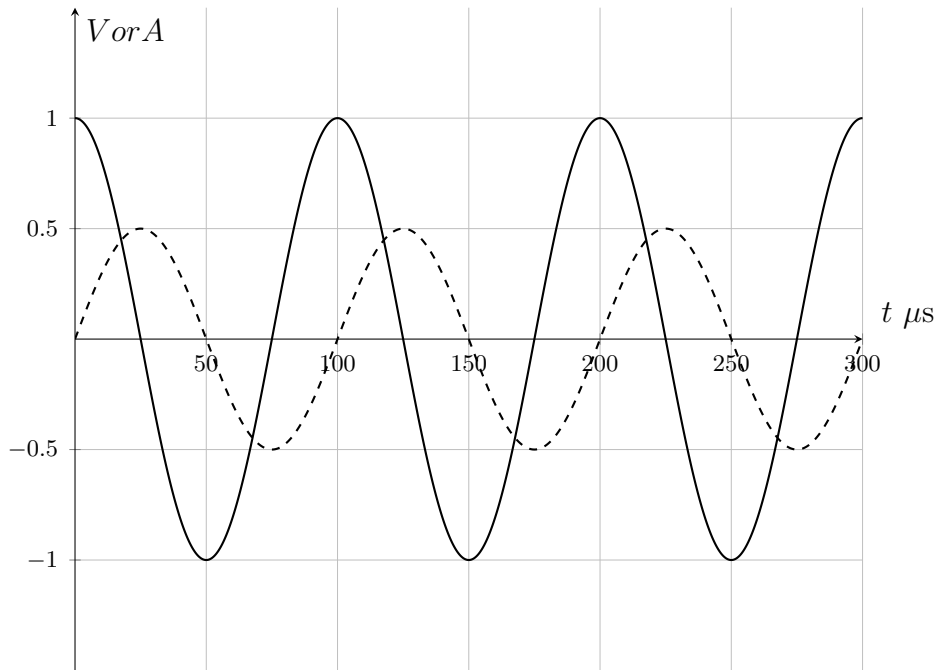
$$i_L = -i_C = -C \frac{dv_C}{dt} = -CA \omega_o (-\sin(\omega_o t + \varphi)) \qquad (16)$$

$$\Rightarrow CA \omega_o \sin \varphi = 0 \qquad \Rightarrow \varphi = 0 \qquad (17)$$

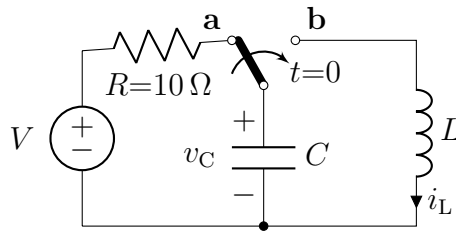
$$\Rightarrow A = V \qquad (18)$$

$$v_c(t) = V \cos \omega_o t$$

$v_c(t)$  (in volts) and  $i_L(t)$  (in amps) are given by the plots shown below for  $t > 0$ . The plot does not indicate which is which.



(5B) (2pts) Identify which curve corresponds to  $v_c(t)$  and which one corresponds to  $i_L(t)$ . Circle the correct variable for each curve in the box below. The circuit from part (A) is repeated for your convenient reference. **To discourage guessing, you will receive 1 pt of partial credit for each answer left blank.**



