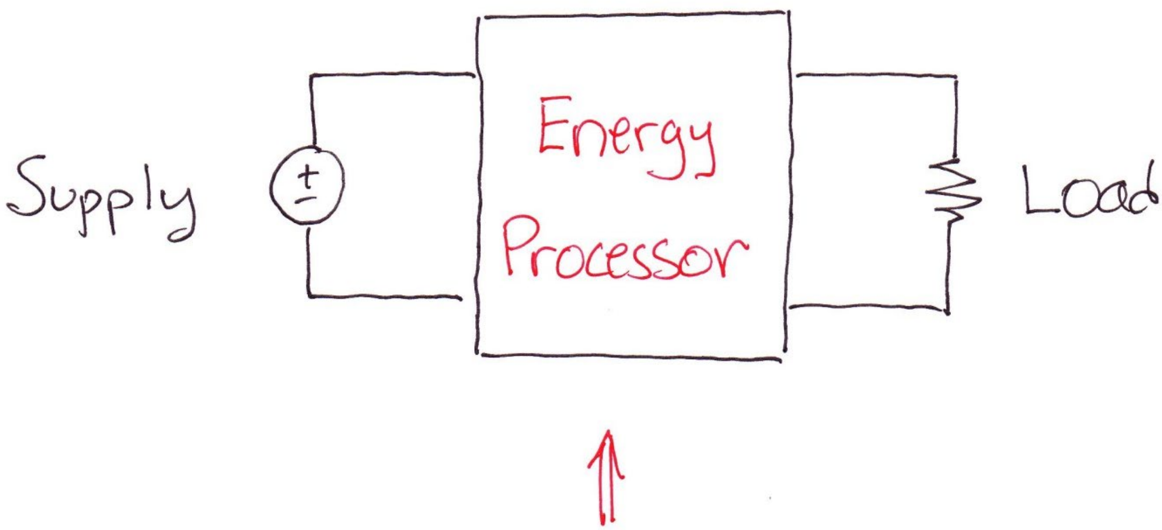
6.002 - Lecture 15

Energy Processing

- Energy-Efficient Delivery
- Simple Power Electronics
- Pulse Width Modulation
- First-Order Review
- Time Averaging
- Audio Example

