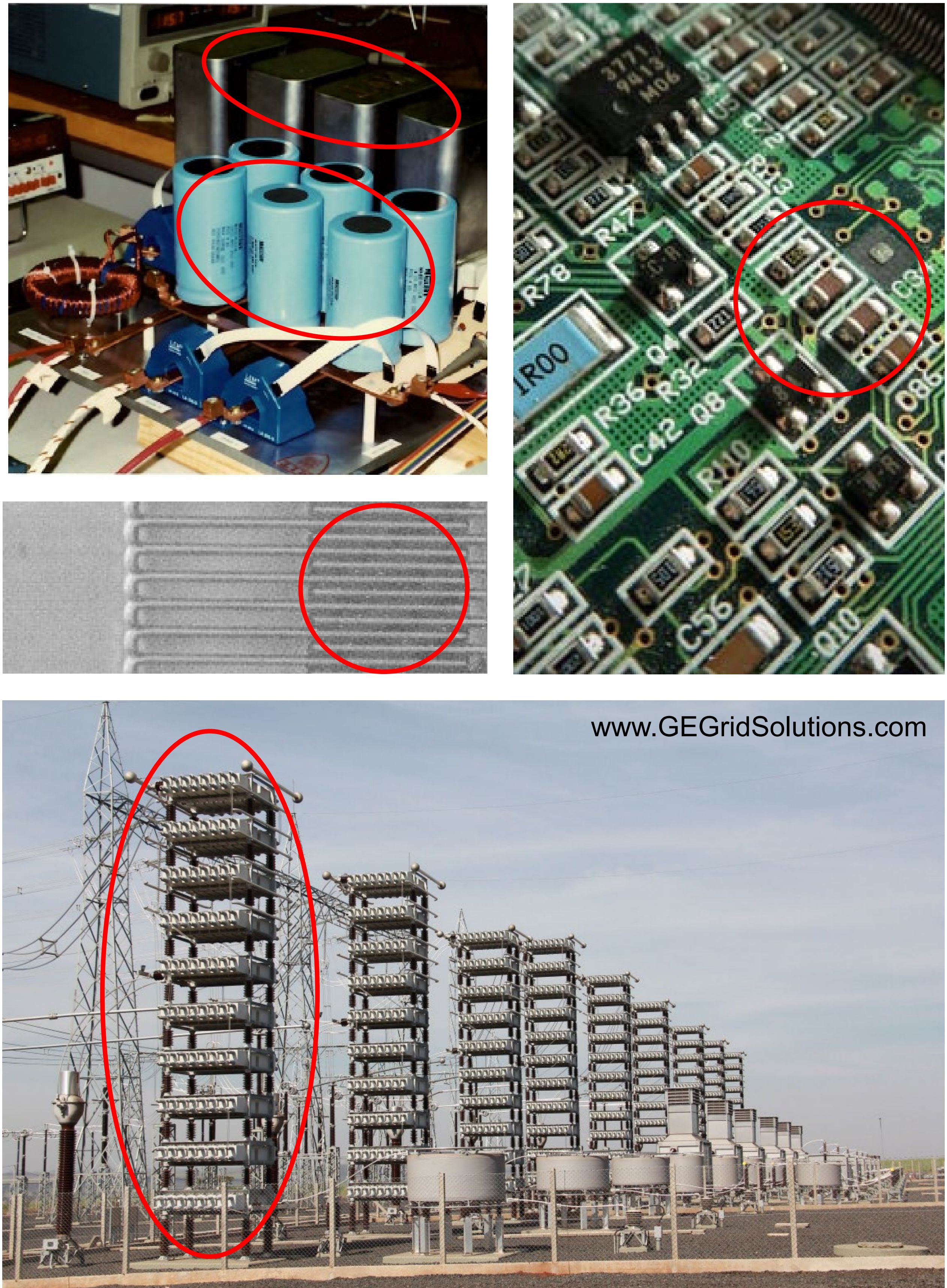
6.002 - Lecture 10

Capacitors

- Capacitors are Everywhere
- Uses
- Basic Physics
- First-Order ODEs


R78
R47
Q4
R36
R32
C42
Q8
C56
Q10
www.GEGridSolutions.com

Breadboard Capacitance

https://learn.sparkfun.com/
tutorials/how-to-use-a-breadboard

https://wiki.analog.com/university/courses/electronics
/electronics-lab-breadboard-coupling

