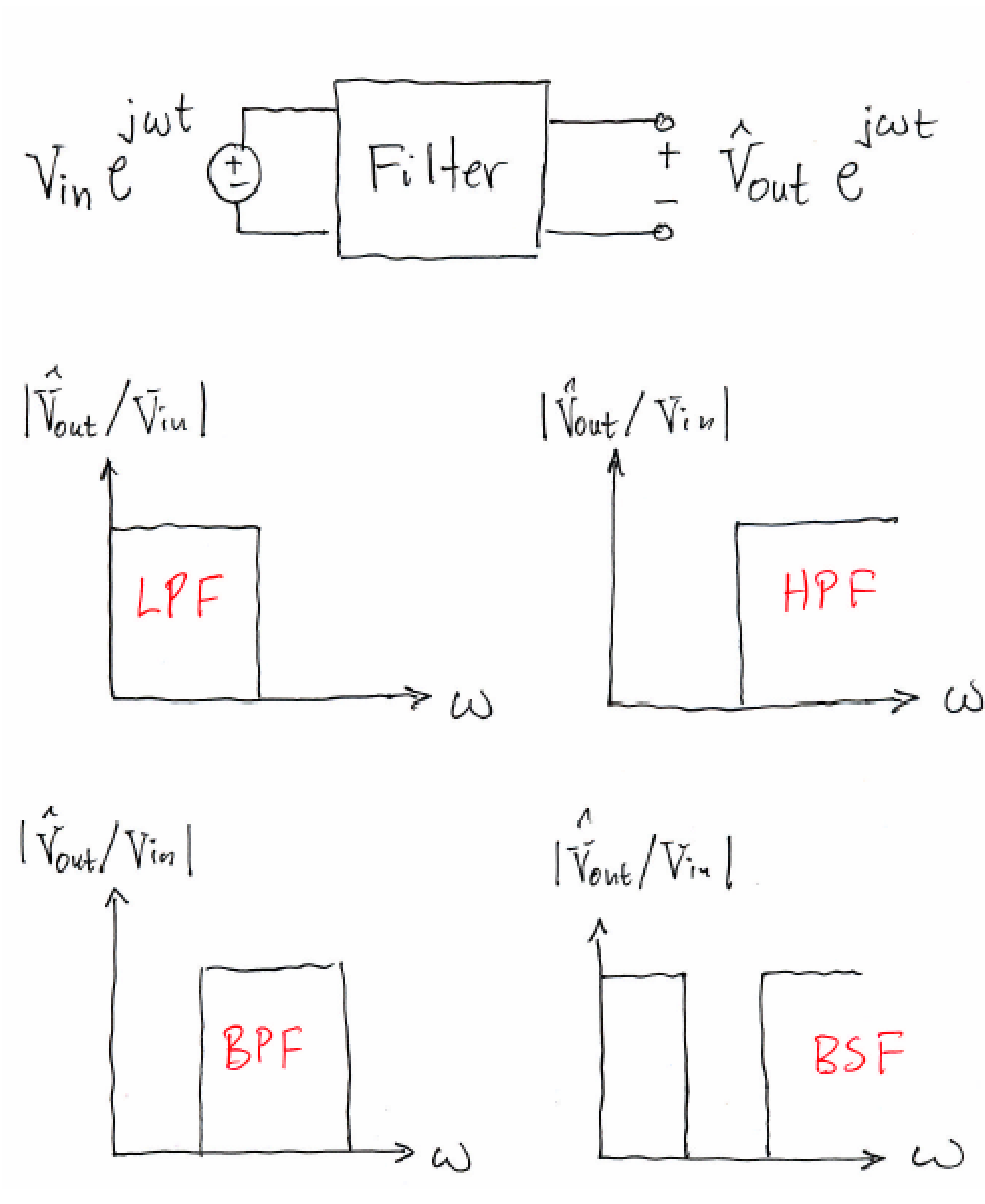
6.200 - Lecture 23

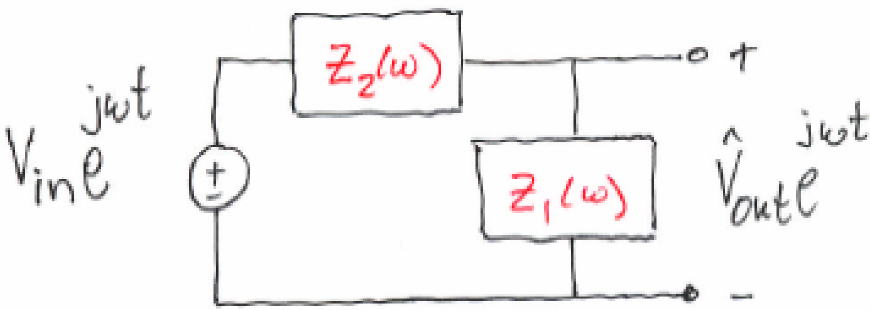
Filtering:

- Review Simple Objectives (LPF , HPF , BPF , BSF)
- Review Simple Synthesis
- LC Resonators
- Tesla Coil

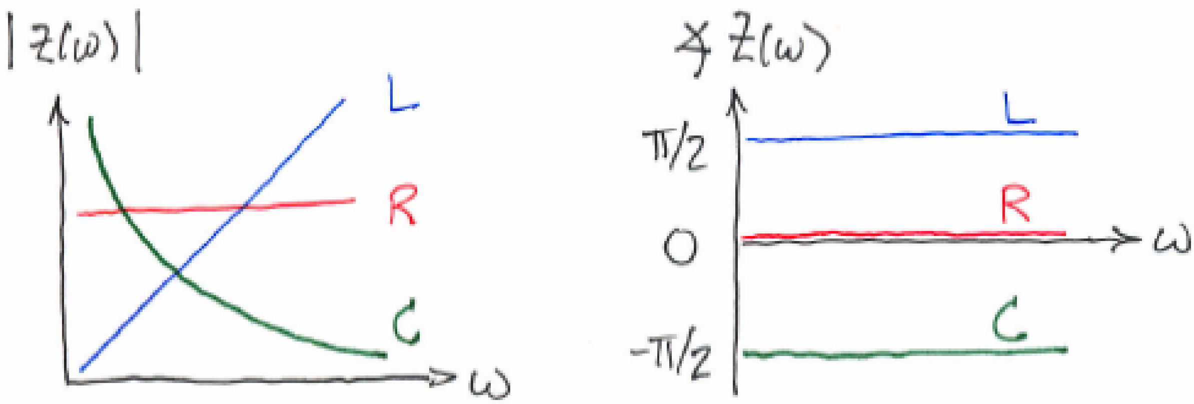
Simple Filtering Objectives



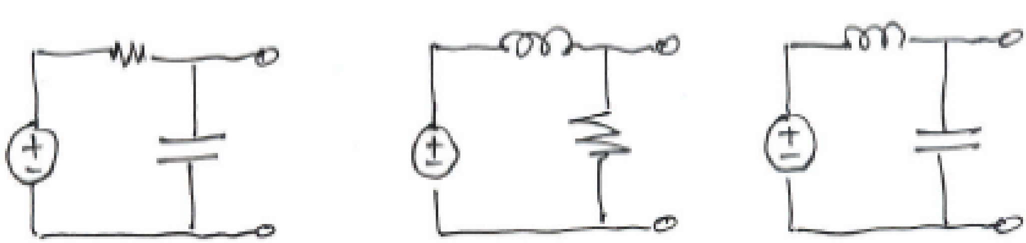
Simple Filtering Synthesis



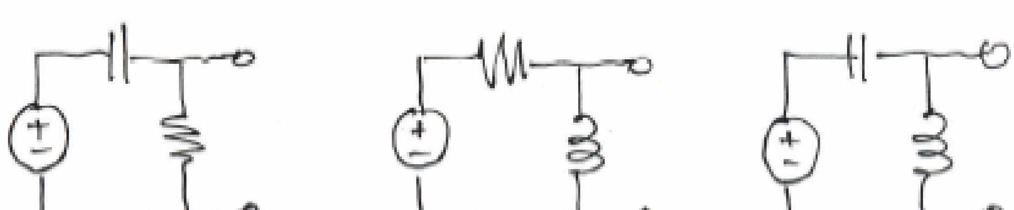
$$\frac{\hat{V}_{out}}{V_{in}} = \frac{Z_1(\omega)}{Z_2(\omega)+Z_1(\omega)}$$



