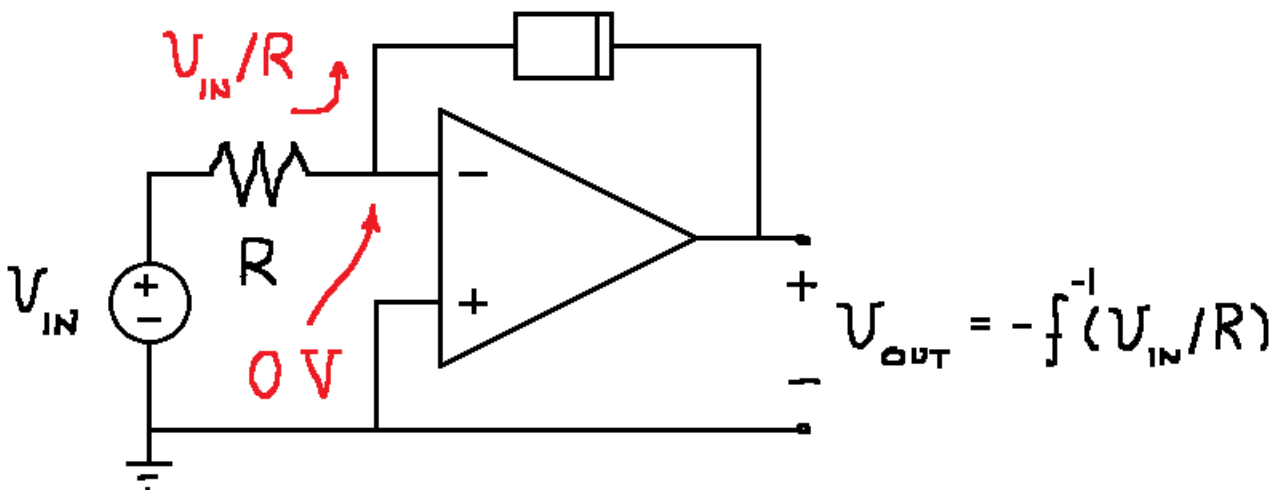
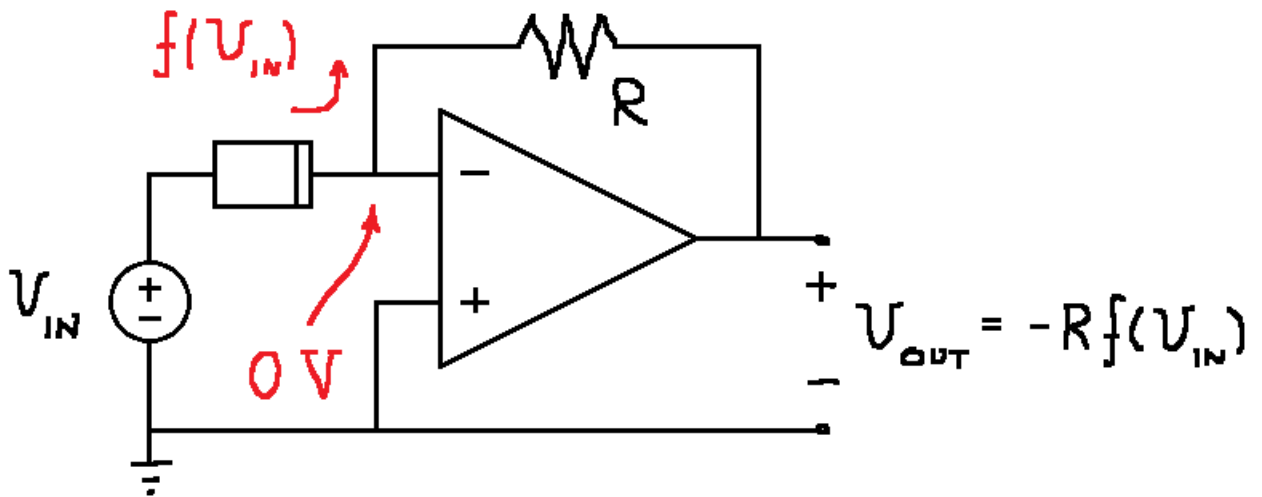
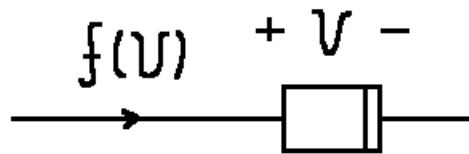