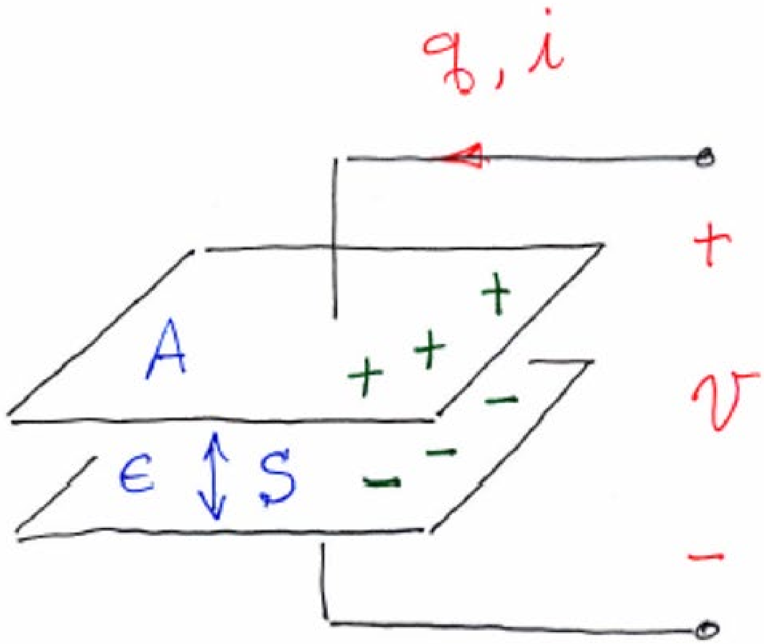
Uses & Memory

Capacitor Uses:
- Energy Storage
- Memory
- Time-Domain Dynamics
- Frequency-Domain Filtering
- Timing
- Resonators
- Sensing & Energy Transduction

Memory Devices
(Excluding Thermal Memory)
- Capacitors
- Inductors

Memoryless Devices:
(Absent Thermal Memory)
- Sources
- Resistors
- Ideal Diodes
- Ideal Transistors
- Ideal Transformers

Ideal Capacitor



Farads

$$q = C\,v$$

Coulombs Volts

$$C = \frac{\epsilon A}{S}$$

$$\underbrace{q = \int_{-\infty}^{t} i\,dt \qquad i = \frac{dq}{dt}}_{\text{Memory}} = \frac{d}{dt}(Cv) = C\frac{dv}{dt}$$

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$$P = vi = C\underbrace{v\frac{dv}{dt}}_{} = \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}Cv^2}_{}\Big)$$

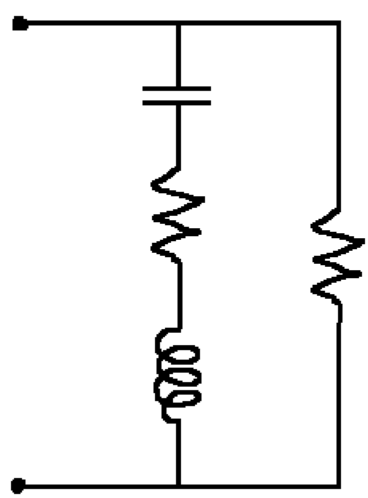
Two signs Possible
⇒ Reversible

Stored Energy

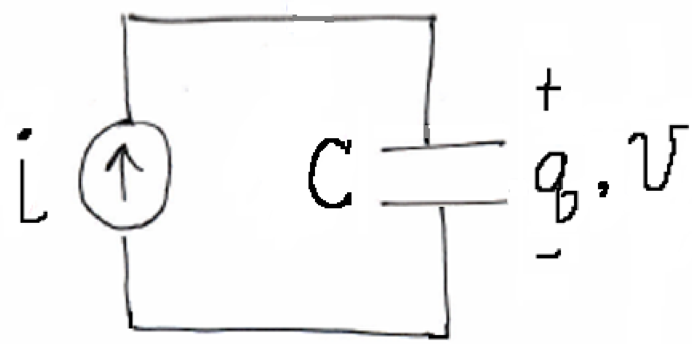
Common Nonidealities

- Electrode series resistance
- Electrode series inductance
- Parallel dielectric conductance

