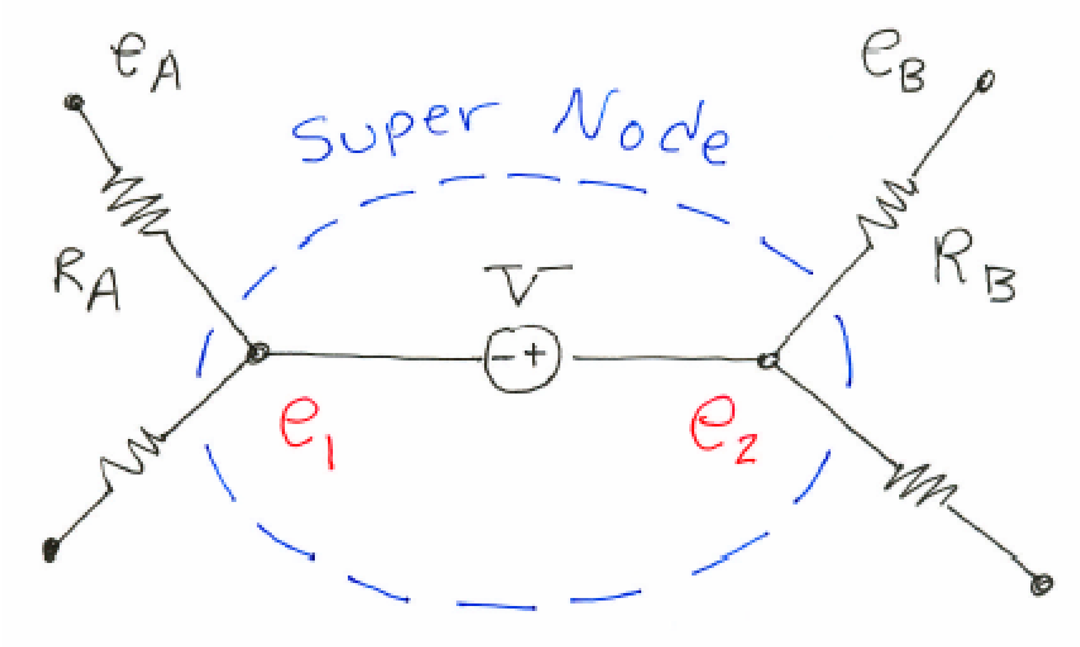
6.002 - Lecture 04

Circuit Analysis Simplifications

- (Floating Voltage Sources)
- Linearity
- Superposition

Super Nodes

Super nodes treat floating voltage sources.

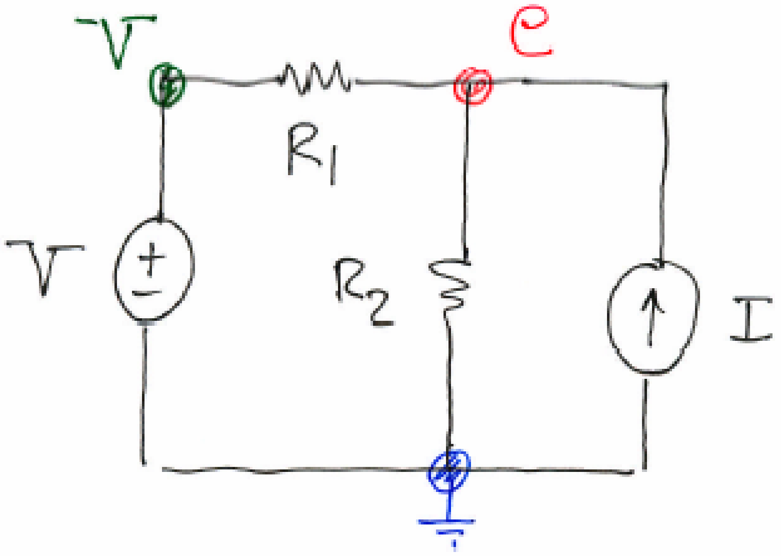


$$e_2 = e_1 + V$$

Write only one KCL for each super node.

$$\frac{e_1 - e_A}{R_A} + \frac{\overbrace{e_1 + V}^{e_2} - e_B}{R_B} + \cdots = 0$$

