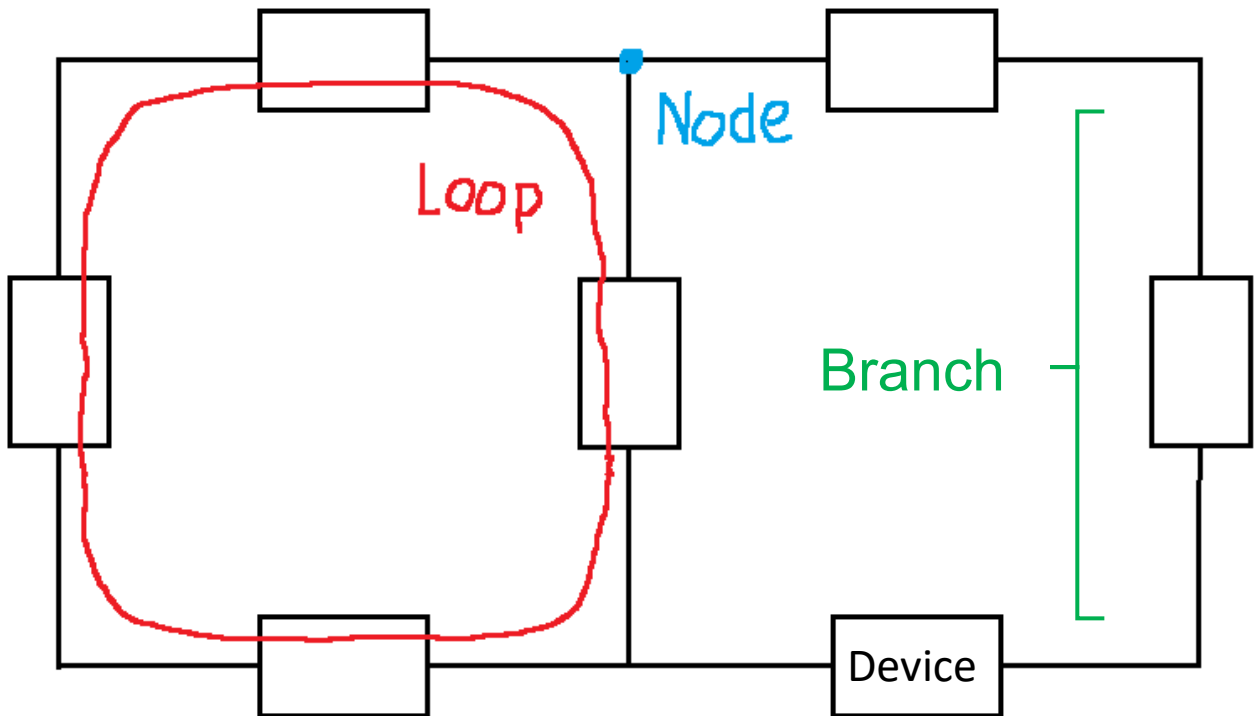


6.002 - Lecture 02

(Brute Force) Circuit Analysis

- Review: Labeling Devices
- Review: Nodes and Loops
- Brute Force Analysis
- Series and Parallel Resistors
- Voltage and Current Dividers

Circuit Terminology



Device: a multi-terminal lumped-parameter piece of electrical hardware having identifiable voltages across (between) its terminals and currents through its terminals. Most devices in 6.200 have only two terminals.

Circuit: a collection of devices joined together at their terminals, presumably designed for a useful purpose.

Node: the topological point at which one or more device terminals are joined together. Nodes can be spatially distributed, occupying more than a point in real space.

Branch: the topological pathway between two electrically neighboring nodes. Branches are occupied by devices.