A lumped-parameter model can often represent these non ideal behaviors.



Dynamics & Memory



$$q = Cv$$

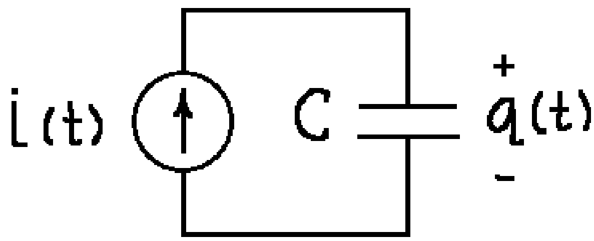
$$i = dq/dt$$

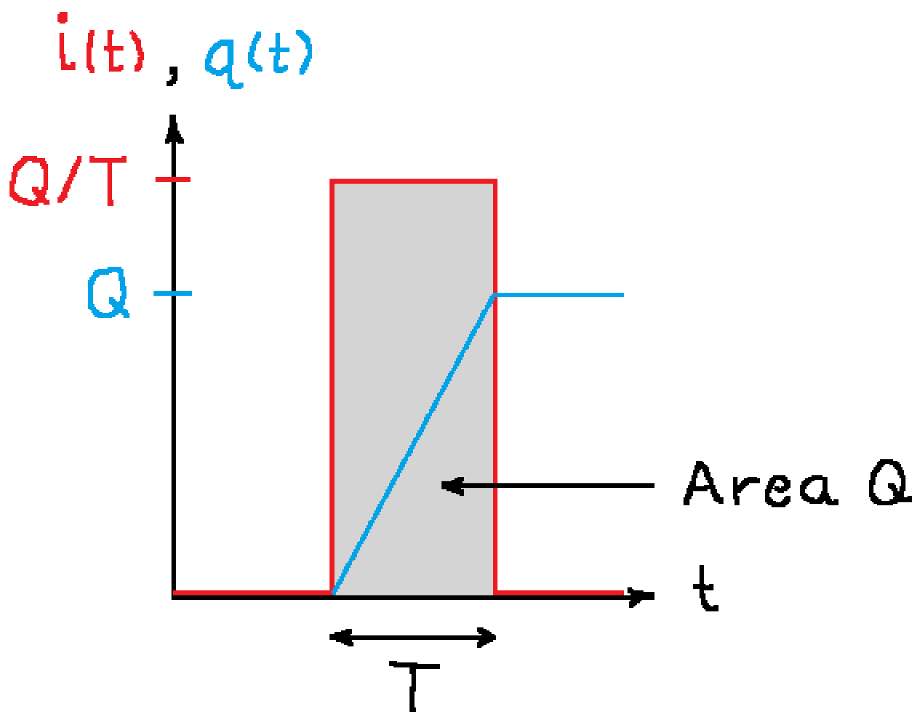
$$q(t) = \int_{-\infty}^{t} i(t)\,dt = \int_{-\infty}^{t_0} i(t)\,dt + \int_{t_0}^{t} i(t)\,dt$$

$$= \underbrace{q(t_0)}_{\text{Memory}} + \int_{t_0}^{t} i(t)\,dt$$

- Charge (and voltage) depends on the entire current history.
- Charge summarizes (memorizes) the current history relevant to the future.