6.002 - Lecture 09

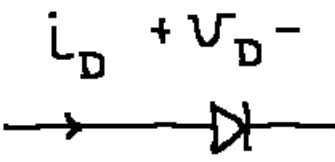
More Op Amp Examples

- Diode (Nonlinear Resistor)
- Log & Exponential Amps
- Multipliers & Dividers
- How An Op Amp Works

Nonlinear Functions



Exponential Diode

Exponential Diode: 

$$i_D = I_s \left[e^{V_D / n V_T} - 1 \right]$$

I_s = Saturation Current $\approx 1 \text{ pA} \dots 10 \text{ nA}$

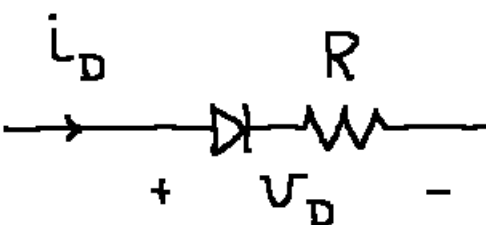
$$V_T = k_B T / q$$

$$k_B = 1.38 \cdot 10^{-23} \text{ J/K}$$

T = Temperature

$$q = 1.6 \cdot 10^{-19} \text{ C}$$

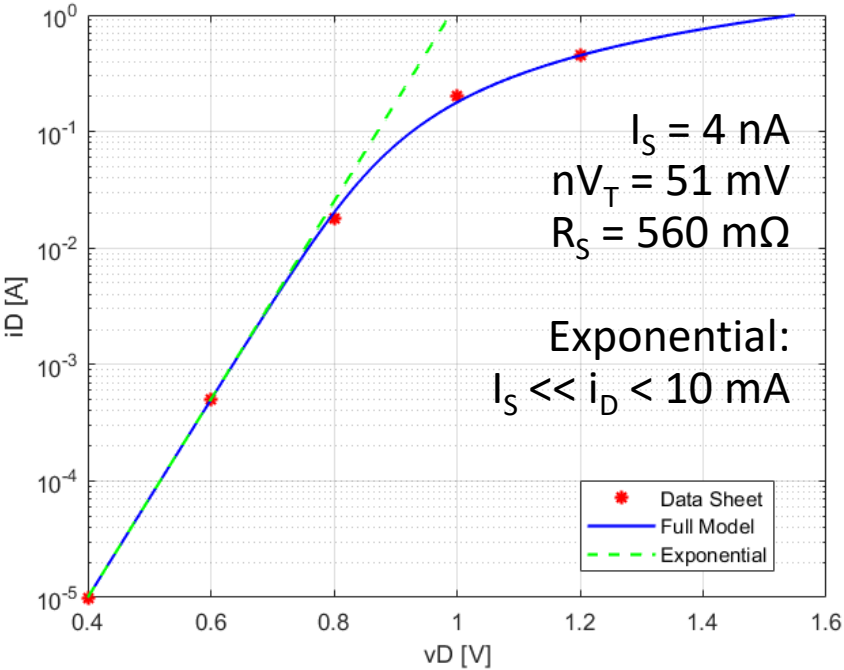
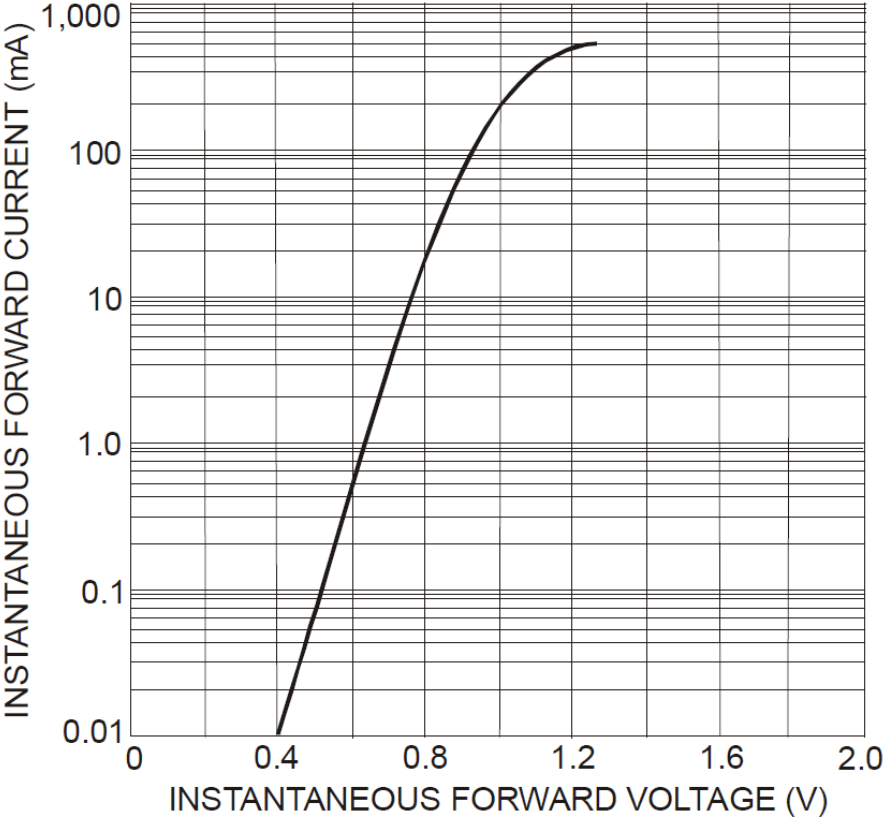
n = Ideality Factor $\approx 1 \dots 2$