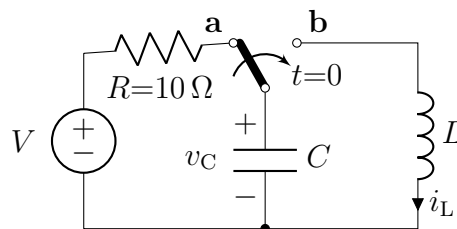
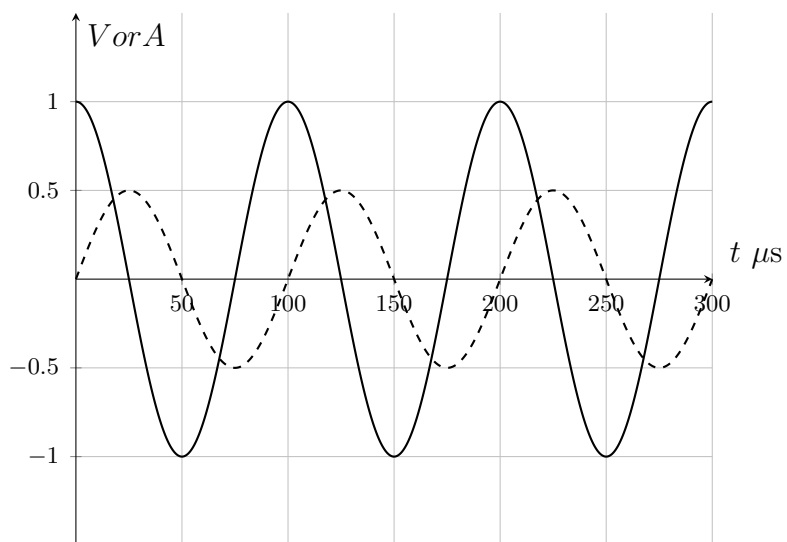
$$v_C \sim \cos \omega_0 t$$

$$i_L \sim \sin \omega_0 t$$

Circle the correct variable for each graph indicated.

Solid:	$i_L$	<input type="radio"/> $v_C$	Dashed:	<input type="radio"/> $i_L$	<input type="radio"/> $v_C$
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(5C) (2pts) Using the plot (repeated along with the circuit below for your convenience), determine  $V$ , the strength of the voltage source in volts?



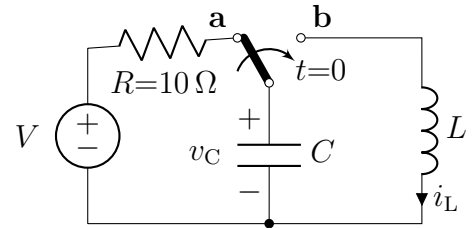
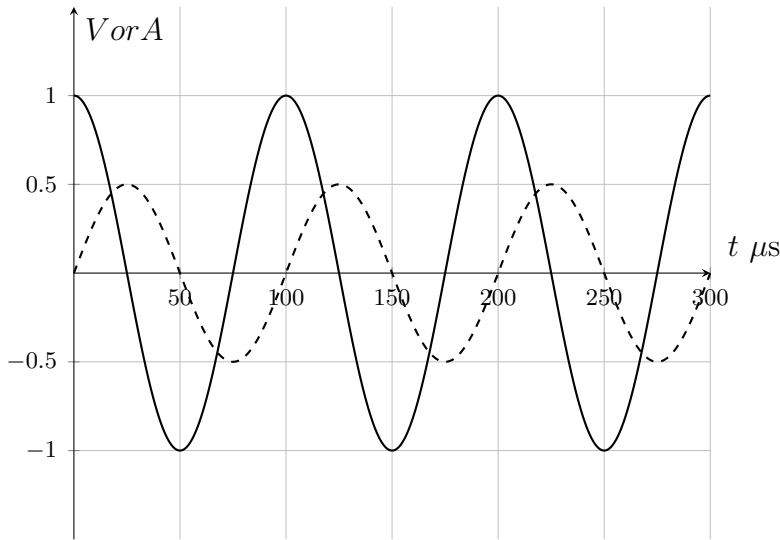
$$v = A \cos \omega_0 t, \text{ where } A \equiv V$$

$$\Rightarrow V = 1V$$

$$V = 1V$$

[V]

(5D) (4pts) Again using the plot, again repeated along with the circuit below for your convenience, write a numerical expression for the values of  $L$  and  $C$ . Note: you need not calculate the final value.



$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$T = 100 \mu s = 10^{-4} s$$