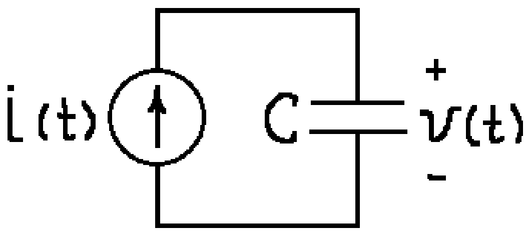
State Continuity

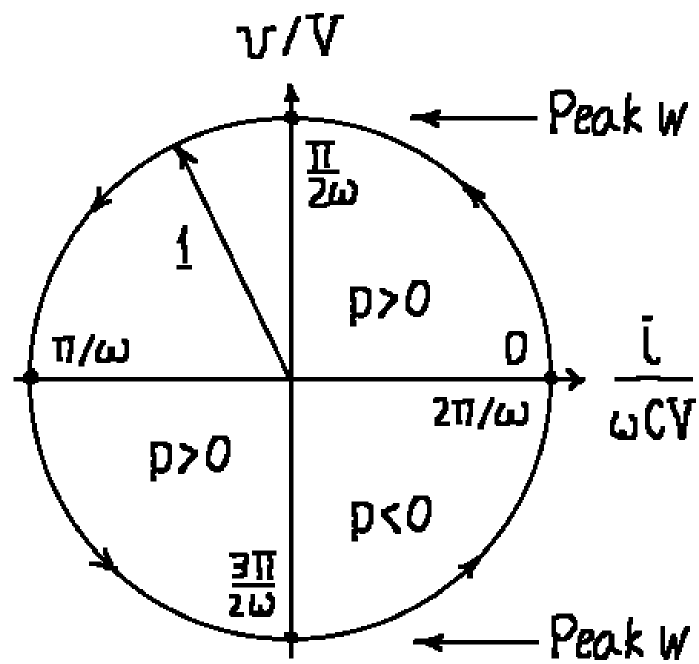




- Lim T → 0 : q(t) → step and i(t) → ∞
- Without infinite current, q(t) cannot step and so is continuous