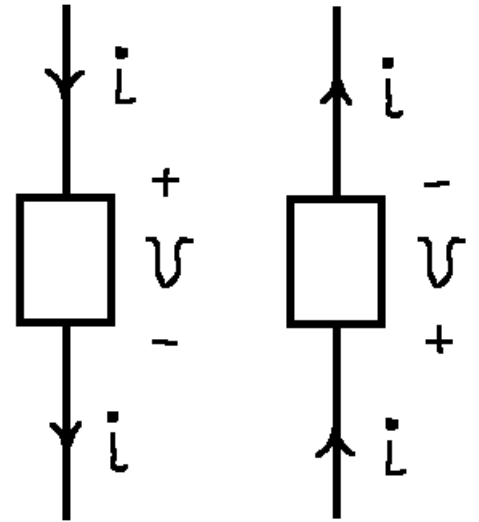
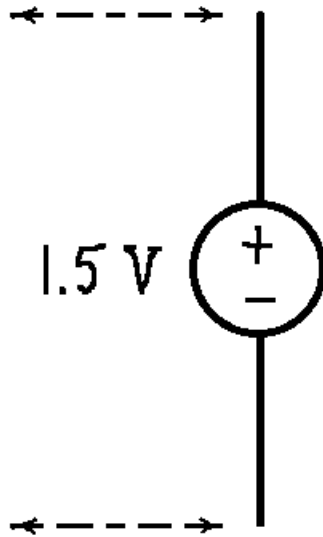
Loop: A closed-path through the branches that traverses no branch more than once.

Circuit Analysis: determination of all branch (device) voltages and currents.

Labeling

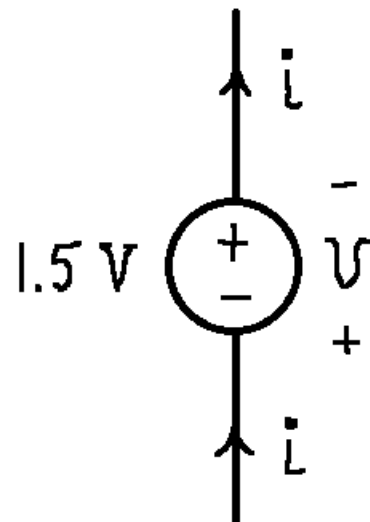
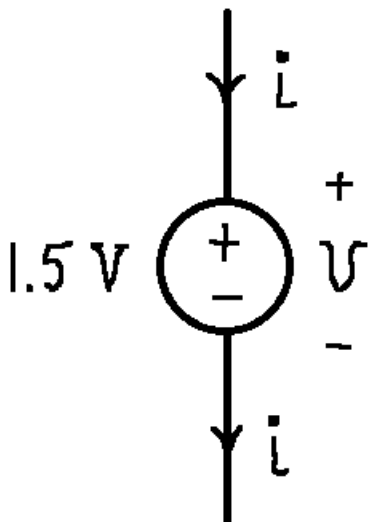
Intrinsic

Extrinsic



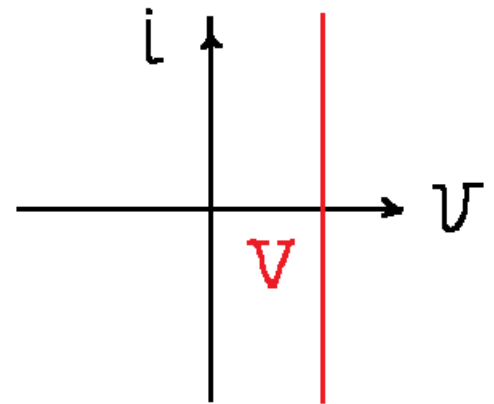
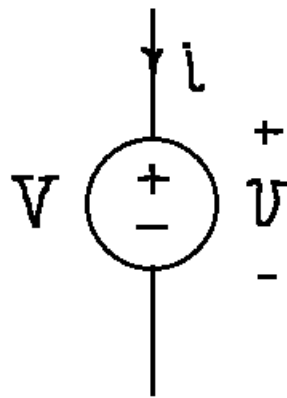
$$U = 1.5 \text{ V}$$

$$U = -1.5 \text{ V}$$

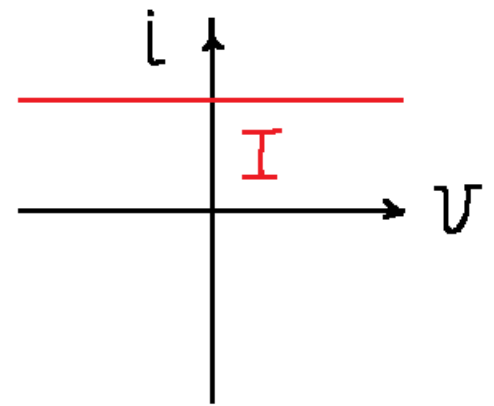
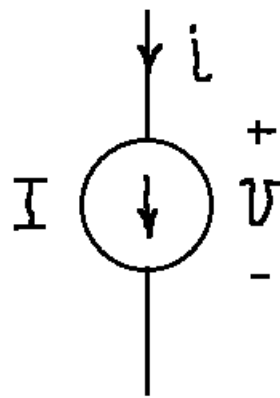