$$\Rightarrow f = 10^4 Hz$$

$$\Rightarrow \omega_o = 2\pi \cdot 10^4 \frac{rad}{s}$$

$$Z_c = \frac{|v_{max}|}{|i_{max}|} = 2 = \sqrt{\frac{L}{C}}$$

$$L = Z_c / \omega_o = 2 / 2\pi \cdot 10^4 = \frac{1}{\pi} \cdot 10^{-4} H$$

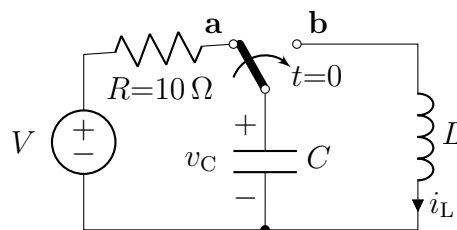
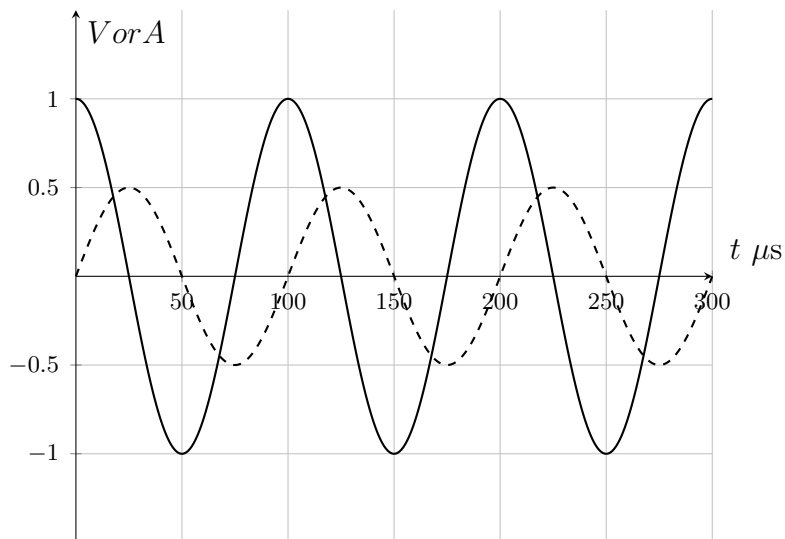
$$C = \frac{1}{\omega_o Z_c} = \frac{1}{4\pi \cdot 10^4} F$$

$$L = \frac{1}{\pi} \cdot 10^{-4} H$$

$$C = \frac{1}{4\pi} \cdot 10^{-4} F$$



(5E) (4pts) Write a numerical expression for the total energy (with units) stored in the inductor and capacitor after  $225 \mu s$  have elapsed? Note: you do not need to evaluate the final expression. Again, the graph and circuit are repeated for your convenience.



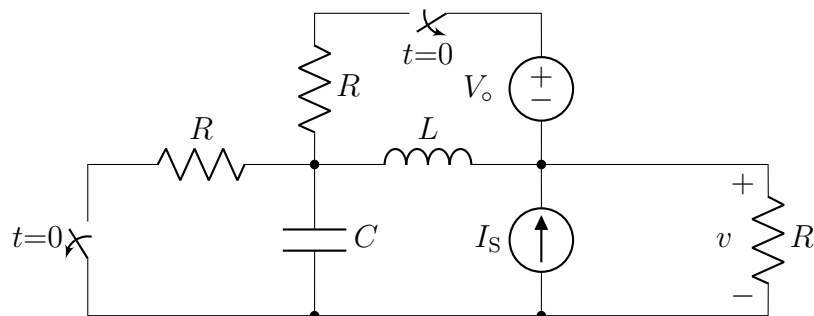
Let's just use the initial energy, when all energy is in the capacitor and  $v_c = 1 \text{ V}$ .

$$E = \frac{1}{2}cv^2$$

$$\text{Total Energy} = \frac{1}{2} \left( \frac{1}{4\pi} \cdot 10^{-4} \right) J$$

### Problem 6, 12 pts: Switching It Up

Consider this circuit:

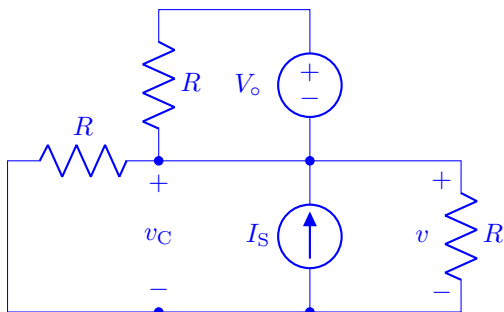


Initially, the switches are closed (i.e. form a connection) for a very long time the system is in the steady state. Then at  $t = 0$  both of the switches open (i.e. the connections break).

(6A) (4pts) Solve for  $v$  at  $t = 0_-$ , the moment before the switches open.