Power & Energy



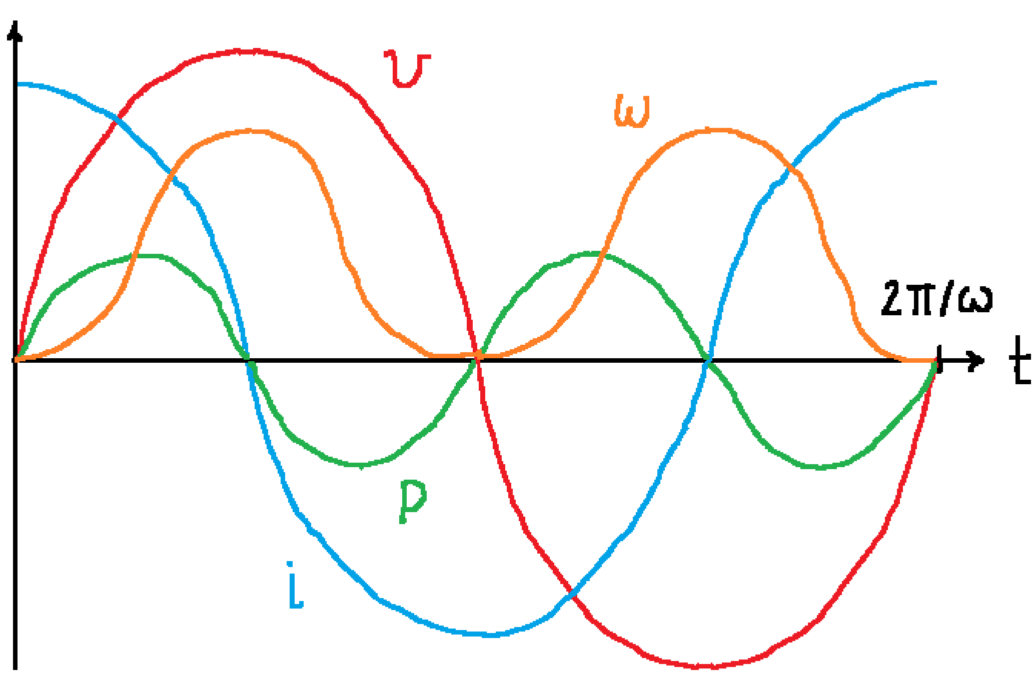


$$v = V\sin(\omega t)$$

$$i = \omega C V\cos(\omega t)$$

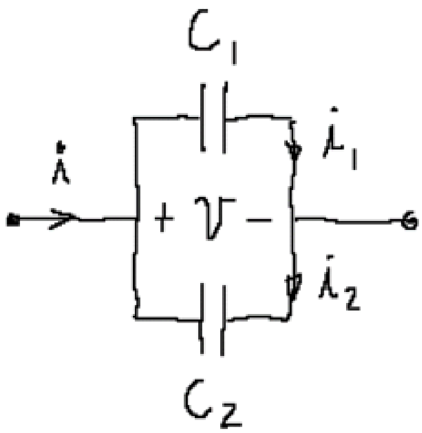
$$p = \omega C V^2 \sin(2\omega t)/2$$

$$w = C V^2 \sin^2(\omega t)/2$$



Capacitor Combinations

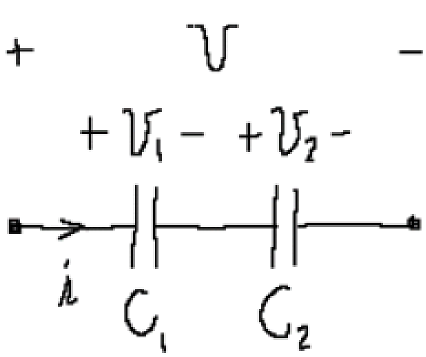
Parallel



$$i = i_1 + i_2$$
$$= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$
$$= \underbrace{(C_1 + C_2)}\frac{dv}{dt}$$

Effective capacitance ⇒
parallel capacitances add

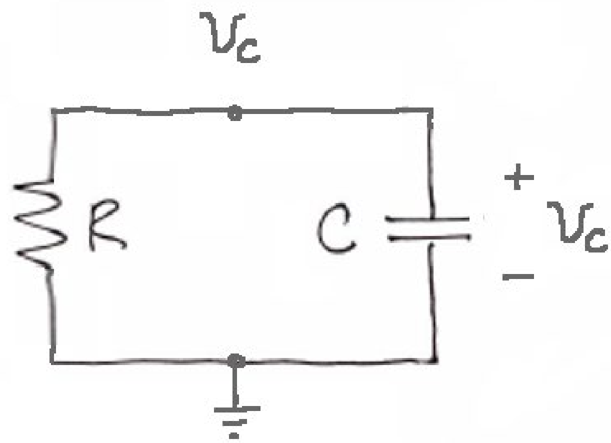
Series



$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$$
$$= \frac{1}{C_1} i + \frac{1}{C_2} i$$
$$= \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)} i$$