Devices

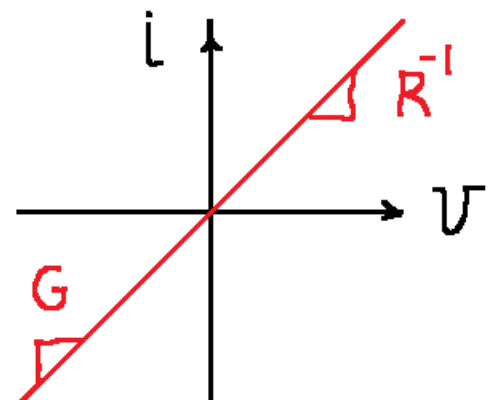
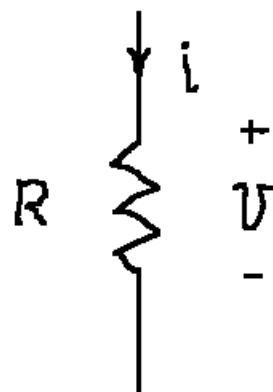
Voltage Source



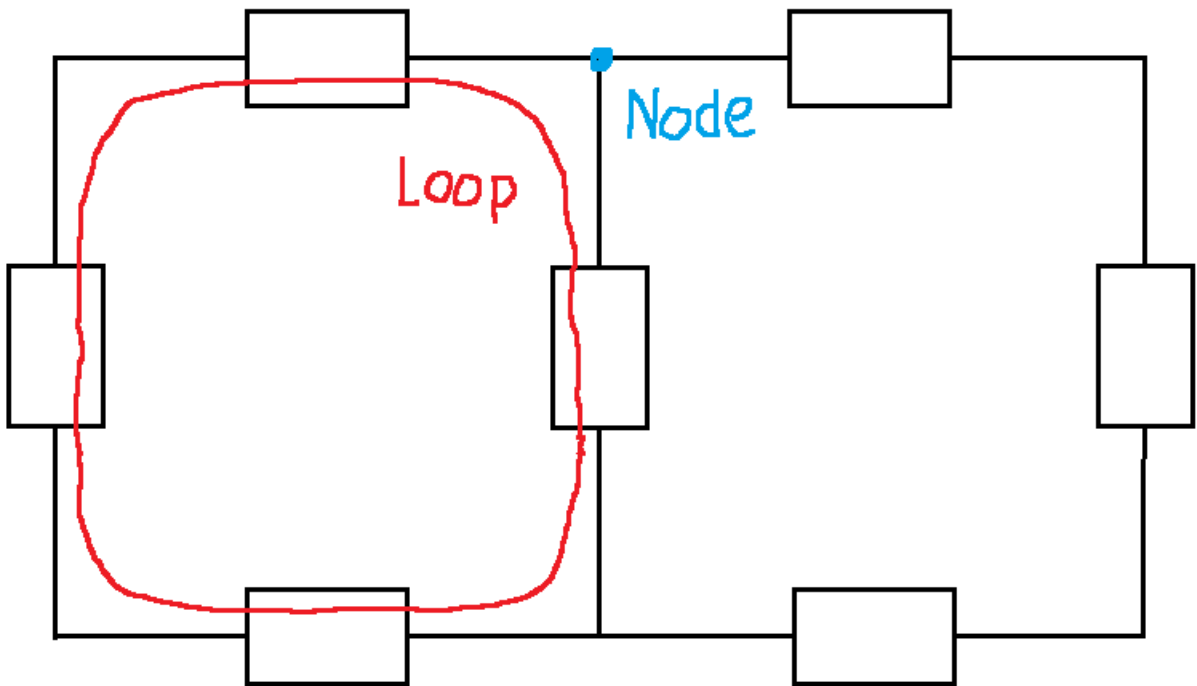
Current Source



Linear Resistor



Connections



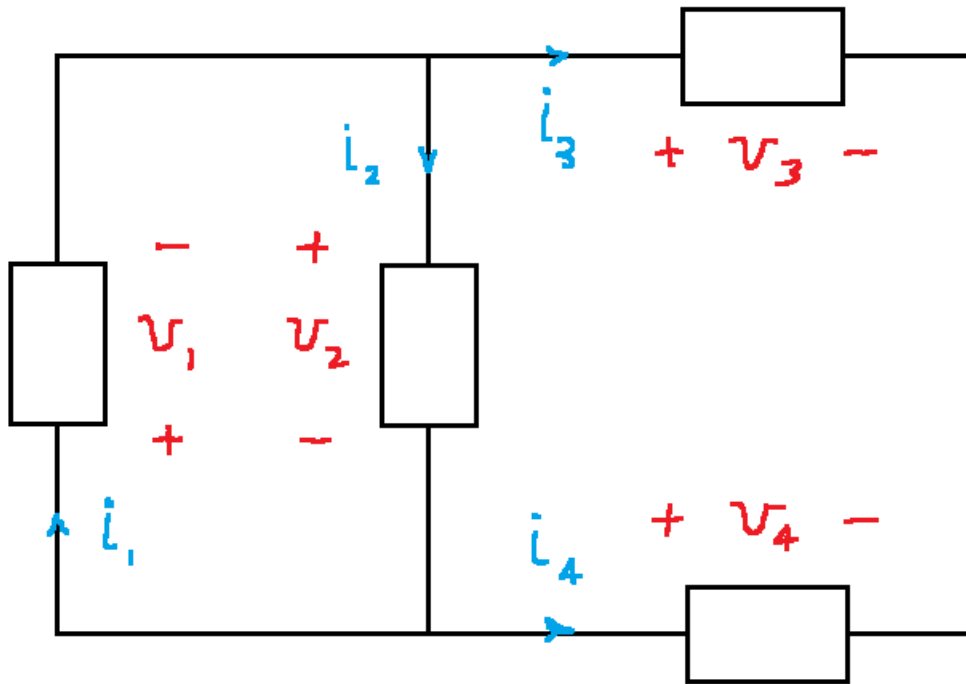
KVL: $\sum v = 0$
Around Loop