$C \rightarrow$  open

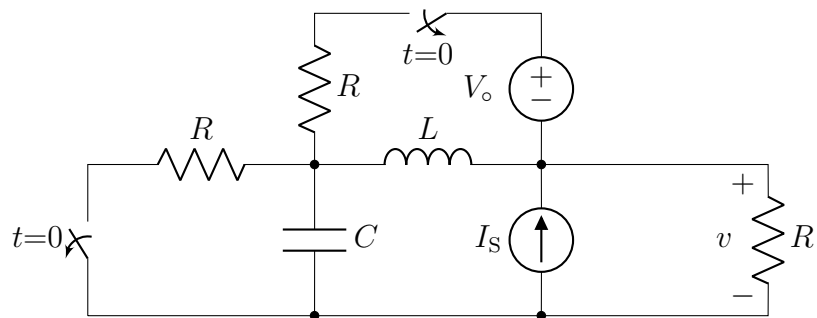
$L \rightarrow$  short



Use superposition:  $V_o$  has no influence on  $v$  due to short across it, so  $I_s$  is only effect and  $v = I_s/2R$ .

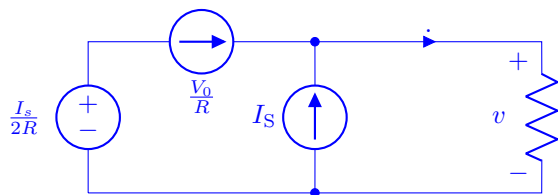
$$v(t = 0_-) = \frac{I_s R}{2}$$

(6B) (4pts) Determine the branch voltage  $v$  at the instant  $t = 0_+$ , immediately after the switches open. The circuit is repeated below for your convenience.



$$v_c(0_-) = v = \frac{I_s R}{2} = v_c(0_+)$$

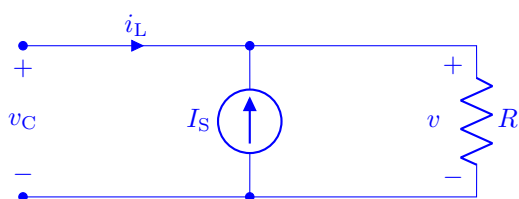
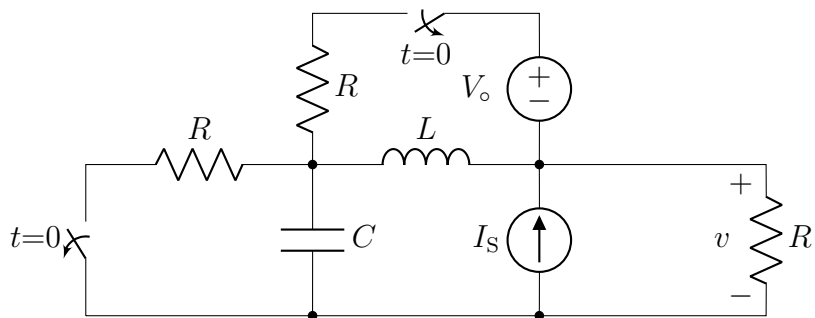
$$i_L(0_-) = \frac{V_o}{R} - \frac{I_s R}{2}$$



