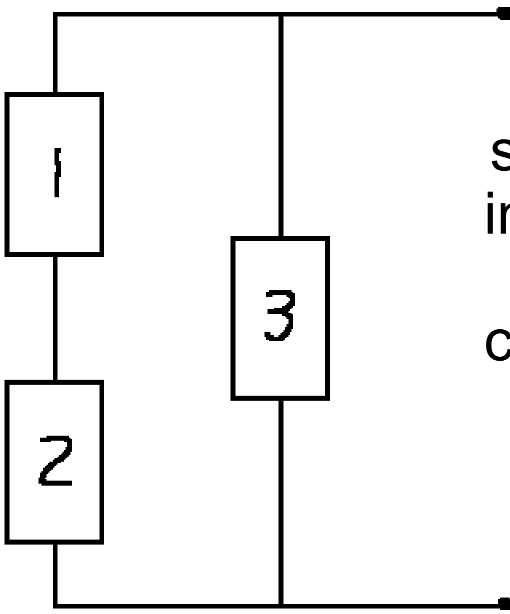
6.002 - Lecture 03

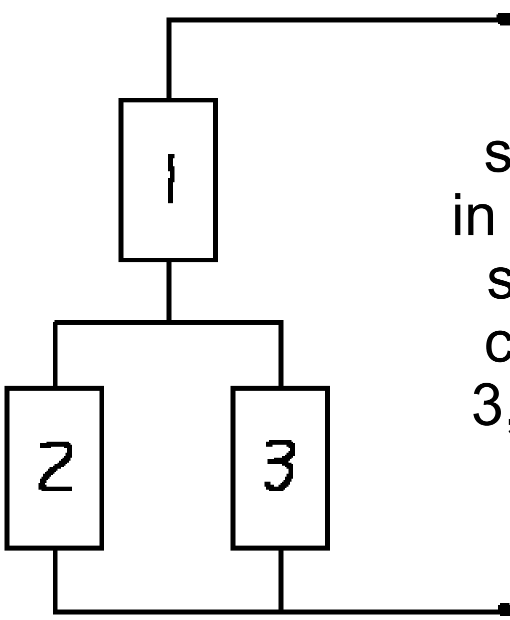
Circuit Analysis Simplifications

- Parallel & Series Reductions
- Dividers
- Node Analysis

Series? Parallel?

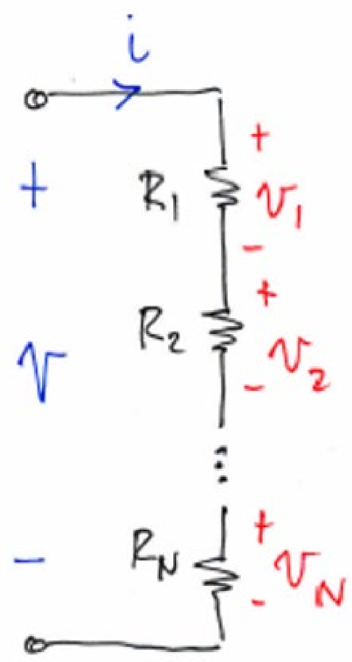


Devices 1 and 2 share the same current, hence they are in series. Device 3 shares the same voltage with the series combination of Devices 1 and 2, hence it is in parallel with the series combination of Devices 1 and 2.



Devices 2 and 3 share the same voltage, hence they are in parallel. Device 1 shares the same current with the parallel combination of Devices 2 and 3, hence it is in series with the parallel combination of Devices 2 and 3.

Series Resistors (Common i)

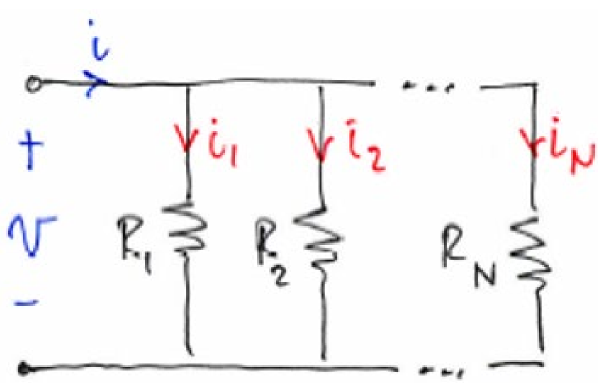


$$
\begin{aligned}
v &= v_1 + v_2 + \dots v_N \\
&= R_1 i + R_2 i + \dots R_N i \\
&= (R_1 + R_2 + \dots R_N)\, i \\
&\equiv R i
\end{aligned}
$$

$$R = R_1 + R_2 + \dots R_N$$

$$v_n = R_n / (R_1 + R_2 + \dots R_N)$$

Parallel Resistors (Common v)