Challenge: Efficient Energy Delivery

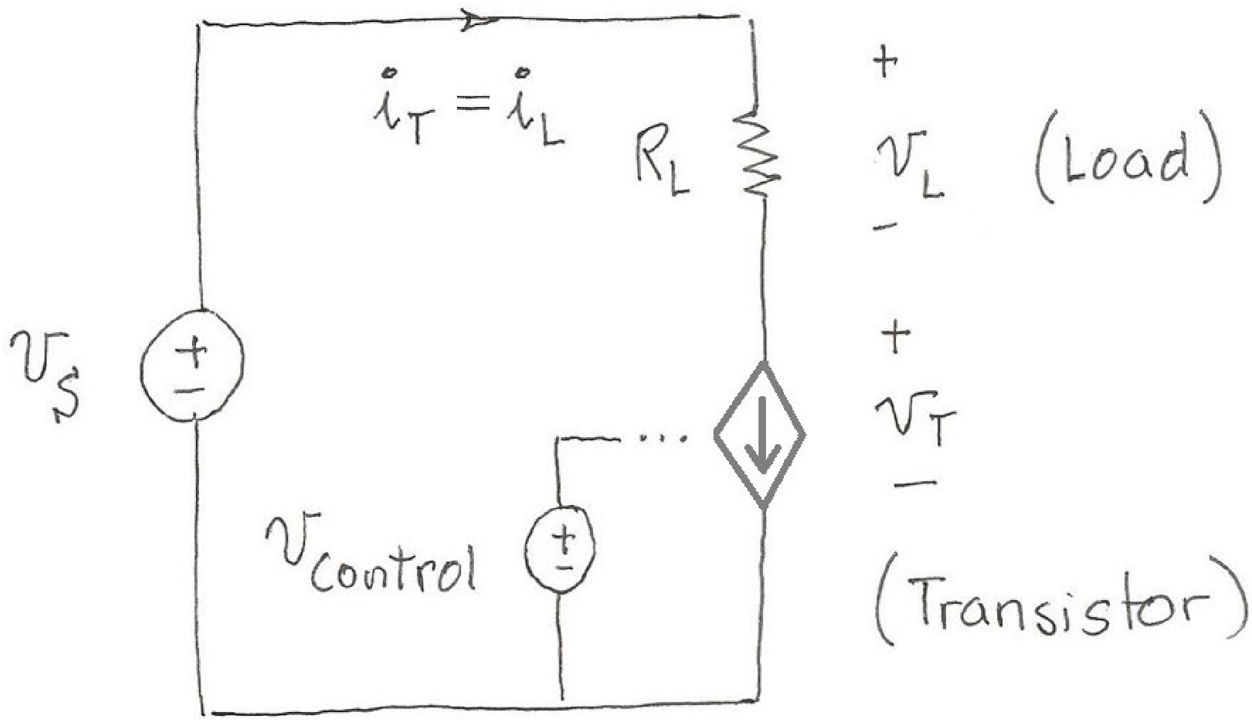


What goes here for greatest efficiency?

Complexities:
- Voltage mismatch
- Time-varying supply voltage
- Time-varying load voltage and power demand
- Varying load characteristics

Efficiency Benefits:
- Reduced energy cost
- Simplified thermal management

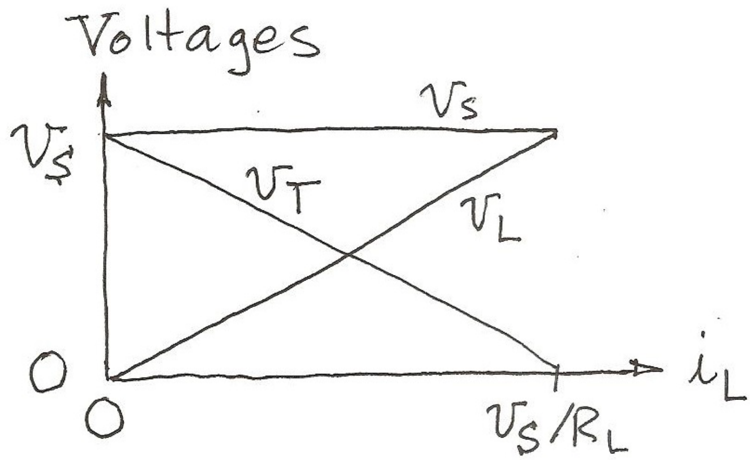
A Simple Energy Processor
(But, is it a good one?)

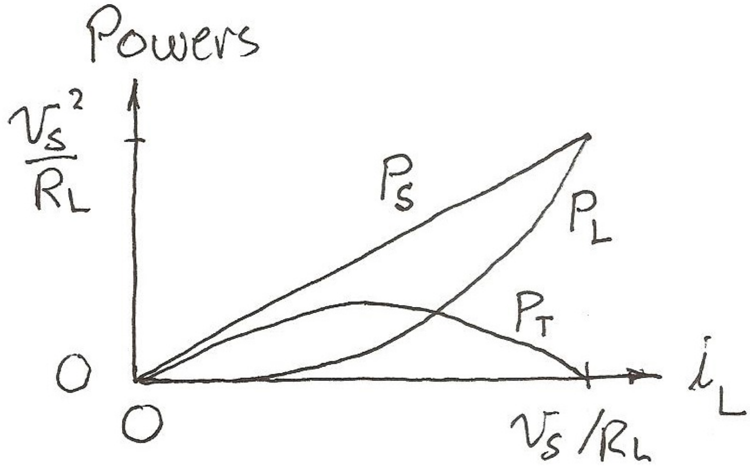


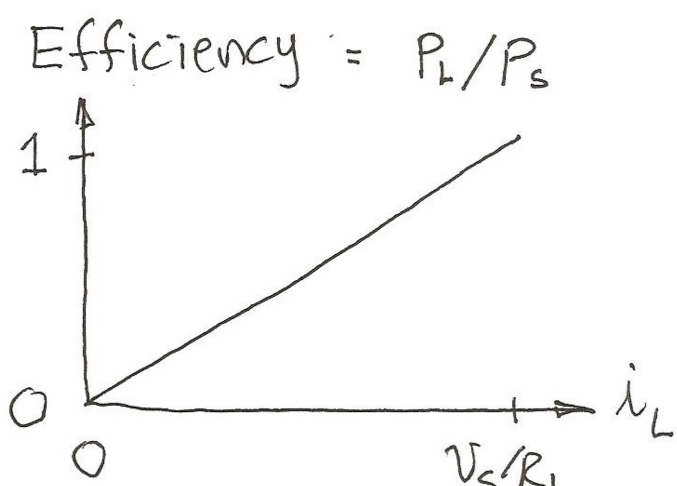
Design and Operation:

- Repurpose the amplifier studied earlier.
- Design the amplifier such that $\min\{v_S\} \geq \max\{v_L = R_L * i_L\}$.
- What if the design is not possible?
- Choose $v_{Control}$ to provide i_L and v_L.

