MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

QUIZ 2 - 6.002 Circuits and Electronics

April 13, 2022

Total Points: 100 **Time Limit: 120 minutes**

YOUR NAME____

YOUR KERBEROS ID____

RECITATION: 11 am 12 pm 1 pm

General Instructions:

1. Please do all of your work in the spaces provided in this examination booklet. If you need additional sheets, be sure to put your name and the name of the examination on each sheet. Place your answer for each question in the space provided on this booklet.

2. The exam consists of 4 problems on pages 2-13. Please make sure you have all of the pages. Use the space immediately following each question to show your work and the answer to the question.

3. All sketches must be adequately labeled

4. You will be graded on both your solution (that is, the work shown) and your final answer. It is possible to get the right answer, but not receive full credit if your reasoning is unclear. A few words of explanation are required.

5. Indicate units on all numerical answers.

6. The exam is closed books but calculators and two two-sided sheet of notes and formulas is allowed.

Grade: **Problem 1: (/25)**

Problem 2: (/25)

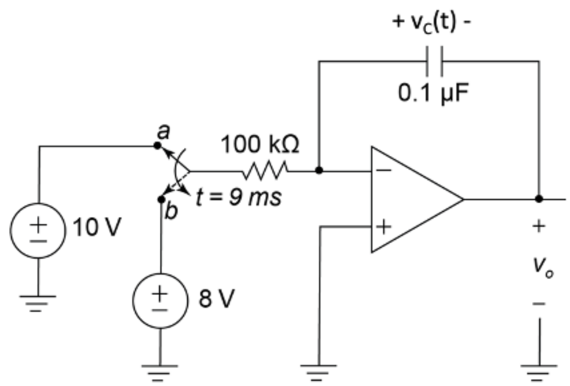
Problem 3: (/25)

Problem 4: (/25)

Total grade:

Problem 1. First Order RC Circuit with an Op-Amp [25 pts]

Consider the circuit below with an ideal op-amp. The switch makes contact with terminal *a* at t = 0. At that instant, the voltage across the capacitor is $v_C(t=0) = 5$ V. The switch remains at terminal *a* for 9 ms and then moves instantaneously to terminal *b*, where it remains. In this problem we will find the output voltage (v_o) at t = 20 ms.



To find the output voltage (v_o) at t = 20 ms, follow the steps below:

a) [7pts] Find an expression for v_o during the time the switch is at terminal *a*.

v_o=

b) [8 pts] Find an expression for v_o during the time the switch is at terminal *b*.

v_o=

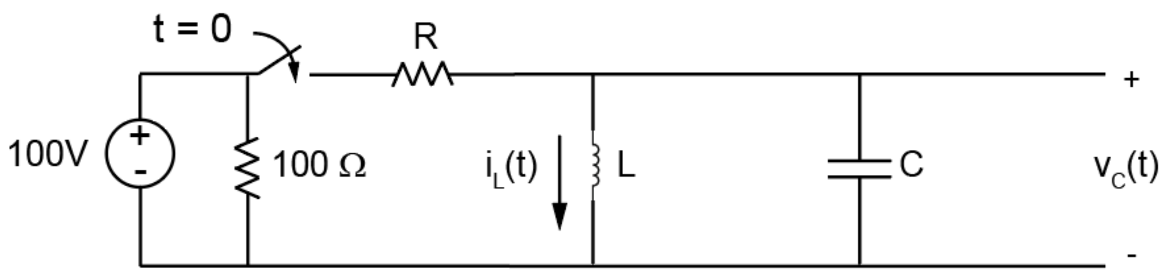
c) [5 pts] Find v_o at t = 20 ms.

v_o (t = 20 ms) =

d) [5 pts] Based on your results from Parts a – b, what does this op-amp circuit do.