Simple Nodal Analysis

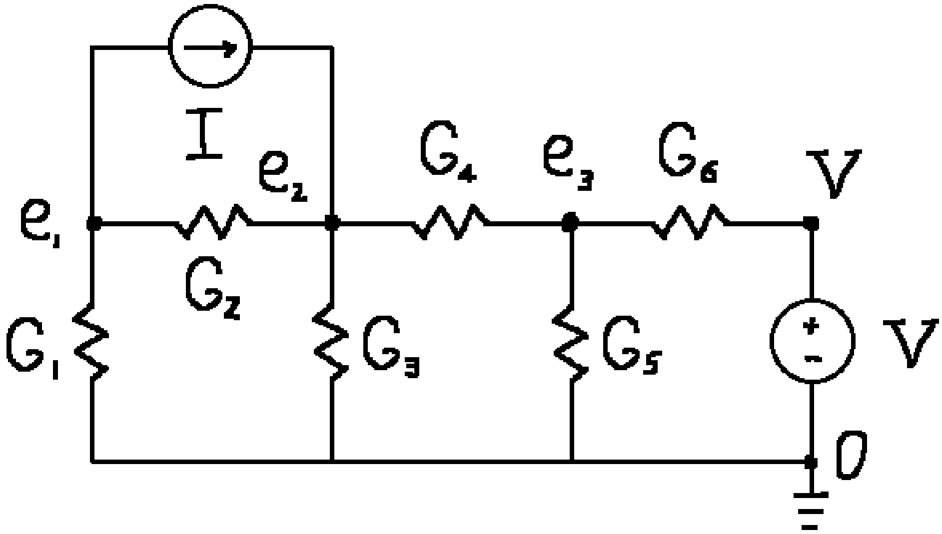


$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

$$e\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{V}{R_1}+I \quad \cdots\to \quad \overbrace{\overset{\substack{\text{Conductance}\\ \text{Matrix}\\ \downarrow}}{G}\,\underset{\substack{\uparrow\\ \text{Node}\\ \text{Voltage}\\ \text{Vecter}}}{\bar{e}} = \overset{\substack{\text{Coefficient}\\ \text{Matrix}\\ \downarrow}}{H}\,\underset{\substack{\uparrow\\ \text{Source}\\ \text{Vector}}}{\bar{i}}}^{\text{Linear in } \bar{e} \text{ and } \bar{i}.}$$