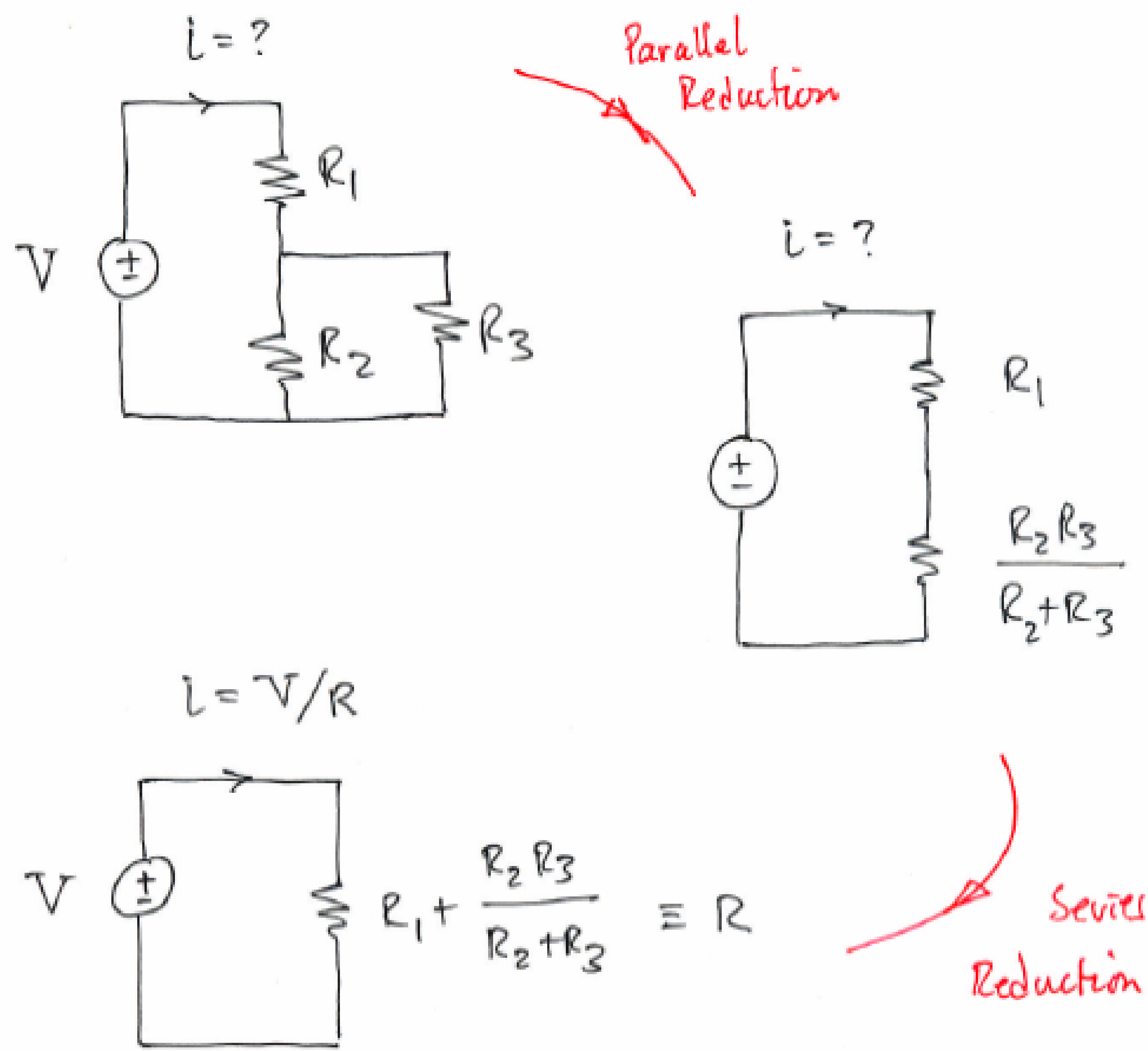


$$
\begin{aligned}
i &= i_1 + i_2 + \dots i_N \\
&= G_1 v + G_2 v + \dots G_N v \\
&= (G_1 + G_2 + \dots G_N)\, v \\
&\equiv G v
\end{aligned}
$$