← Either Direction

KCL: $\sum i = 0$
Into Node

← Or Out From

KVL & KCL



$$\text{KVL: } -V_1 - V_2 = 0 \text{ or } V_1 = -V_2$$

$$V_4 + V_2 - V_3 = 0 \text{ or } V_4 + V_2 = V_3$$

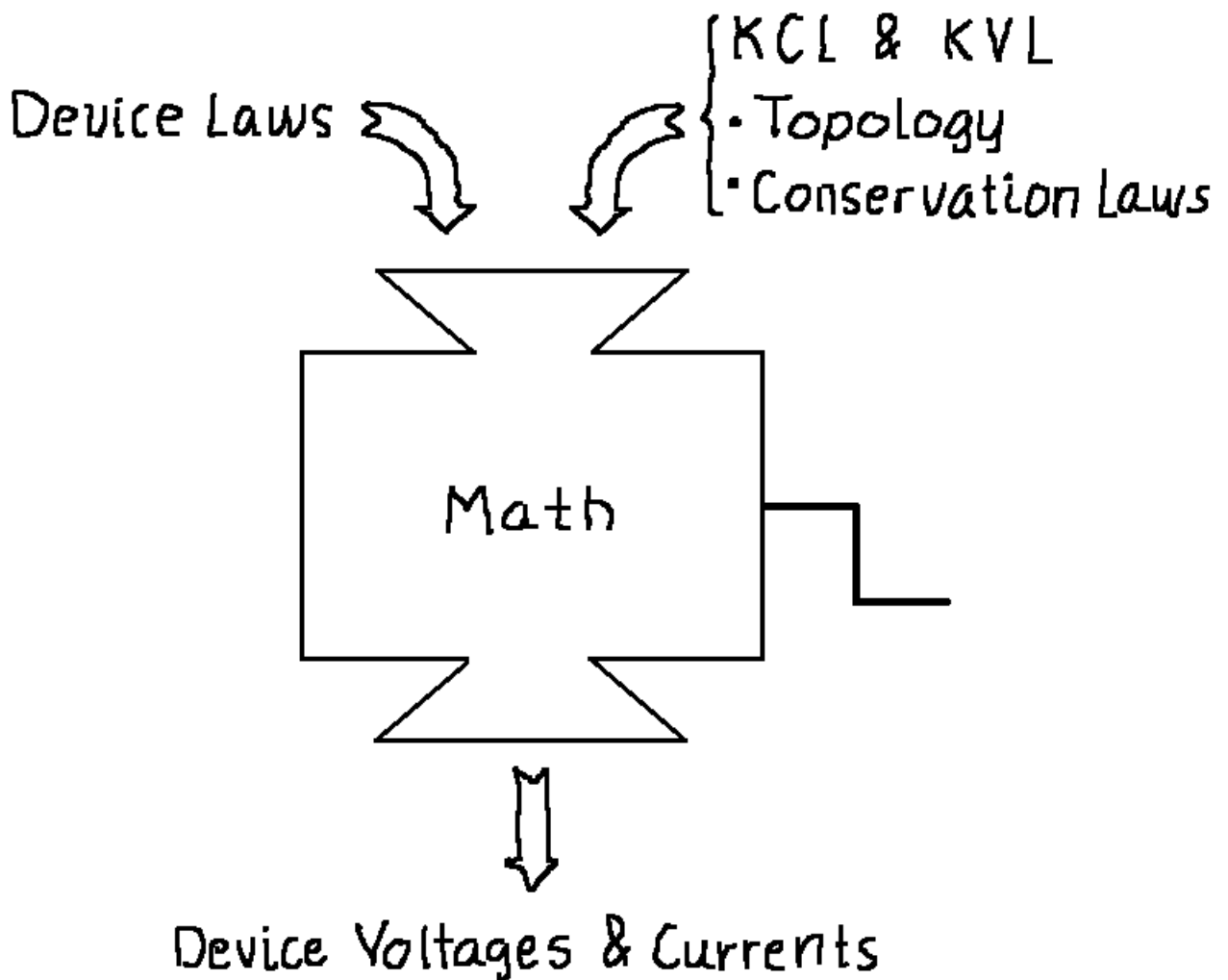
$$-V_1 - V_3 + V_4 = 0 \text{ or } V_4 = V_1 + V_3$$