$$v = \left(\frac{V_o}{R} + I_s\right)R$$

$$v(t = 0_+) = V_o + \frac{I_s \cdot R}{2}$$

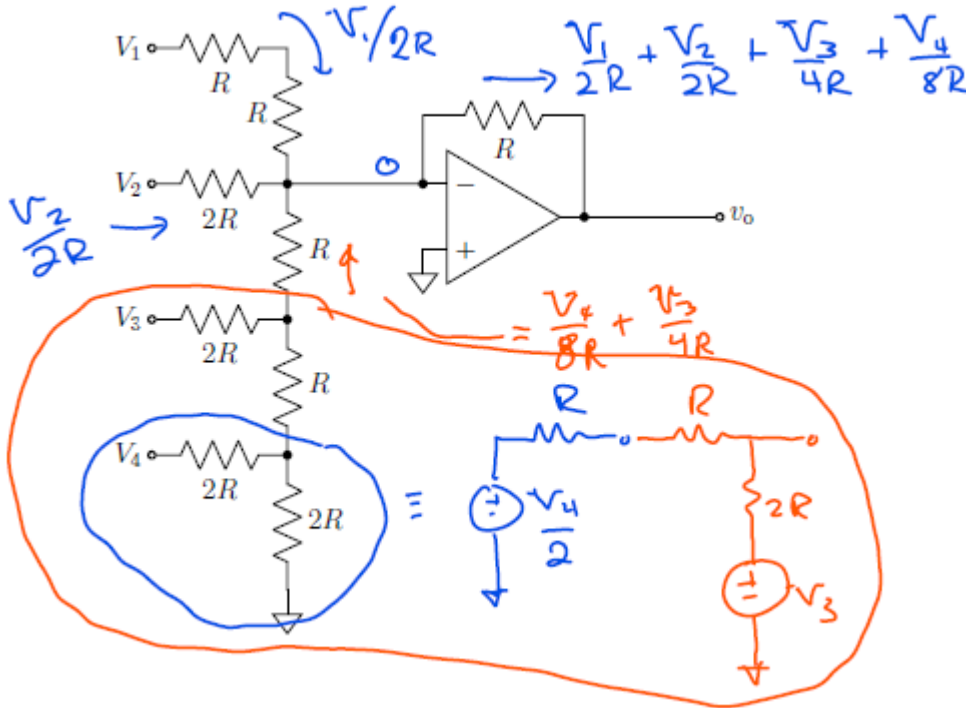
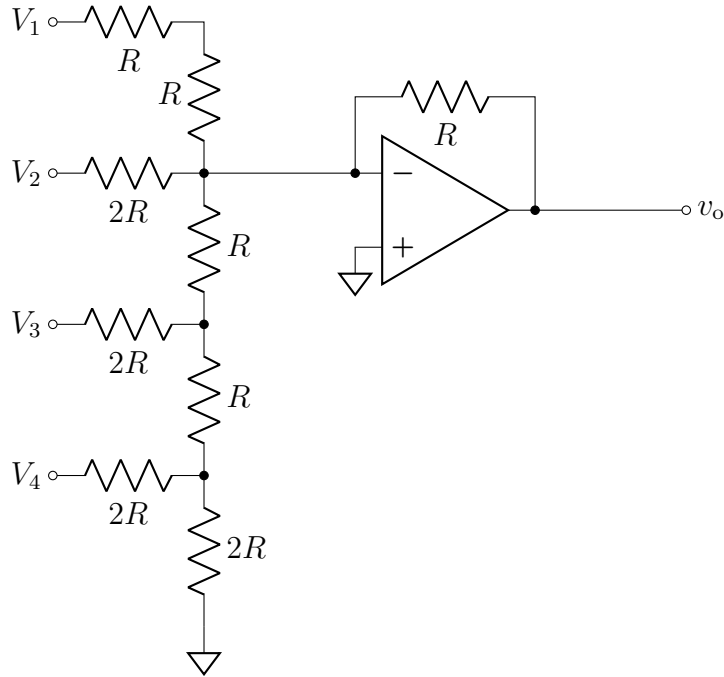
(6C) (4pts) Now assuming the switches remain open for all  $t > 0$ , determine  $v$  as  $t$  approaches  $\infty$ . Again, the circuit is repeated below for your convenience.



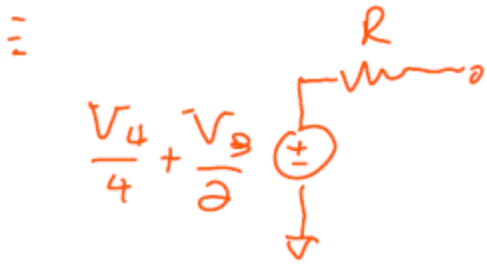
$$v(t = \infty) = I_s R$$

**Problem 7, 8 pts: Summer Vacation**

Consider the circuit shown below. You may make the ideal op-amp assumption, assume stability, and ignore output limitations of the op-amp.



(7A) (8pts) Provide an expression for  $v_o$ .



$$v_o = -\frac{V_1}{2} - \frac{V_2}{2} - \frac{V_3}{4} - \frac{V_4}{8}$$

*Worksheet (intentionally blank)*