$$G = G_1 + G_2 + \dots G_N$$

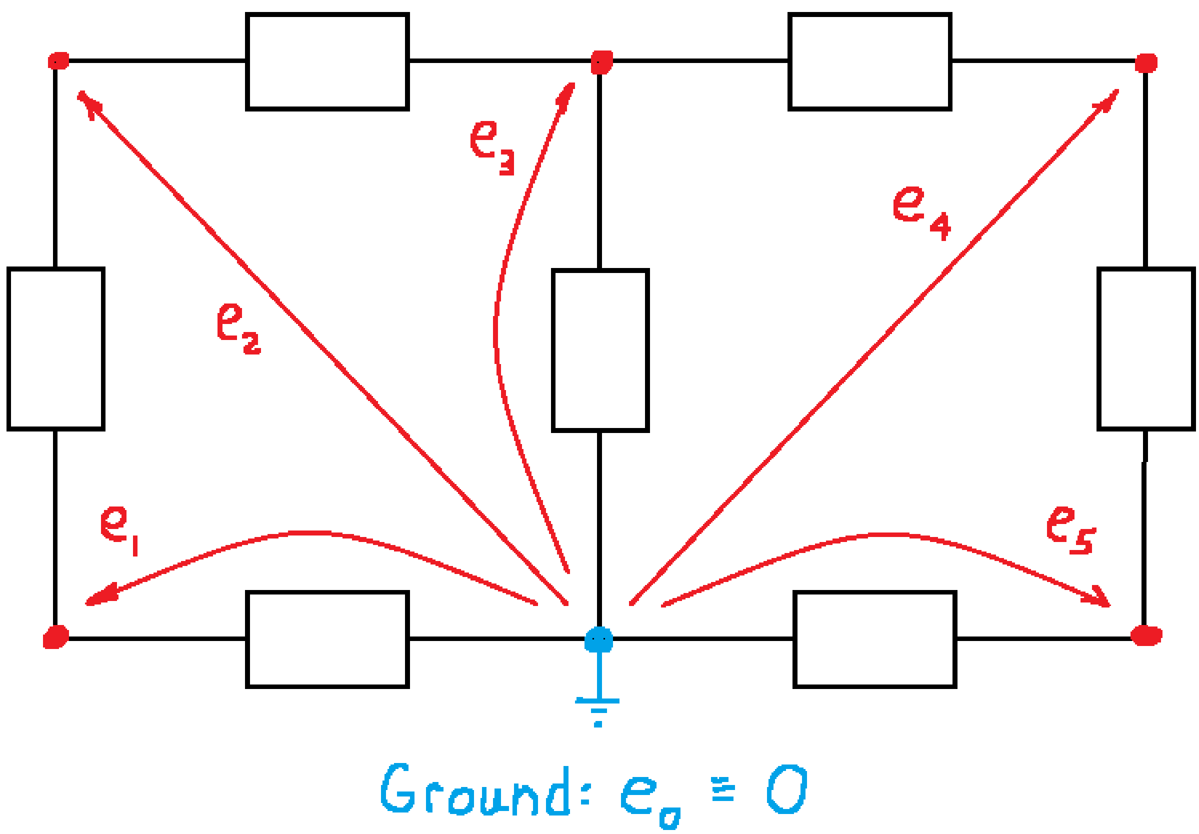
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}$$

Simplification Process



Use voltage and current dividers, and device laws to determine remaining branch variables.

Node Voltages



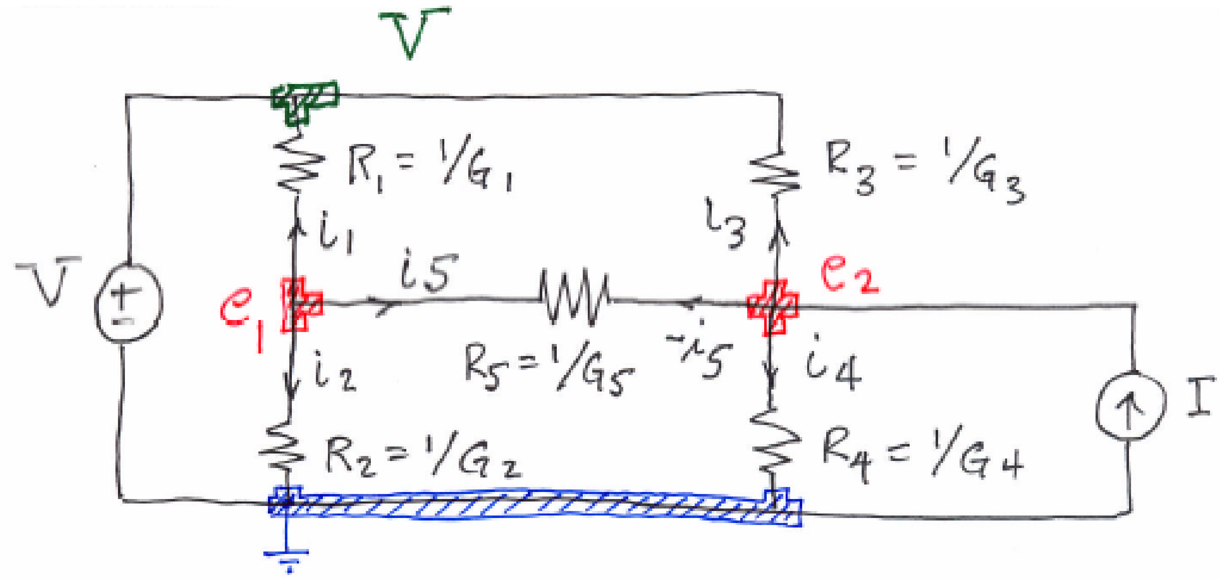
A node voltage *e* is defined as the potential difference between the corresponding node (+) and the ground (-). The ground is assigned an absolute potential of zero. Therefore the node voltage is also the absolute potential of the corresponding node.