$$\text{KCL: } i_1 - i_2 - i_3 = 0 \text{ or } i_1 = i_2 + i_3$$

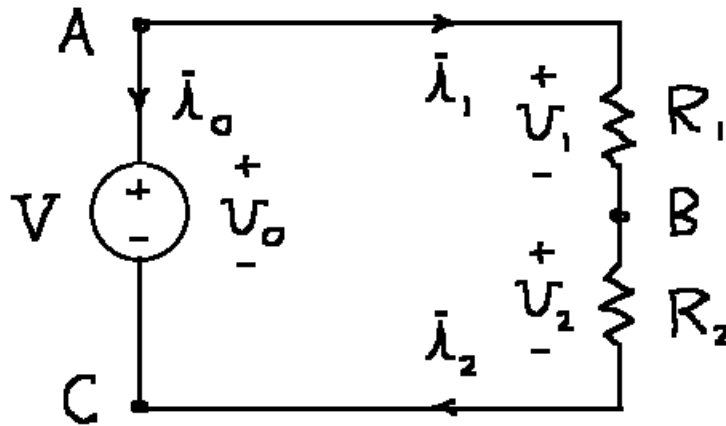
$$i_3 + i_4 = 0 \text{ or } i_3 = -i_4$$

$$-i_1 + i_2 - i_4 = 0 \text{ or } i_2 = i_1 + i_4$$

Brute Force Analysis



Brute Force Analysis #1



Devices

$$v_0 = V$$

$$v_1 = R_1 i_1$$

$$v_2 = R_2 i_2$$

KCL

$$A: i_0 + i_1 = 0$$

$$B: i_2 - i_1 = 0$$

$$C: -i_0 - i_2 = 0$$

One redundant KCL

KVL

$$v_0 - v_1 - v_2 = 0$$

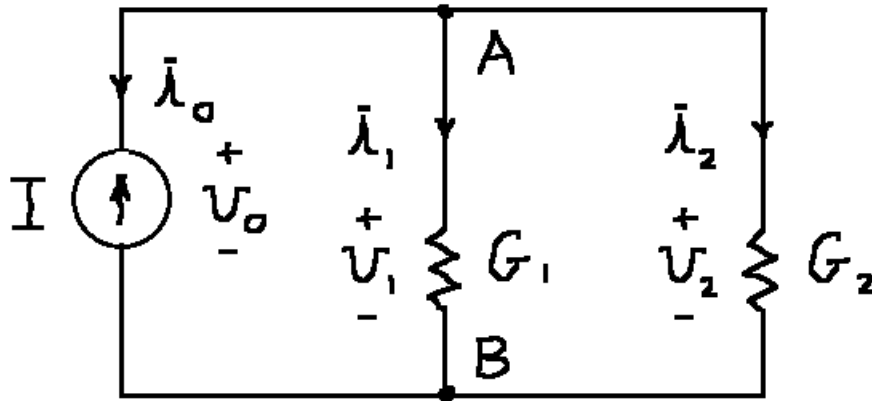
Solution

$$i_1 = i_2 = -i_0 = V / (R_1 + R_2) \equiv i$$

$$v_0 = V ; v_1 = R_1 i = V \frac{R_1}{R_1 + R_2} ; v_2 = R_2 i = V \frac{R_2}{R_1 + R_2}$$