Practical Diode: 

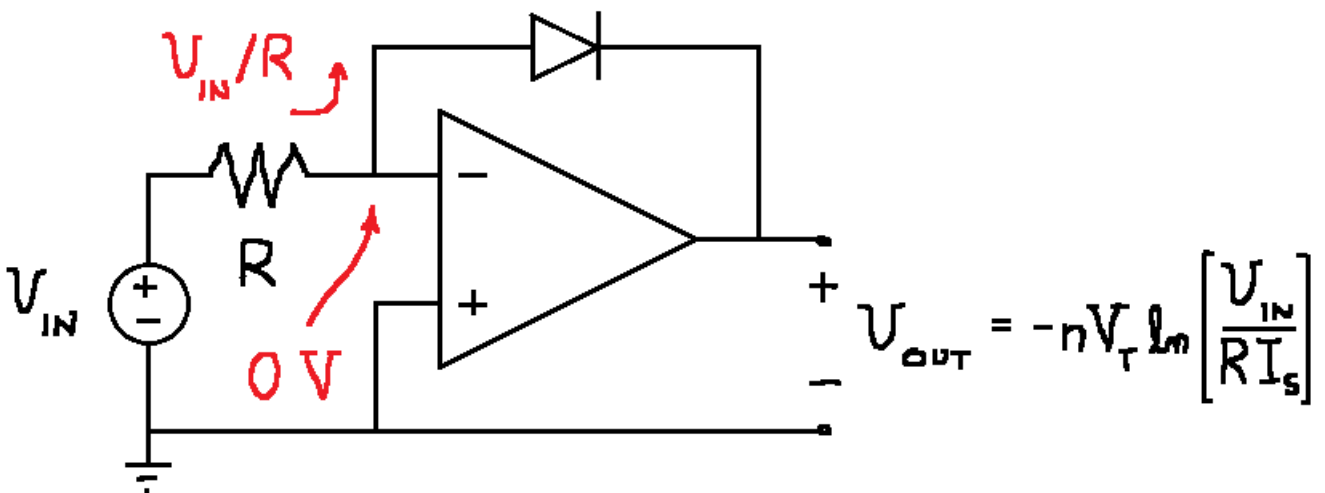
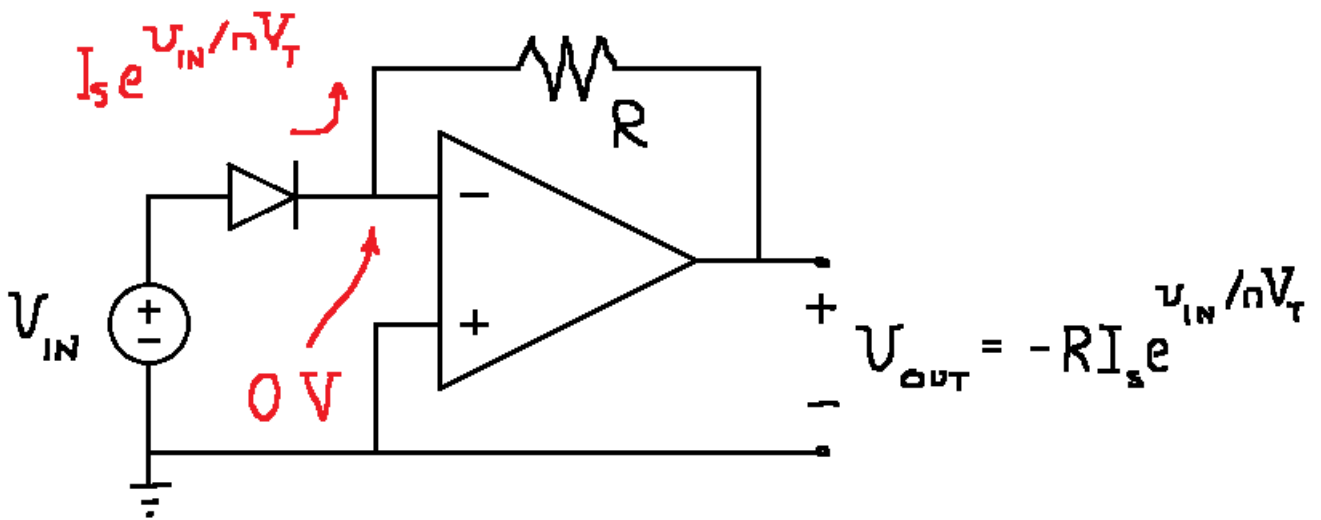
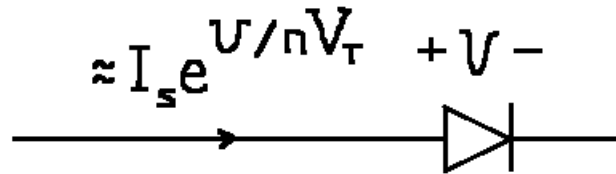
$$V_D = n V_T \ln(i_D / I_s + 1) + R i_D$$