(Simplified) Node Analysis

The following assumes an absence of floating voltage sources.

1) Draw the circuit neatly.
2) Select a reference node from which all other node voltages are measured. Define its voltage to be zero. This node is the “ground” node.
3) Label all voltage-sourced node voltages with their sourced voltage.
4) Label all remaining un-sourced nodes with their unknown node voltages. These are the primary analytic unknowns.
5) Write KCL for each node with an unknown node voltage, and immediately back substitute the device laws and KVL. The resulting equations are now in terms of the node voltages.
6) Solve the KCL equations for the unknown node voltages.
7) Back solve for all branch voltages and then branch currents as desired.

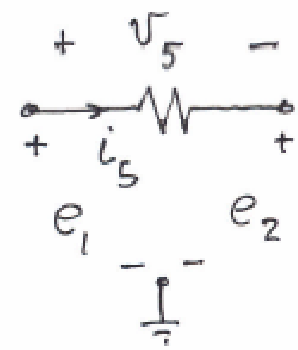
Node Analysis Example



e_1 Node: $i_1 + i_2 + i_5 = 0$

$$G_1(e_1 - V) + G_2(e_1 - 0) + G_5(e_1 - e_2) = 0$$

$$e_1(G_1 + G_2 + G_5) + e_2(-G_5) = V(G_1)$$



$$e_1 = v_5 + e_2$$

$$v_5 = e_1 - e_2$$

e_2 Node: $i_3 + i_4 + (-i_5) - I = 0$

$$G_3(e_2 - V) + G_4(e_2 - 0) + G_5(e_2 - e_1) - I = 0$$

$$e_1(-G_5) + e_2(G_3 + G_4 + G_5) = V(G_3) + I$$

Solution

$$\begin{bmatrix} G_1+G_2+G_5 & -G_5 \\ -G_5 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix}$$

$$\frac{\begin{bmatrix} G_3+G_4+G_5 & G_5 \\ G_5 & G_1+G_2+G_5 \end{bmatrix}}{(G_1+G_2+G_5)(G_3+G_4+G_5) - G_5^2} \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

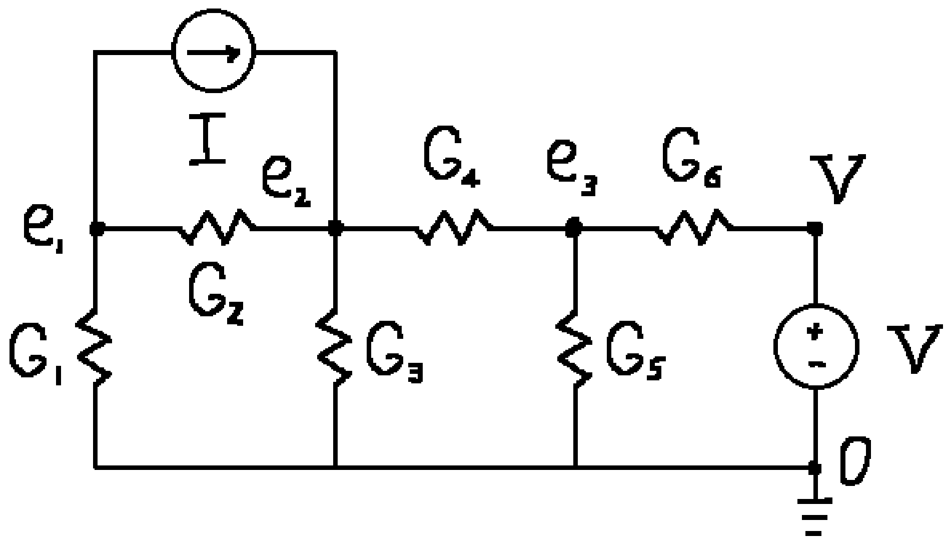
$$e_1 = \frac{(G_3+G_4+G_5)G_1 V + G_5(G_3 V + I)}{G_1G_3 + G_1G_4 + G_1G_5 + G_2G_3 + G_2G_4 + G_2G_5 + G_5G_3 + G_5G_4}$$