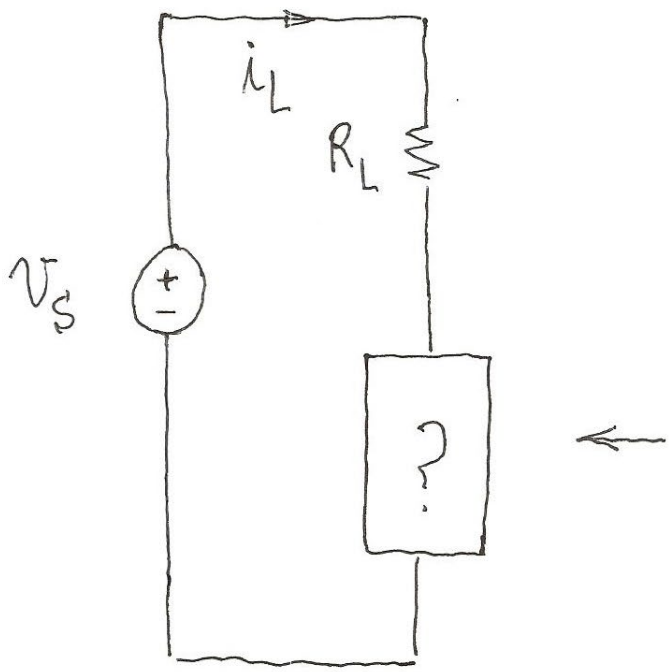
Energy Efficiency







The Challenge Revisited

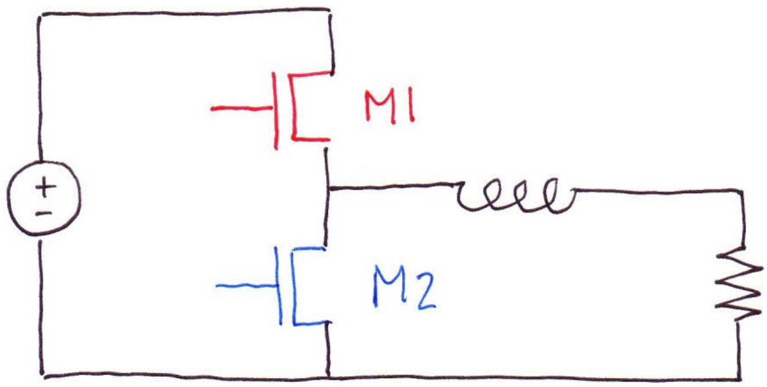


This "device" must experience the voltage $v_S - R_L i_L$, and so must dissipate the power $(v_S - R_L i_L)\, i_L$ if it is dissipative.

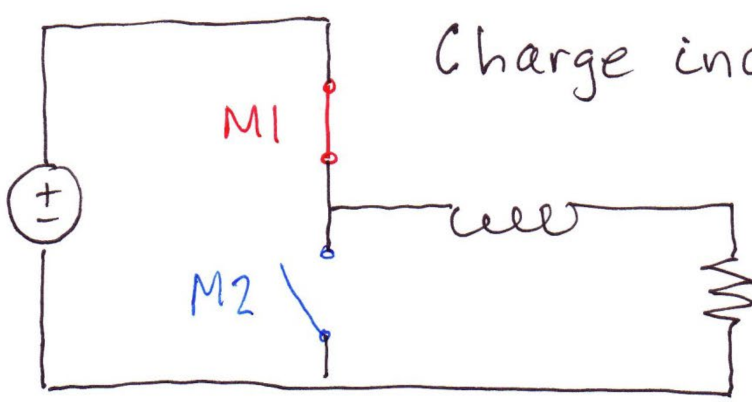
What about non-dissipative devices?