Effective reciprocal capacitance
⇒ series reciprocal capacitances add

RC Node Analysis



Node Method $\Rightarrow$ $$\frac{v_C}{R} + C\frac{dv_C}{dt} = 0$$

$$\underbrace{RC}_{\text{Time}}\frac{dv_C}{dt} + v_C = 0$$

Well-Posed Problem $\Rightarrow$ $v_c(0)$ Given

Solution $\Rightarrow$ $v_c(t) = v_c(0)\, e^{-t/RC}$

ODE Solution

$$RC\frac{dv_c}{dt} + v_c = 0$$

$$\frac{dv_c}{v_c} = -\frac{1}{RC}dt$$

$$\int_{v_c(0)}^{v_c(t)} \frac{dv_c}{v_c} = -\frac{1}{RC}\int_{0}^{t} dt'$$

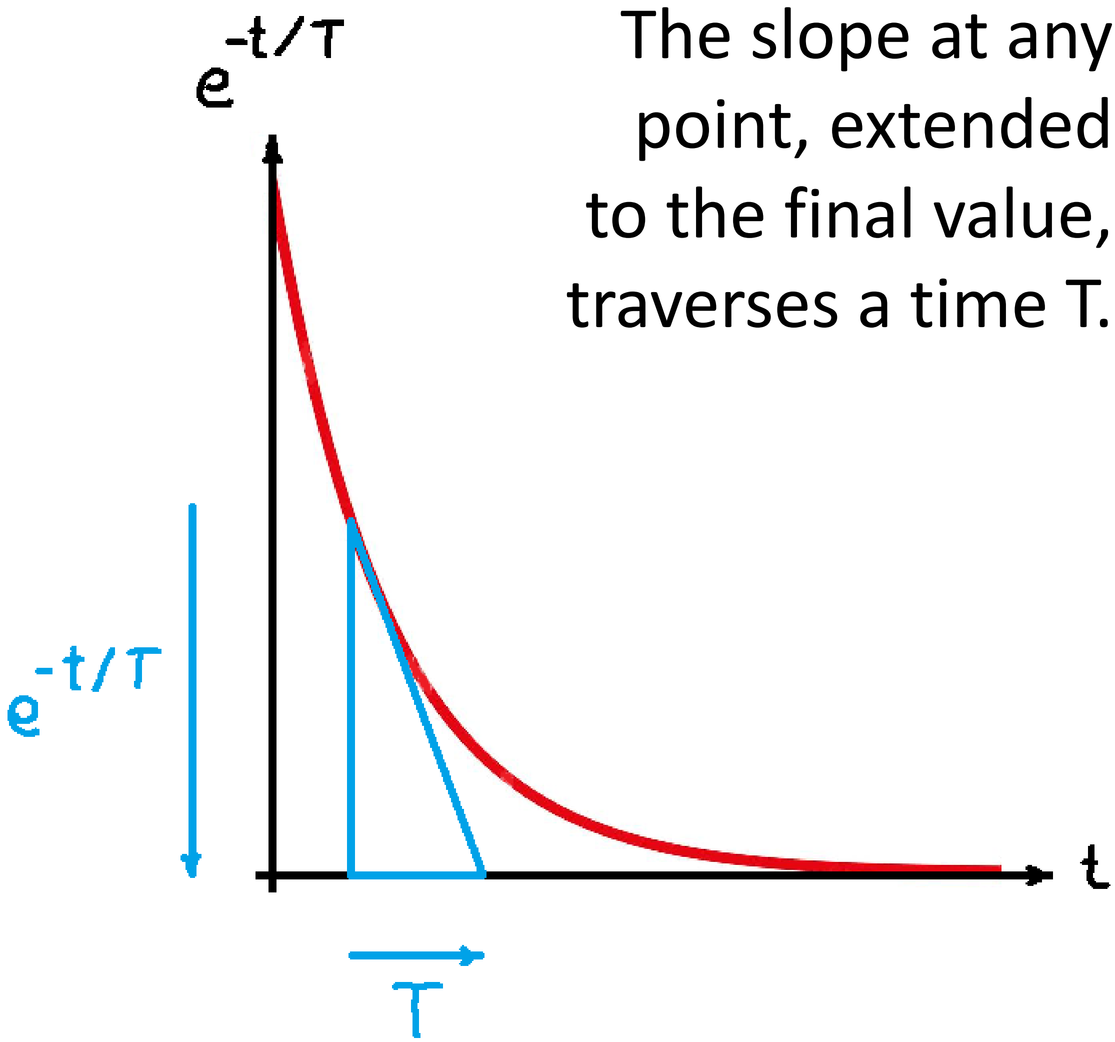
$$\ln(v_c(t)) - \ln(v_c(0)) = -\frac{t}{RC}$$

$$\ln\left(\frac{v_c(t)}{v_c(0)}\right) = -\frac{t}{RC}$$

$$v_c(t) = v_c(0)\,e^{-t/RC}$$

Exponential Decay

$$\frac{d}{dt} e^{-t/T} = \frac{-e^{-t/T}}{T} = \text{Slope} = \frac{\text{Rise}}{\text{Run}}$$



The slope at any point, extended to the final value, traverses a time T.