$$e=\frac{R_2}{R_1+R_2}V+\frac{R_1R_2}{R_1+R_2}I$$

Another Nodal Analysis



e_1 KCL: $G_1(e_1-0)+G_2(e_1-e_2)+I=0$

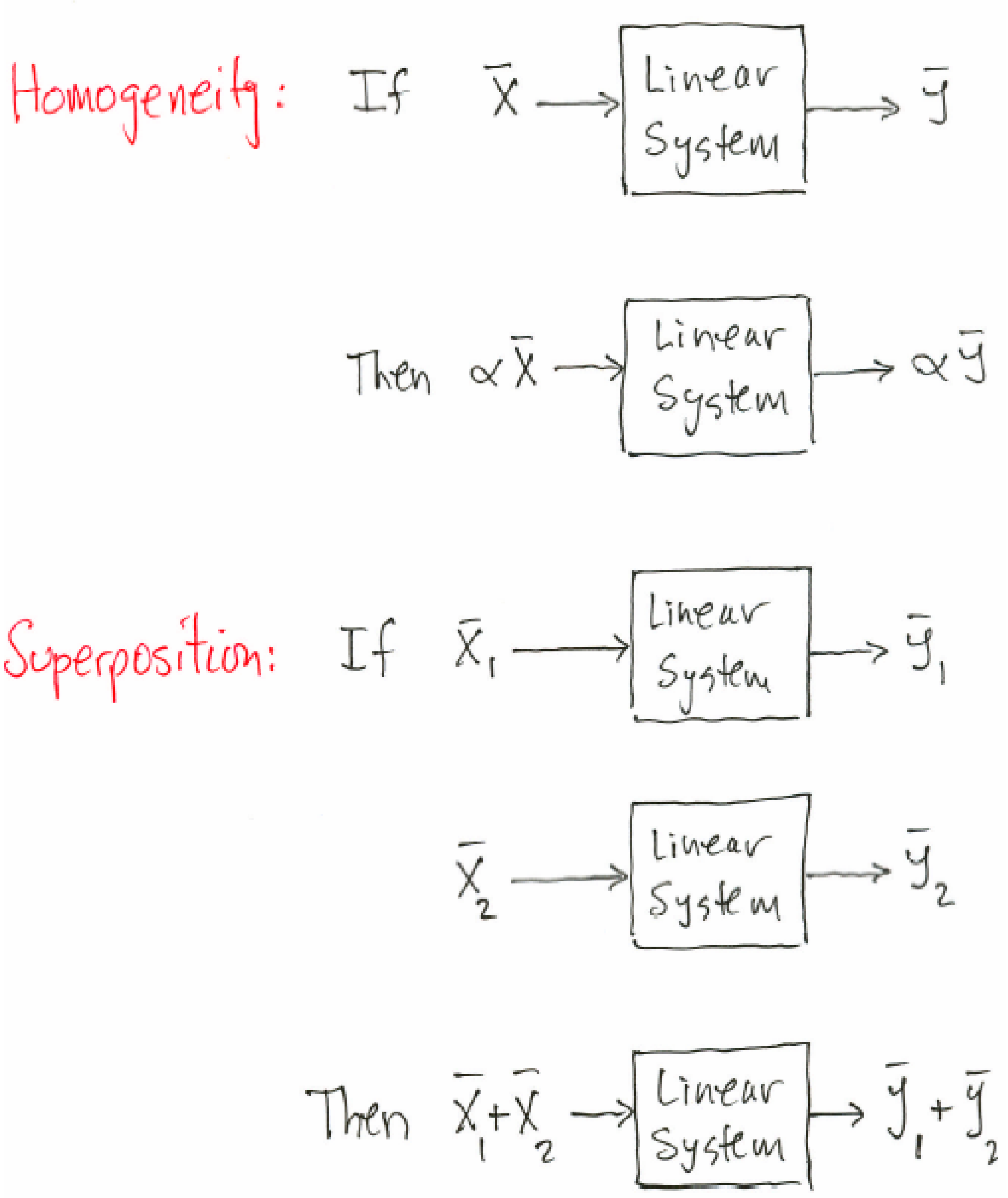
e_2 KCL: $G_3(e_2-0)+G_2(e_2-e_1)+G_4(e_2-e_3)-I=0$

e_3 KCL: $G_5(e_3-0)+G_4(e_3-e_2)+G_6(e_3-V)=0$

$$\begin{bmatrix} \boxed{G_1}\boxed{+G_2} & \boxed{-G_2} & 0 \\ \boxed{-G_2} & \boxed{G_2}\boxed{+G_3}\boxed{+G_4} & \boxed{-G_4} \\ 0 & \boxed{-G_4} & \boxed{G_4}\boxed{+G_5}\boxed{+G_6} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \boxed{-1} \\ \boxed{+1} \\ 0 \end{bmatrix} I + \begin{bmatrix} 0 \\ 0 \\ \boxed{G_6} \end{bmatrix} V$$