$$\text{Note: } v_0 = v_1 + v_2 = i(R_1 + R_2) \Rightarrow \frac{v_0}{i} = \underbrace{R_1 + R_2}_{\text{Effective } R}$$

Brute Force Analysis #2



Devices

KCL

KVL

$$i_0 = -I$$

$$A: i_0 + i_1 + i_2 = 0$$

$$v_0 - v_1 = 0$$

$$i_1 = G_1 v_1$$

$$B: -i_0 - i_1 - i_2 = 0$$

$$v_0 - v_2 = 0$$

$$i_2 = G_2 v_2$$

One redundant KCL

$$v_1 - v_2 = 0$$

One redundant KVL

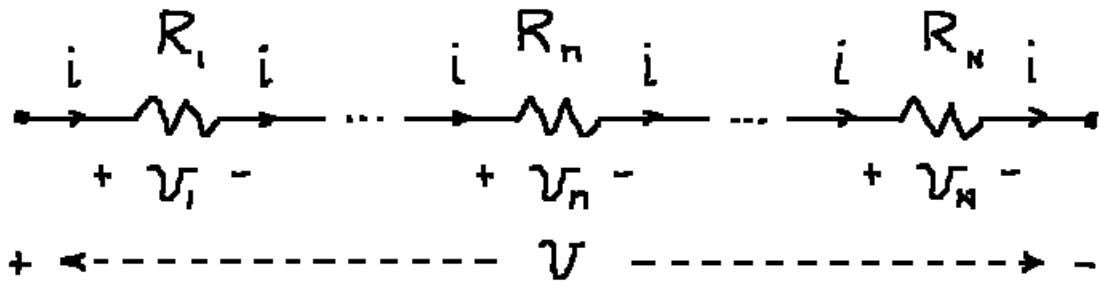
Solution

$$v_0 = v_1 = v_2 = I / (G_1 + G_2) = V$$

$$i_0 = -I; i_1 = G_1 V = I \frac{G_1}{G_1 + G_2}; i_2 = G_2 V = I \frac{G_2}{G_1 + G_2}$$

$$\text{Note: } -i_0 = i_1 + i_2 = V(G_1 + G_2) \Rightarrow \frac{V}{-i_0} = \underbrace{G_1 + G_2}_{\text{Effective } G}$$

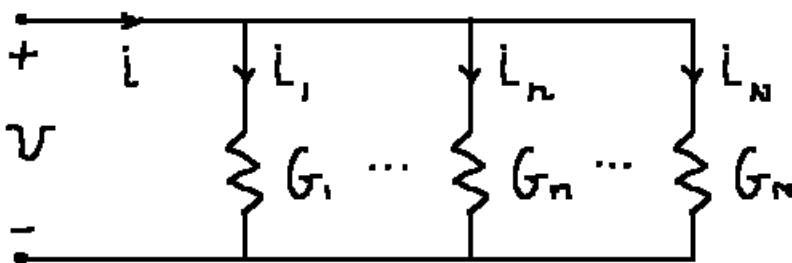
Resistor Combinations



$$V = \sum_n v_n = \sum_n R_n i = i \sum_n R_n \Rightarrow v/i = R = \sum_n R_n$$

Series resistances add: $R = \sum_n R_n$

Voltage divider: $v_n = R_n i = (R_n / R) V$



$$i = \sum_n i_n = \sum_n G_n V = V \sum_n G_n \Rightarrow i/V = G = \sum_n G_n$$

Parallel conductances add: $G = \sum_n G_n$; $R^{-1} = \sum_n R_n^{-1}$

Current divider: $i_n = G_n V = (G_n / G) i$

Summary

- Intrinsic and extrinsic labels
- Brute force analysis
- Series resistances add
- Parallel conductances add
- Voltage dividers
- Current dividers