* Capacitors
* Inductors
* Switches (via Transistors)

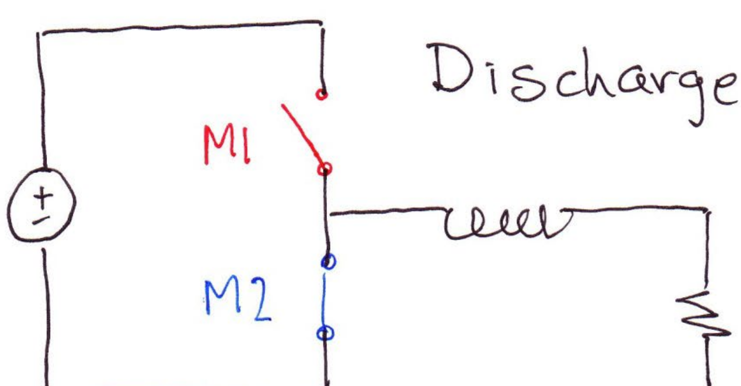
An Alternative Energy Processor



Operate transistors as switches

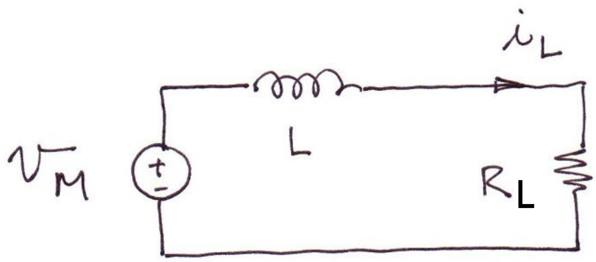


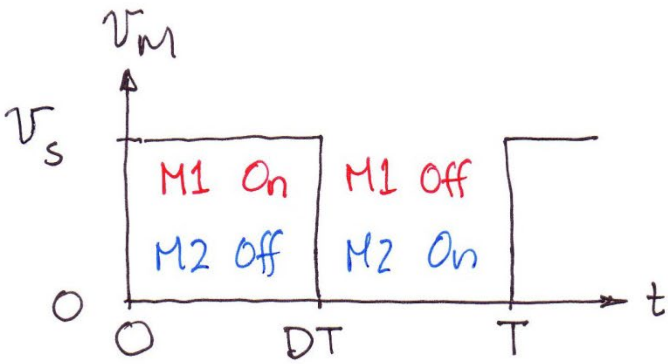
Charge inductor



Discharge inductor

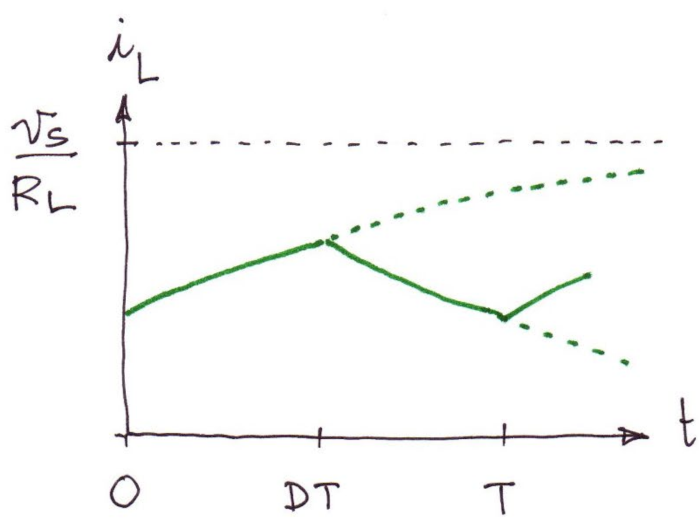
Cyclic Steady-State Operation





$T \equiv$ Switching Period

$D =$ Duty Cycle



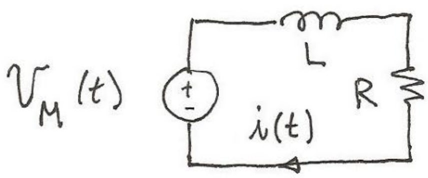
$\tau = \frac{L}{R_L}$

* What is the average current?

* What is the current ripple?

* What is the dynamic behavior?