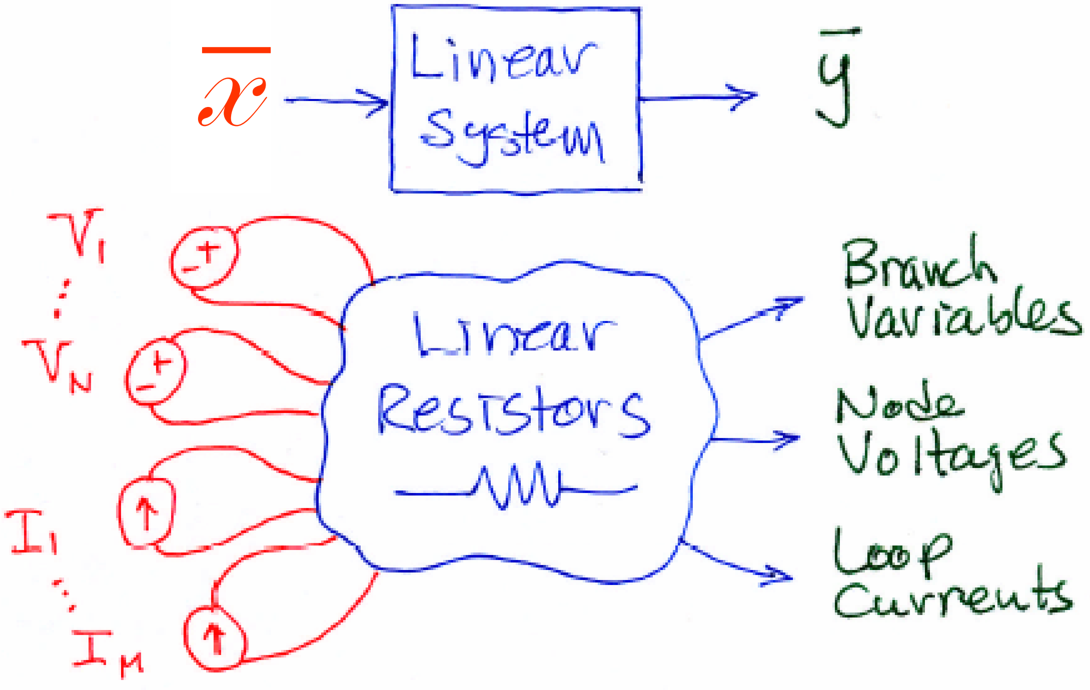
Homogeneity & Superposition

Homogeneity and superposition
are properties of linear systems.



Linear Networks and Systems

Linear system ⇒ all stimuli are external inputs. For example, $y = x_1 + x_2$ is linear from $\bar{x} \to y$, but $y = x_1 + x_2 + 1$ is not.

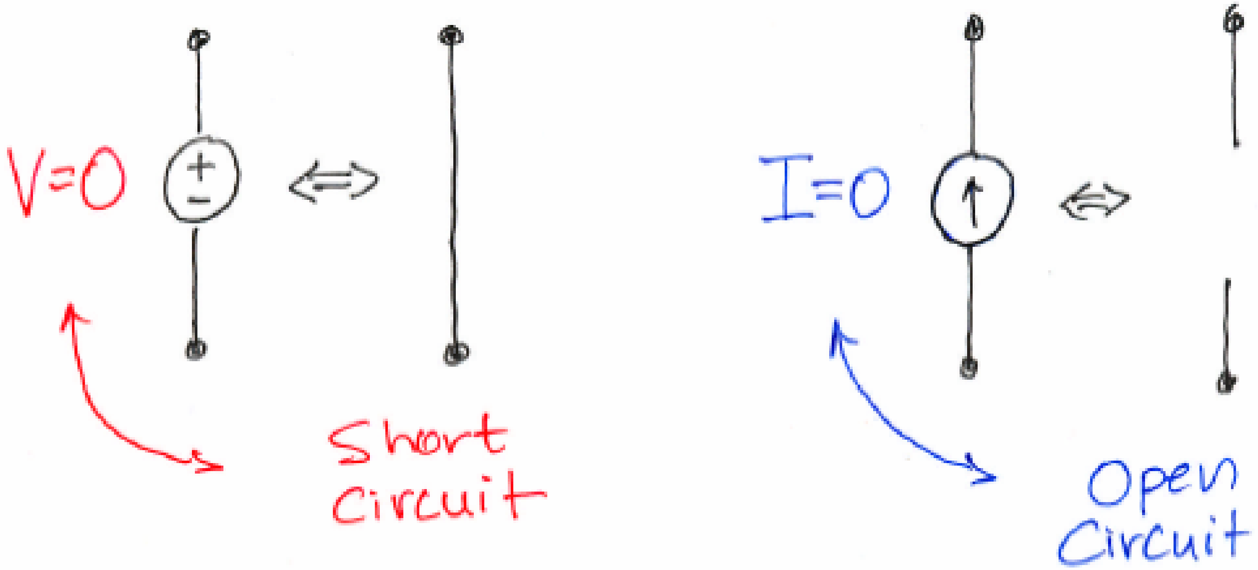


No source should be left within the network for the purposes of superposition. Superposition is carried out over all sources.