R = Body Resistance $\approx \dots 1 \Omega \dots$

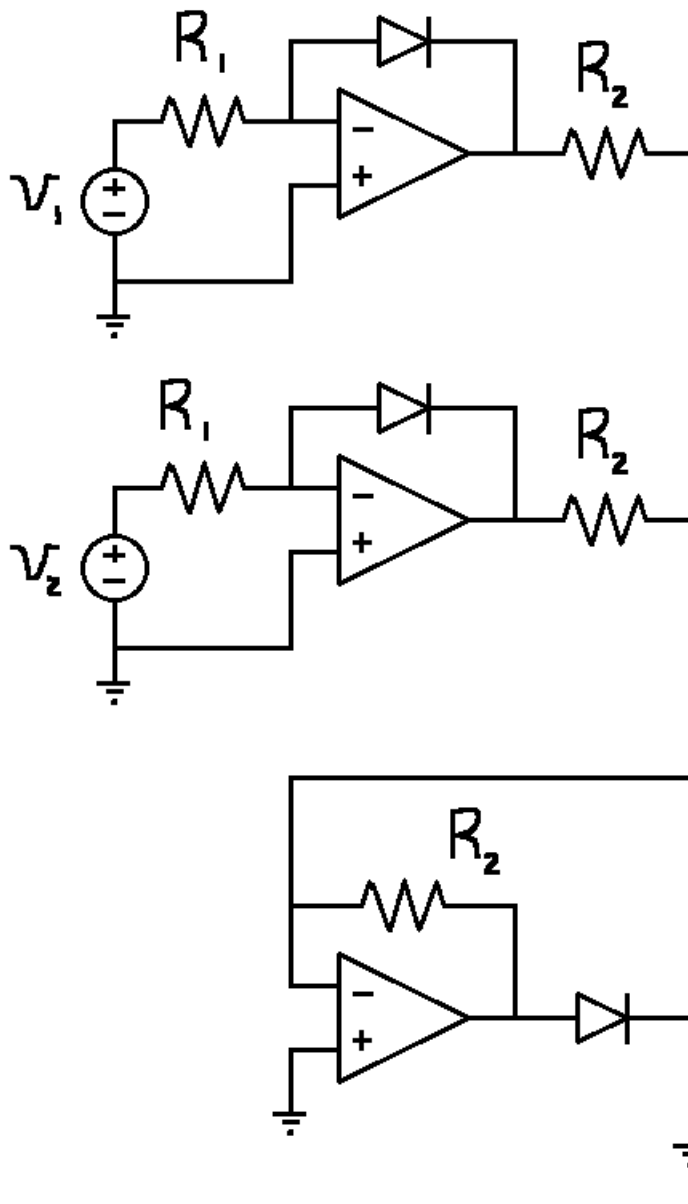
Exponential Diode – 1N4148



Exponential & Log Amps



Multiplier



$$R_1 = 500 \text{ k}\Omega$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_3 = 1 \text{ k}\Omega$$

$$I_s \approx 4 \text{ nA}$$