Inductor-Resistor Network Review



$$\text{KVL} \Rightarrow \quad L\frac{di}{dt} + Ri = v_M$$

$$\tau\frac{di}{dt} + i = \frac{v_M}{R} \quad ; \quad \tau = \frac{L}{R}$$

$v_M = 0 \quad t \geq 0$

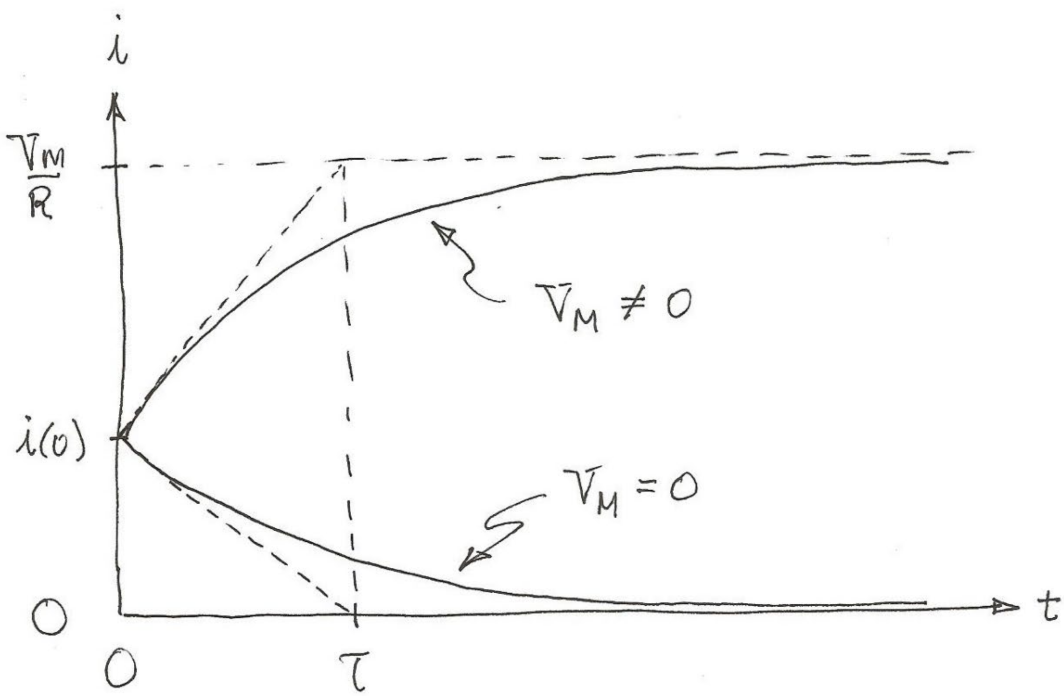
$i(0)$ Given

$$\Rightarrow \quad i(t) = i(0)\, e^{-t/\tau}$$

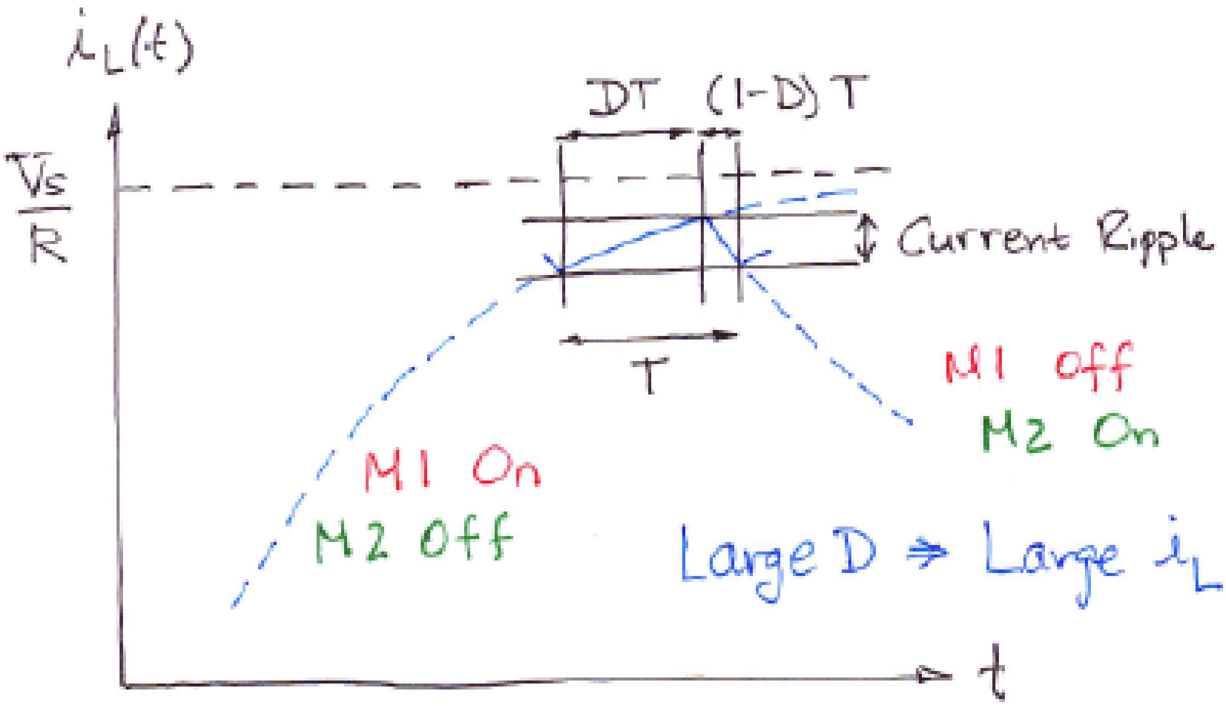
$v_M = V_M \quad t \geq 0$

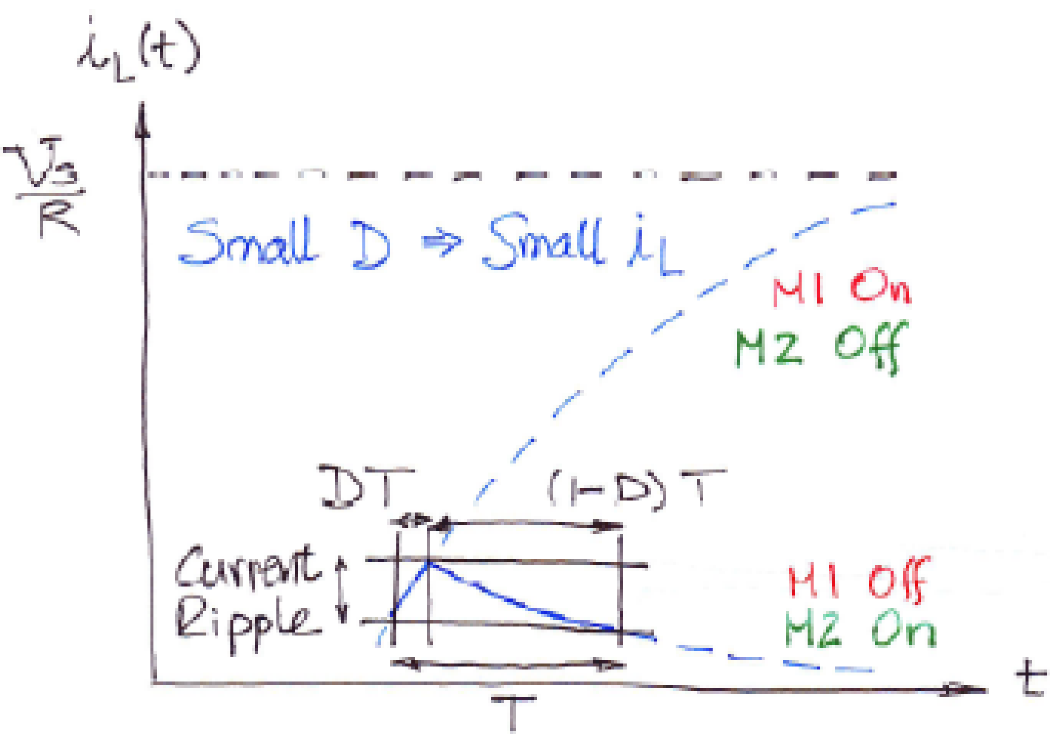
$i(0)$ Given

$$\Rightarrow \quad i(t) = i(0)\, e^{-t/\tau} + \frac{V_M}{R}\left(1 - e^{-t/\tau}\right)$$



Intuitive Operation Preview





Cyclic Steady-State: Averages

KVL $\Rightarrow$ $L \dfrac{d i_L(t)}{dt} + R i_L(t) = v_M(t)$

Average $\Rightarrow$ $\bar{x} \equiv \dfrac{1}{T} \displaystyle\int_{t-T}^{t} x(s)\, ds$ ... for any t

Average KVL $\Rightarrow$ $\dfrac{1}{T} \displaystyle\int_{t-T}^{t} \left(L \dfrac{d i_L(s)}{ds} + R i_L(s) \right) ds = \dfrac{1}{T} \displaystyle\int_{t-T}^{t} v_M(s)\, ds$

$$\Rightarrow \frac{L}{T}\left(i_L(t) - i_L(t-T) \right) + R \bar{i}_L = \bar{v}_M$$