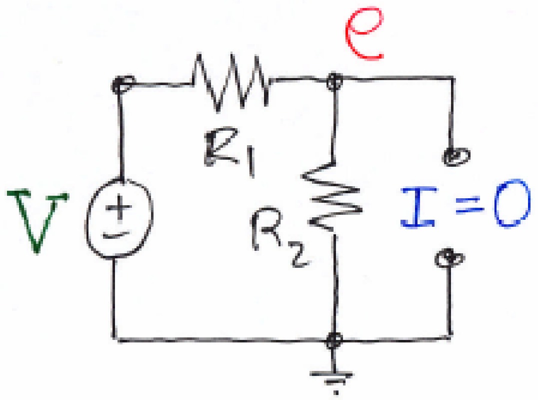
Divide & Conquer

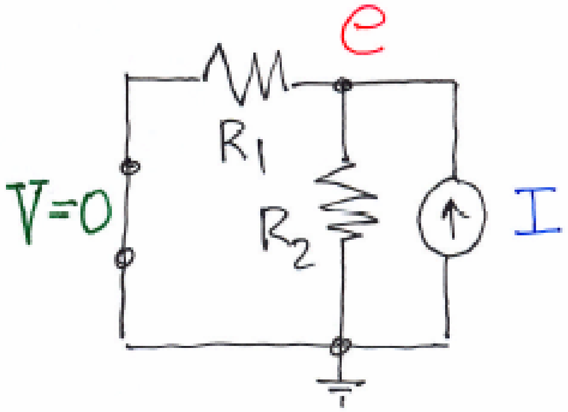
Use superposition to analyze a linear circuit one source at a time. (All sources except one are set to zero each time.)

How to set a source to zero?