$$\begin{aligned}
 & - \frac{R_3 V_1 V_2}{R_1^2 I_s} \\
 & \approx - \frac{V_1 V_2}{1 \text{ V}}
 \end{aligned}$$

Multiplier



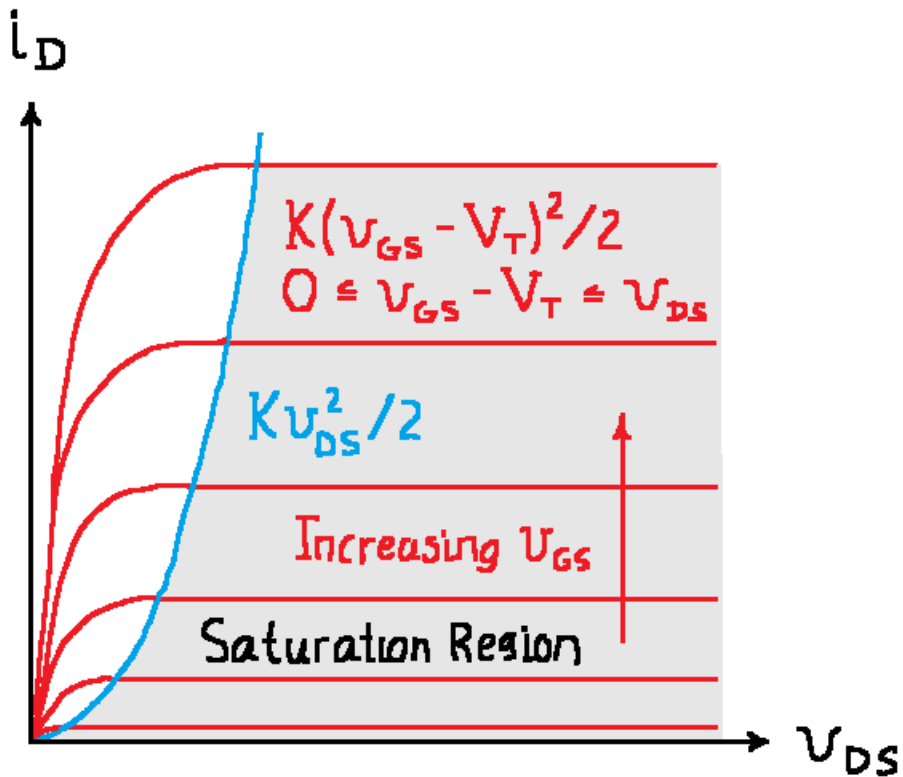
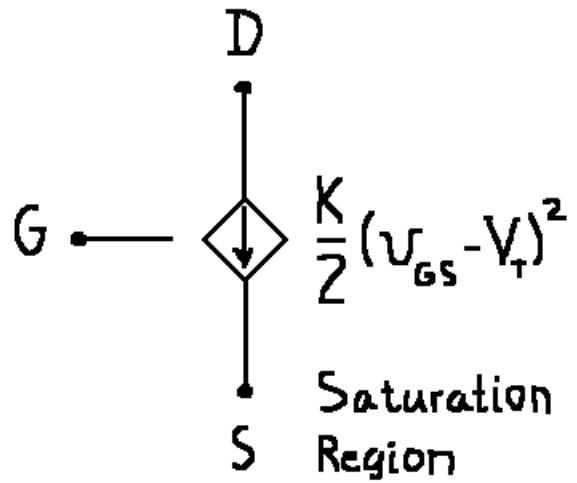
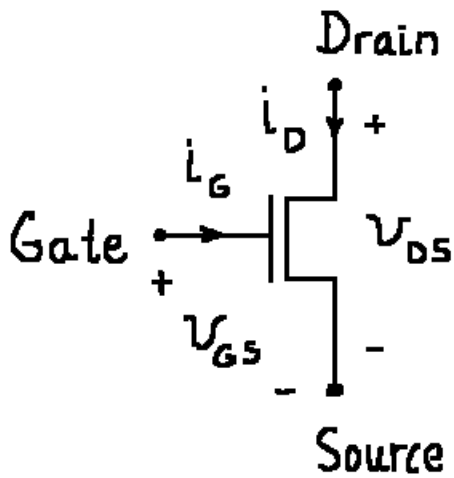
Multiplier performance:

Yellow/Green = Inputs
Blue = Log of product
Pink = Output

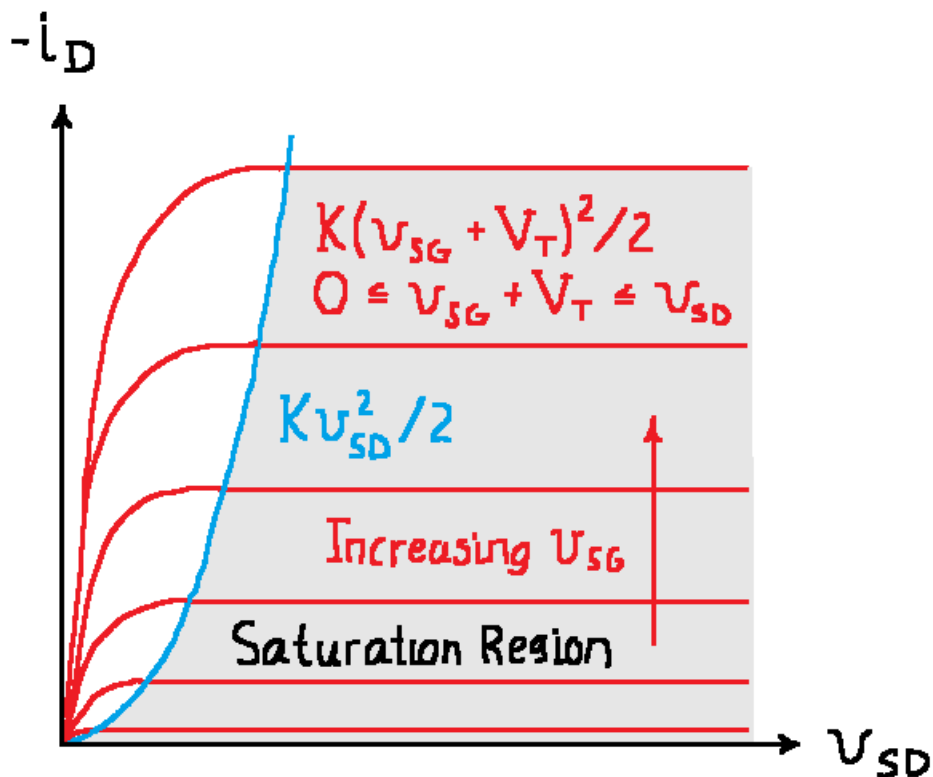
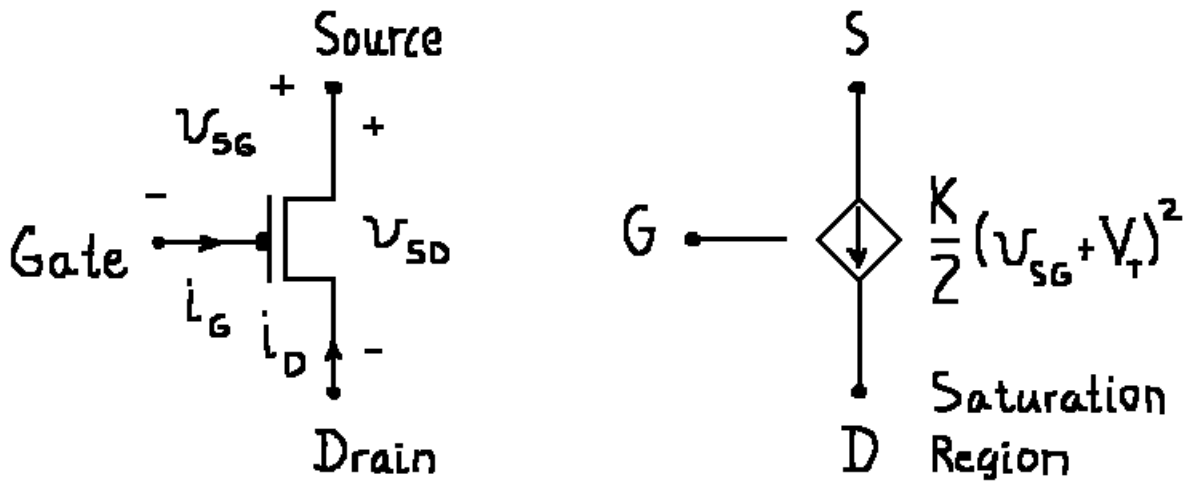
Each input has a 2-V offset and a 2-V peak-to-peak amplitude. The sine input is 100 Hz and the triangle input is 10 Hz.