$$e_2 = \frac{G_5 G_1 V + (G_1+G_2+G_5)(G_3 V + I)}{(\text{Same Denominator})}$$

Demo

$$\left.\begin{aligned} R_1 &= R_4 = 8.2\,k\Omega \\ R_2 &= R_3 = 3.9\,k\Omega \\ R_5 &= 1.5\,k\Omega \\ V &= 3\,V \\ I &= 0\,A \end{aligned}\right\} \quad \begin{aligned} e_1 &= 1.382\,V \\ e_2 &= 1.618\,V \end{aligned}$$

Node Analysis Structure



Omit ground node potential (known to be zero) and ground-node KCL (redundant). The patterns can be used to check work.

$$e_1 \text{ KCL}: G_1(e_1 - 0) + G_2(e_1 - e_2) + I = 0$$

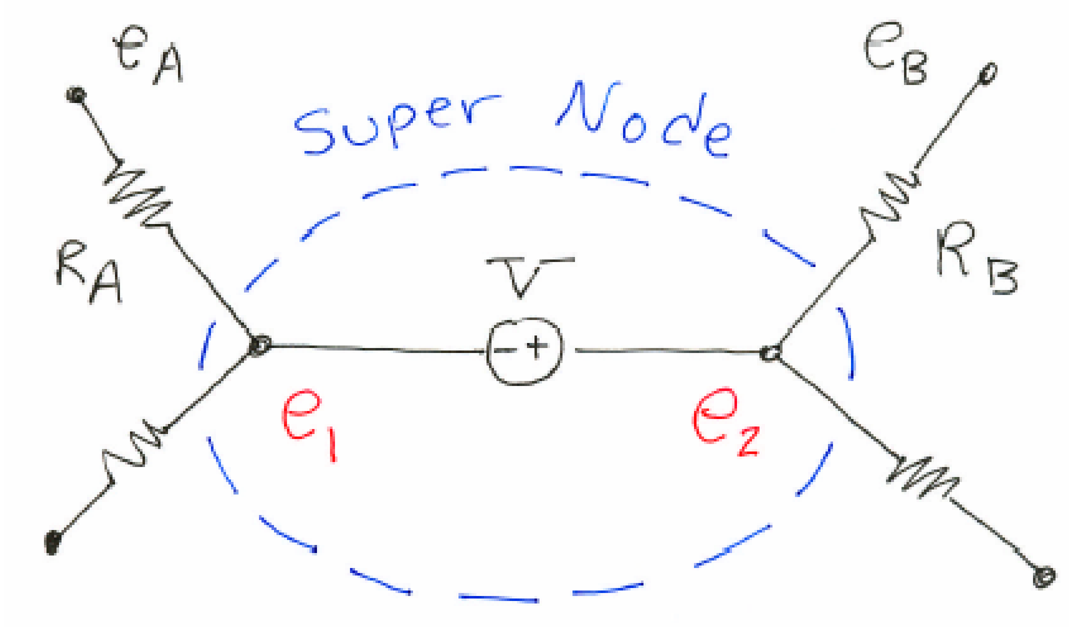
$$e_2 \text{ KCL}: G_3(e_2 - 0) + G_2(e_2 - e_1) + G_4(e_2 - e_3) - I = 0$$

$$e_3 \text{ KCL}: G_5(e_3 - 0) + G_4(e_3 - e_2) + G_6(e_3 - V) = 0$$

$$\begin{bmatrix} \boxed{G_1} + G_2 & -G_2 & 0 \\ -G_2 & G_2 + \boxed{G_3} + G_4 & -G_4 \\ 0 & -G_4 & G_4 + \boxed{G_5} + \boxed{G_6} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \\ 0 \end{bmatrix} I + \begin{bmatrix} 0 \\ 0 \\ \boxed{G_6} \end{bmatrix} V$$

Super Nodes

Super nodes treat floating voltage sources.



$$e_2 = e_1 + V$$

Write only one KCL for each super node.

$$\frac{e_1 - e_A}{R_A} + \frac{\overbrace{e_1 + V}^{e_2} - e_B}{R_B} + \cdots = 0$$