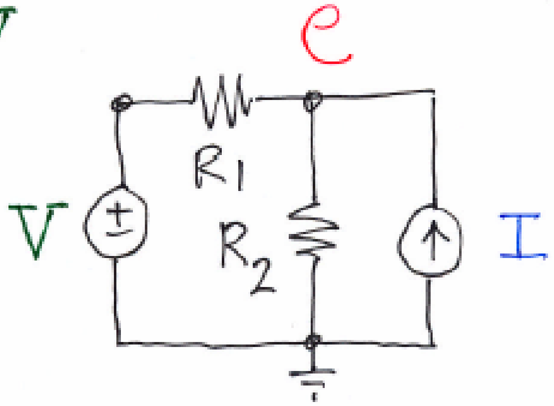


Superposition Examples



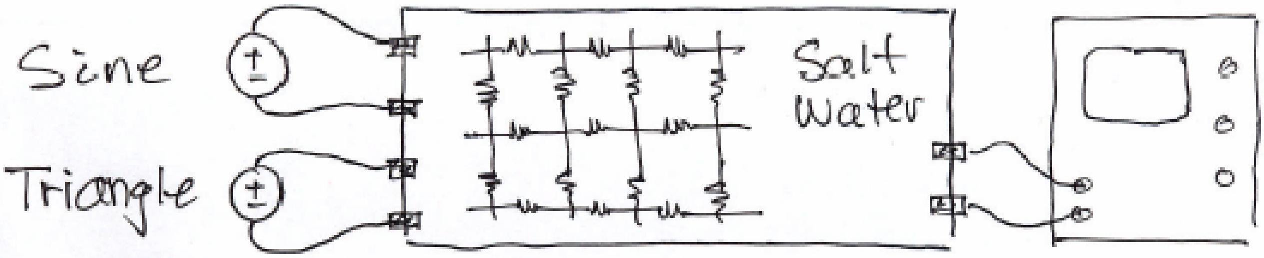


$$e = \frac{R_2}{R_1+R_2} V$$



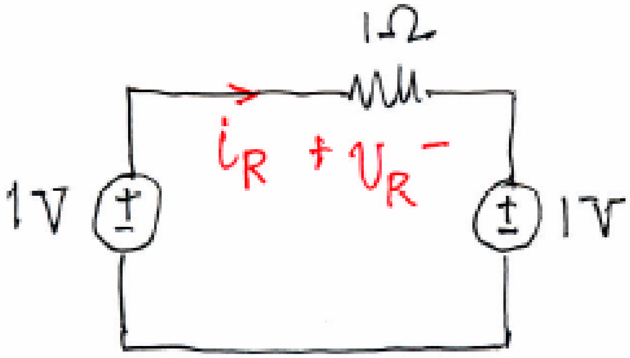
$$e = \frac{R_1 R_2}{R_1+R_2} I$$

$$e = \frac{R_2}{R_1+R_2} V + \frac{R_1 R_2}{R_1+R_2} I$$



No Superposition For Power

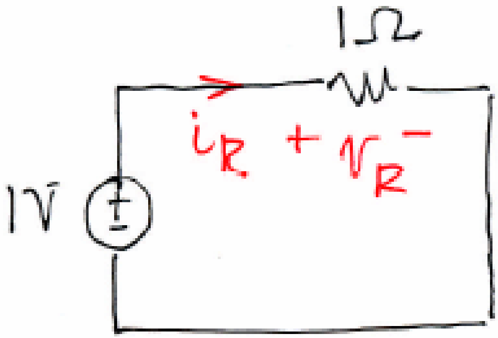
Power is a nonlinear network property, and so the superposition of power is not possible.



$v_R = 0$ V

$i_R = 0$ A

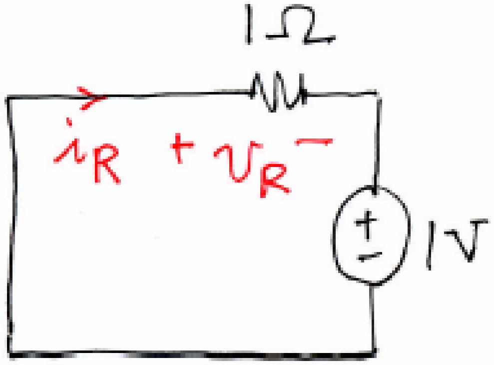
$v_R i_R = 0$ W



$v_R = 1$ V

$i_R = 1$ A

$v_R i_R = 1$ W

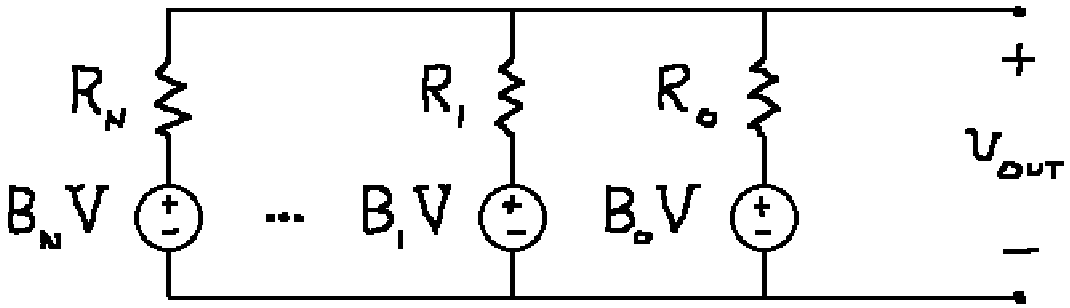


$v_R = -1$ V

$i_R = -1$ A

$v_R i_R = 1$ W

DAC Example



DAC ⇒ Digital Data = $\boxed{B_N \mid \cdots \mid B_1 \mid B_0}$

$$v_{OUT} = KV \sum_n B_n 2^n \; ; \; B_n = \{0, 1\}$$

Use superposition and voltage division:

$$v_{OUT,m} = \frac{\left(\sum_{n \neq m} G_n\right)^{-1} B_m V}{R_m + \left(\sum_{n \neq m} G_n\right)^{-1}} = \frac{B_m V}{1 + R_m \sum_{n \neq m} G_r}$$

$$= \frac{B_m V}{R_m G_m + R_m \sum_{n \neq m} G_n} = \frac{G_m B_m V}{\sum_n G_n}$$

$v_{OUT} = V\left(\sum_n G_n B_n\right) / \left(\sum_m G_m\right)$ ... by superposition

Design ⇒ $G_n = G^* 2^n \Rightarrow K = 1/\left(\sum_m 2^m\right)$