Steady State $\Rightarrow$ $i_L(t) = i_L(t-T)$ ... for every t

$$\Rightarrow \bar{i}_L = \frac{\bar{v}_M}{R_L} = \frac{V_S D}{R_L}$$

PWM Control Variable

$$\Rightarrow \bar{v}_L = R_L \bar{i}_L = V_S D$$

Cyclic Steady-State: Ripples

Define $\Delta i_L \equiv$ peak-peak ripple

M1 On $\Rightarrow$ $V_S \approx \frac{\Delta i_L}{DT} L + R_L \bar{i}_L$

M2 On $\Rightarrow$ $0 \approx \frac{-\Delta i_L}{(1-D)T} L + R_L \bar{i}_L$

Difference $\Rightarrow$ $V_S \approx \frac{L \Delta i_L}{T} \left[\frac{1}{D} + \frac{1}{1-D} \right] = \frac{L \Delta i_L}{T D(1-D)}$

$$\Rightarrow \Delta i_L \approx \frac{V_S T D(1-D)}{L}$$

Minimize ripple:

* maximize L ... expensive
* maximize $\frac{1}{T}$ ... switching losses

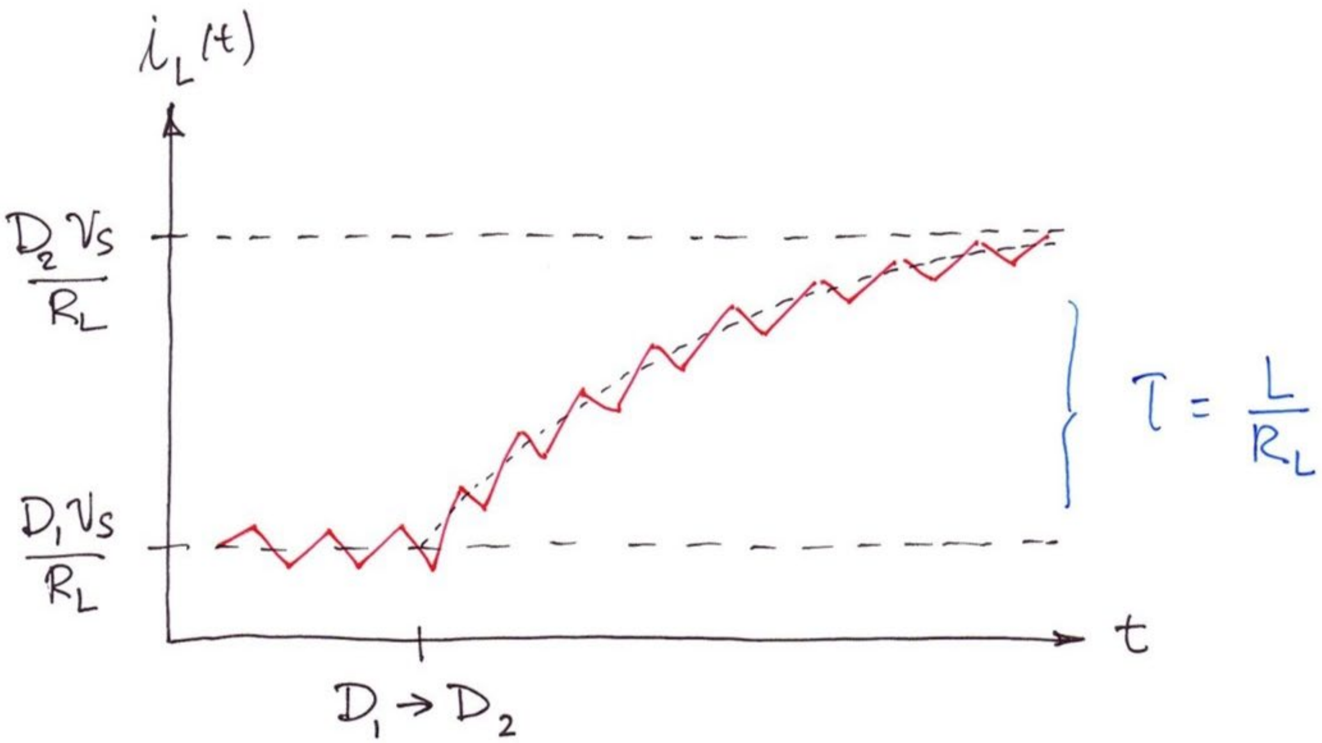
Dynamics

Moving Average $\Rightarrow$ $\bar{x}(t) = \frac{1}{T} \int_{t-T}^{t} x(s)\, ds$

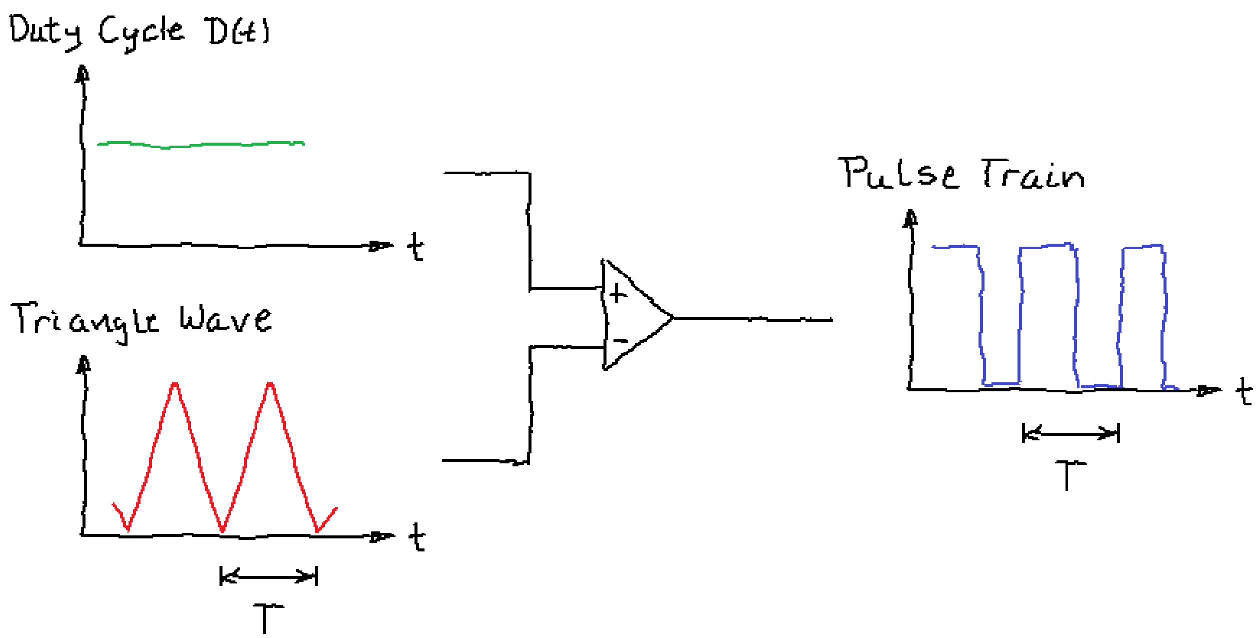
$$\frac{d\bar{x}(t)}{dt} = \frac{x(t) - x(t-T)}{T} = \overline{\frac{dx(t)}{dt}}$$

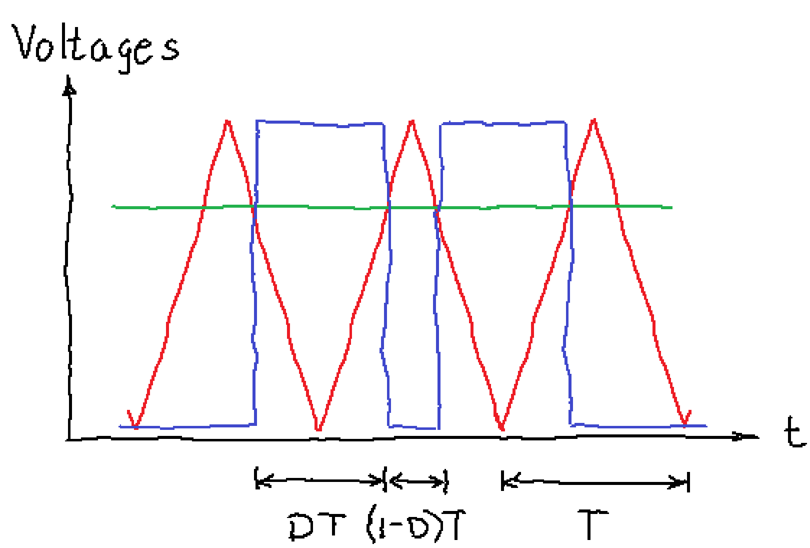
Dynamics $\Rightarrow$ $L \frac{d\bar{i}_L(t)}{dt} + R_L \bar{i}_L(t) = \bar{v}_M(t)$

$$= D(t)\, v_S$$



Implementation: PWM





Implementation: Power Electronics