Problem 2. RLC circuit with graphical interpretation (25 pts)

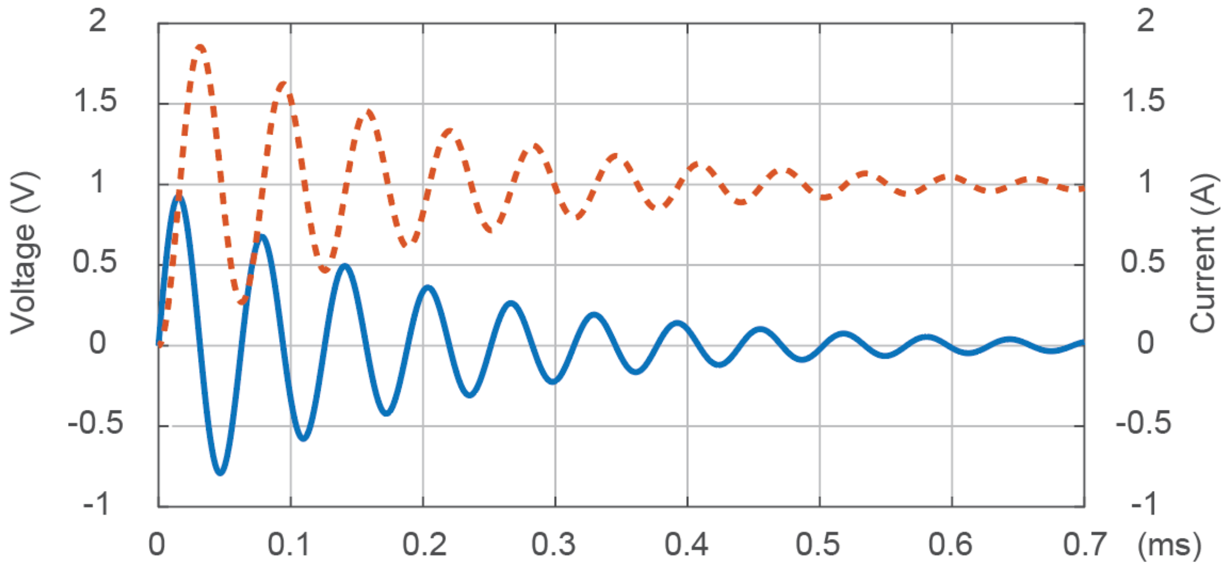
Consider the following circuit. The circuit is at rest for a long time so the inductor current and capacitor voltage are both zero for t < 0. At t = 0, the switch is closed. Please answer the following questions:



a) [2 pts] What are the values of $i_L(t = 0+)$ and $v_C(t = 0+)$ just after the switch closes?

$i_L(t = 0)$ = $v_C(t = 0)$=

b) [6 pts] The figure below shows the inductor current $i_L(t)$ and capacitor voltage $v_C(t)$. Please identify which curve corresponds to $i_L(t)$ and which one corresponds to $v_C(t)$. For full credit, explain your reasoning.



Please circle the solution below:		
$i_L(t)$:	**dotted curve**	**solid curve**
$v_C(t)$:	**dotted curve**	**solid curve**

c) [2 pts] Is this circuit over-damped, under-damped, or critically damped? For full credit, explain your reasoning.

Please circle the solution below:

$The\ circuit\ is$: **under-damped** **critically-damped** **over-damped**

because…

d) [7 pts] Using the voltage and/or current plots, estimate the circuit quality factor Q, natural frequency ω_0, and damping coefficient α.

$\boldsymbol{Q} =$

$\boldsymbol{\omega_0} =$

$\boldsymbol{\alpha} =$

e) [8 pts] Estimate the values of R, L, and C.