The input resistors are 500 kOhm, the summer resistors are 10 kOhm and the output resistor is 1 kOhm. The diode is 1N4148.

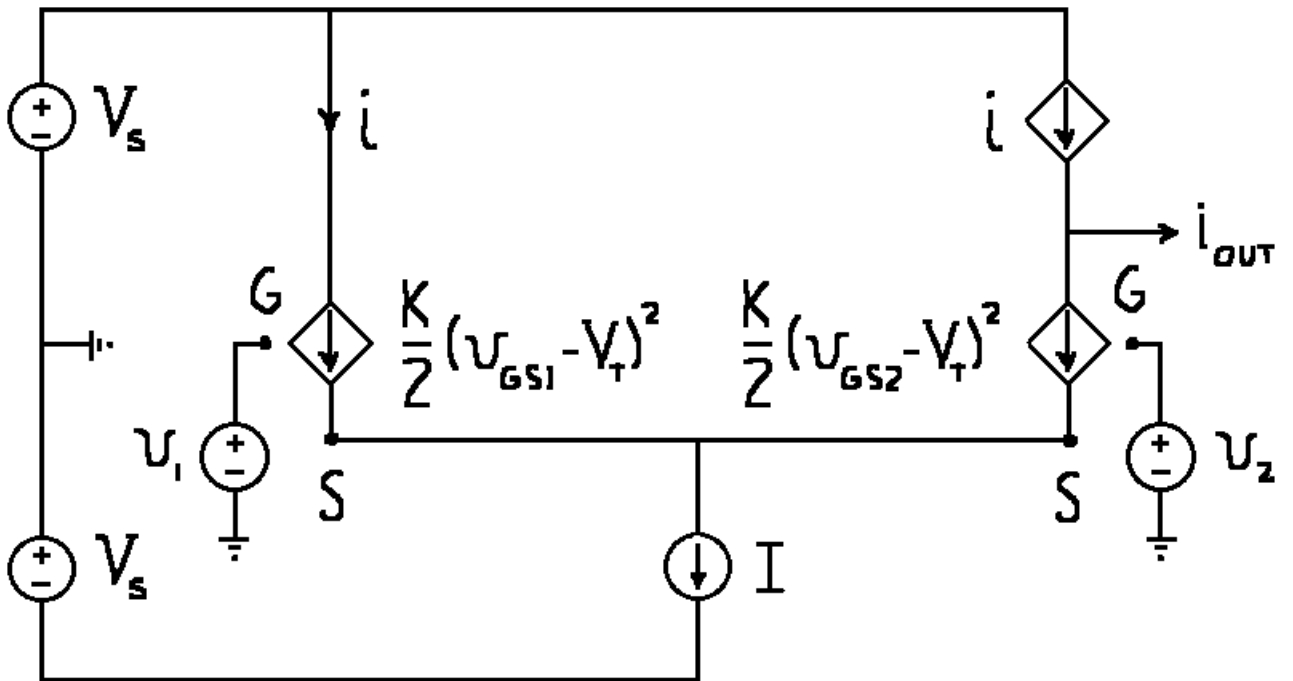
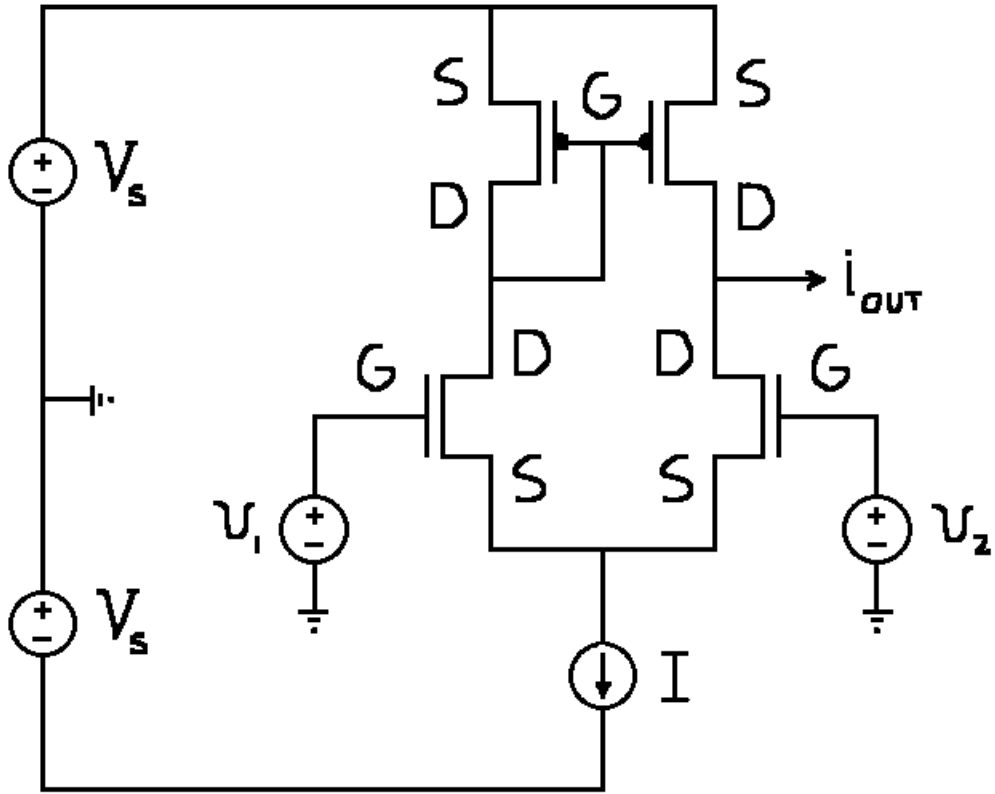
Simplified n-Channel MOSFET



Simplified p-Channel MOSFET



Op-Amp Design



Analysis

$$\text{KCL: } i_{D1} + i_{D2} = I$$

$$\text{KVL: } v_1 - v_{GS1} + v_{GS2} - v_2 = 0$$

$$\text{MOSFET: } i_{D1} = K(v_{GS1} - V_T)^2/2 \rightarrow v_{GS1} - V_T = \sqrt{2i_{D1}/K}$$

$$i_{D2} = K(v_{GS2} - V_T)^2/2 \rightarrow v_{GS2} - V_T = \sqrt{2i_{D2}/K}$$

Define: $\bar{v} = v_1 - v_2$... Differential Input

$$\text{Equilibrium: } \bar{v} = 0$$

$$v_{GS1} = v_{GS2}$$

$$v_{GS1} - V_T = v_{GS2} - V_T = \sqrt{2I/K}$$

$$i_{D1} = i_{D2} = I/2$$

$$i_{OUT} = 0$$

$$\text{Variation: } \delta i_{D1} = \sqrt{2IK} \delta v_{GS1}$$

$$\delta i_{D2} = \sqrt{2IK} \delta v_{GS2}$$

$$\delta i_{OUT} = \sqrt{2IK} (\delta v_{GS1} - \delta v_{GS2})$$

$$= \sqrt{2IK} (\delta v_1 - \delta v_2) = \sqrt{2IK} \delta \bar{v}$$