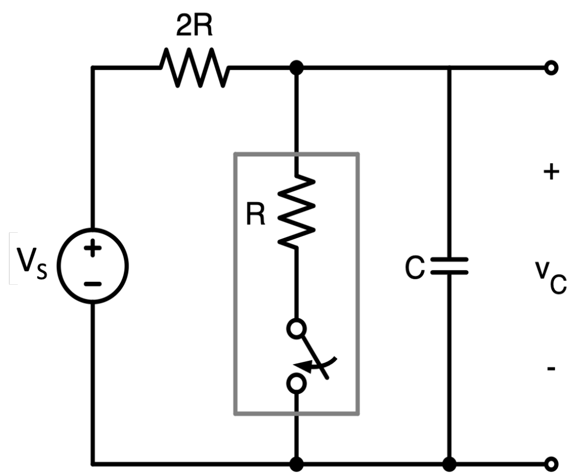
$\boldsymbol{R} =$

$\boldsymbol{L} =$

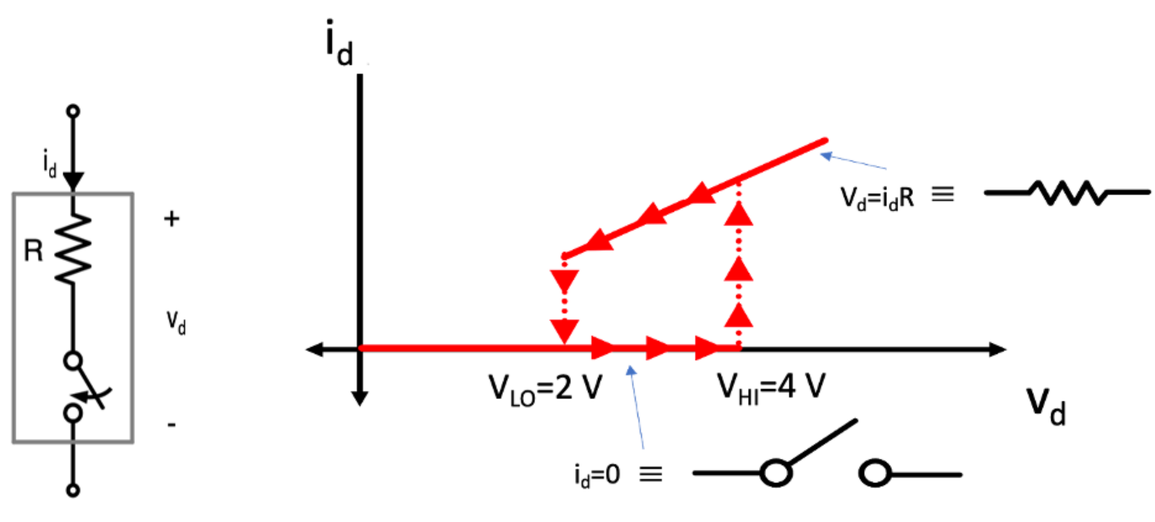
$\boldsymbol{C} =$

Problem 3. Relaxation oscillator [25 pts]

Consider this circuit:



The small grey rectangle contains a device with the *i-v* characteristic shown below. This device has two threshold voltages V_{HI} = 4V and V_{LO} = 2V. Above V_{HI} the switch closes and the device acts as a resistor with resistance *R*, below V_{LO} the switch opens and the device acts as an open circuit. In-between these voltages, the device can exhibit either behavior depending on where it started. Devices with this kind of behavior are said to be "hysteretic".



a) [3 pts] Consider the situation where V_S = 5 V and has been for all time. In this situation, the system will oscillate. Let t = 0 when v_C hits V_{HI} on its upswing. For the time shortly after t = 0, draw a simplified circuit without the switch. That is, if the switch is closed, replace it with a wire; and if it is open, remove that branch. Based on the simplified circuit, redraw it as the combination of a Thevenin equivalent loaded by a capacitor, and specify the Thevenin voltage v_{TH} and resistance R_{TH}.

v_{TH} =
R_{TH} =

b) [7 pts] Derive an expression for $v_C(t)$ for $0 < t < T_1$, and an expression for T_1, which is the time when the hysteretic device changes its state.

In $0 < t < T_1$, $v_C(t) =$

and $T_1 =$

c) [3 pts] Now draw a simplified circuit without the switch during the period $T_1 < t < T$, where T is the time when the hysteretic device changes its state again. That is, if the switch is closed, replace it with a wire; and if it is open, remove that branch. Based on this simplified circuit, redraw it as the combination of a Thevenin equivalent loaded by a capacitor, and specify the Thevenin voltage v_{TH} and resistance R_{TH}.

v_{TH} =

R_{TH} =

d) [7 pts] Find $v_C(t)$ for $T_1 < t < T$, where T is the time when the hysteretic device changes its state again. Since in the time interval $(0, T)$, the hysteretic device changes its state twice, flipping back and forth and completing a full cycle, T is therefore the period of oscillation. Find the expression for T.

In $T_1 < t < T$, $v_C(t)$ =

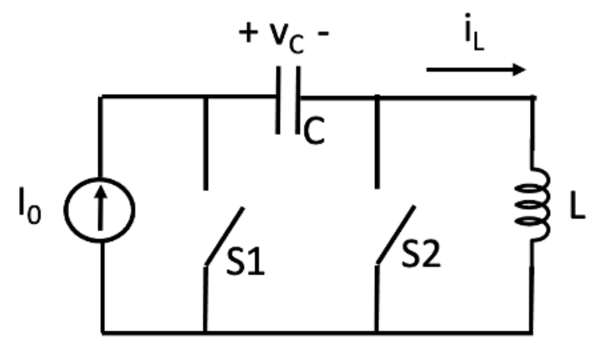
and T =

e) [5 pts] Find the range(s) of V_S for which the circuit does **not** oscillate, and give a brief explanation of your answer.

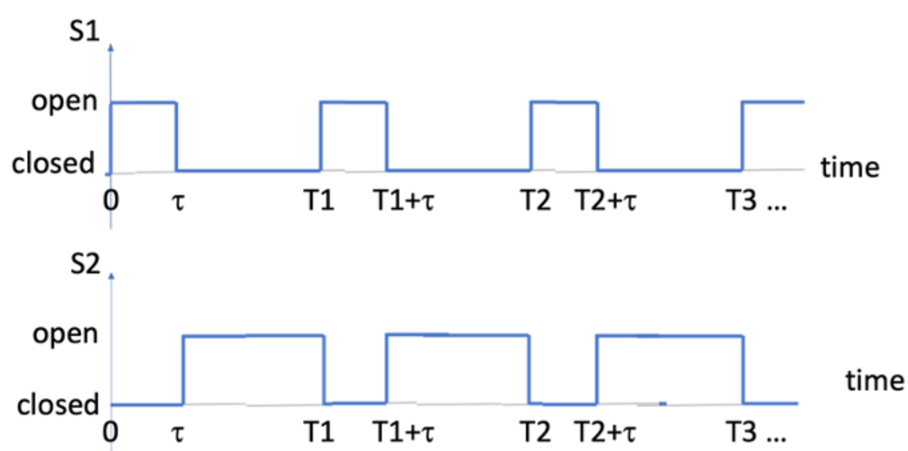
Range(s) of V_S that the circuit does not oscillate:

Problem 4. Boost Converter Energy Problem [25 pts]

In the circuit below, all the components are assumed to be ideal.



The switches S1 and S2 open and close as indicated by the plot below, where τ is a constant and T1, T2, T3… are chosen such that $v_C(T1)=v_C(T2)=\ldots=v_C(TN)=0$.



Assume that both the capacitor and inductor are completely discharged at time t=0.

a) [3 pts] Write an expression for the voltage across the capacitor, $v_C(t)$, for time $0 \le t \le \tau$.

b) [2 pts] Write an expression for the current through the inductor, $i_L(t)$, for time $0 \le t \le \tau$.

c) [5 pts] Write an expression for the current through the inductor, $i_L(t)$, for time $\tau \le t \le T1$.

d) [5 pts] How much energy has been transferred by the current source to the circuit during the time interval t=0 to t=T10?

e) [5 pts] Determine the current through the inductor at t=T10.

f) [5 pts] How would the current through the inductor at time t=T10 change if the inductor is not ideal but has a small series resistance, R, resulting in a network with a quality factor much greater than unity?