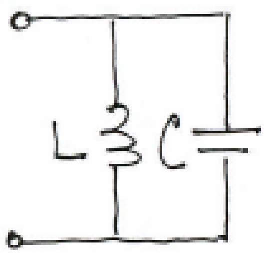
LPF:



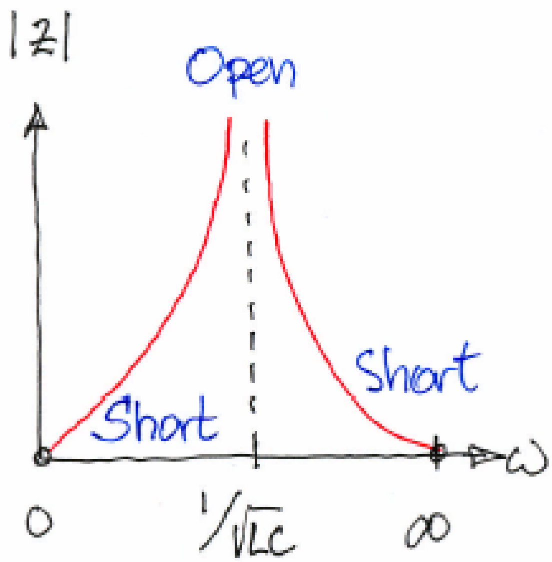
HPF:

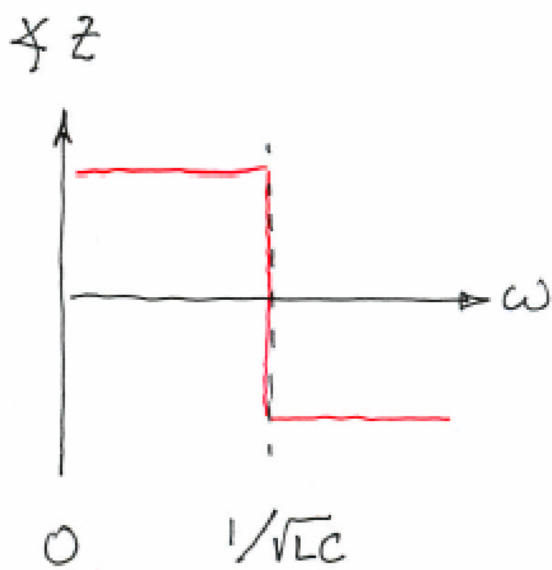


Parallel LC Resonator

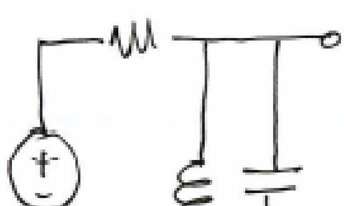


$$Z = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

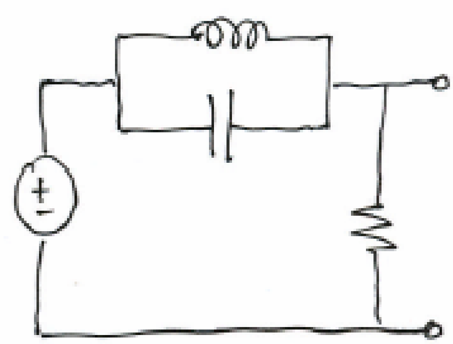




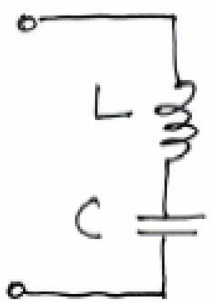
BPF:



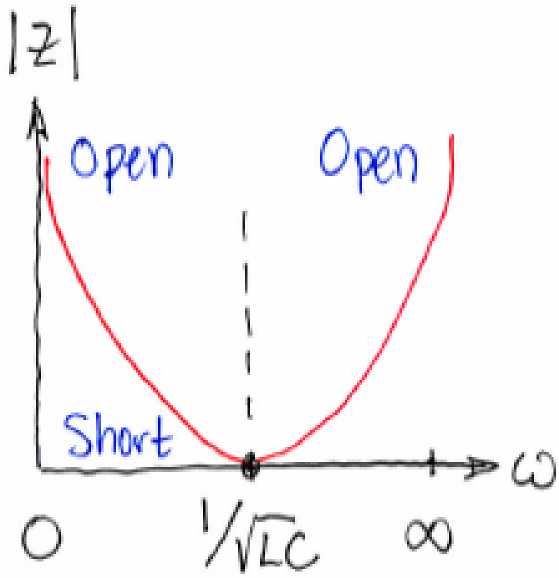
BSF:

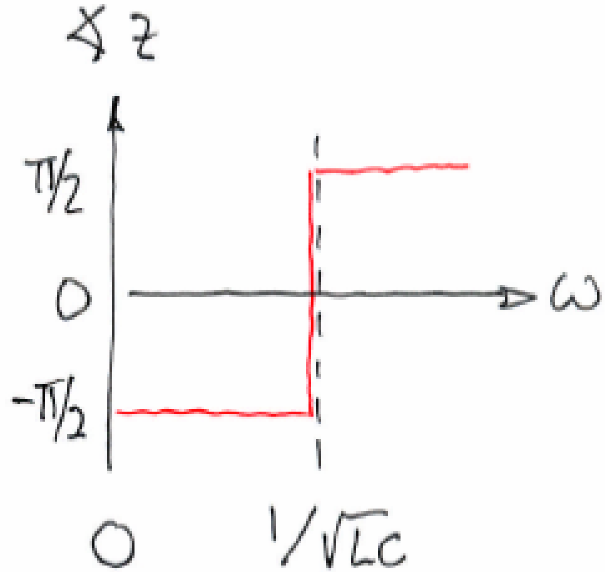


Series LC Resonator

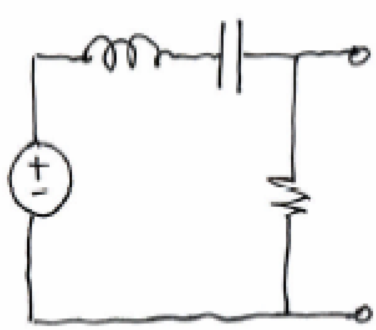


$$Z = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

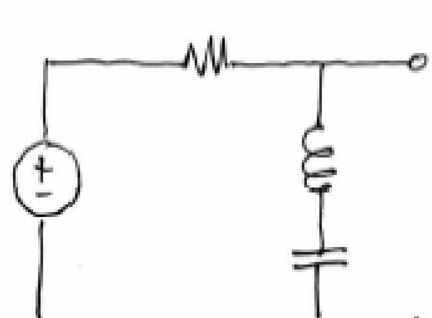




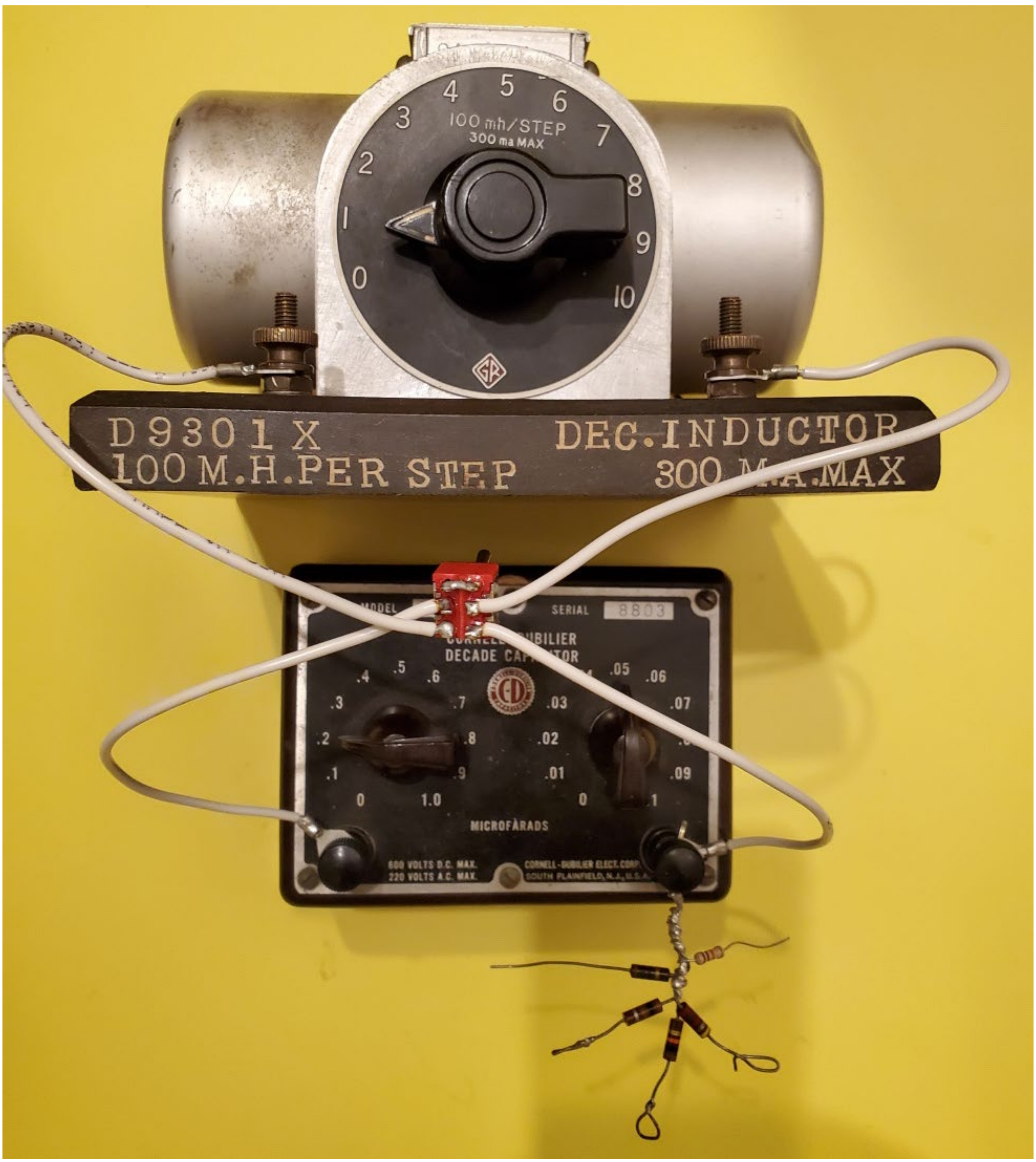
BPF:



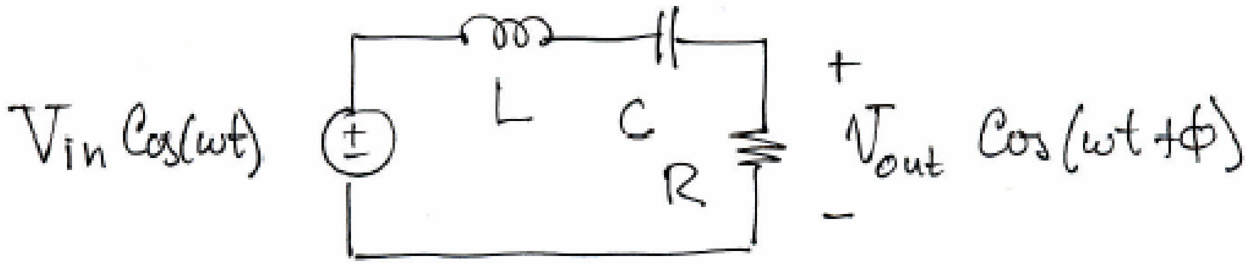
BSF:

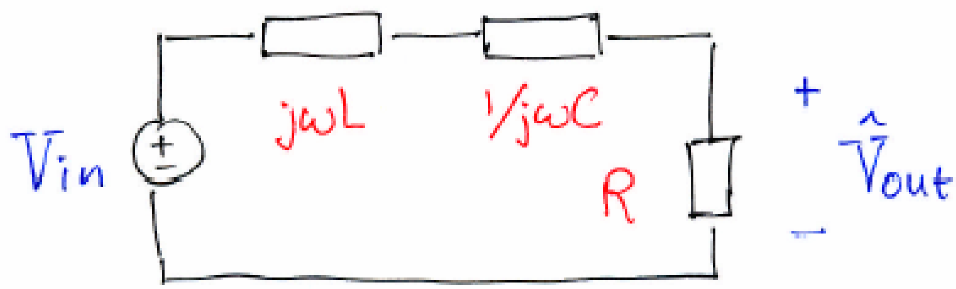


RLC Demos


100 mh/STEP
300 ma MAX
D9301X
DEC.INDUCTOR
100 M.H.PER STEP
SERIAL 8803
DECADE CAP
MICROFARADS
600 VOLTS D.C. MAX.
220 VOLTS A.C. MAX.

Theory: Series-LC BPF



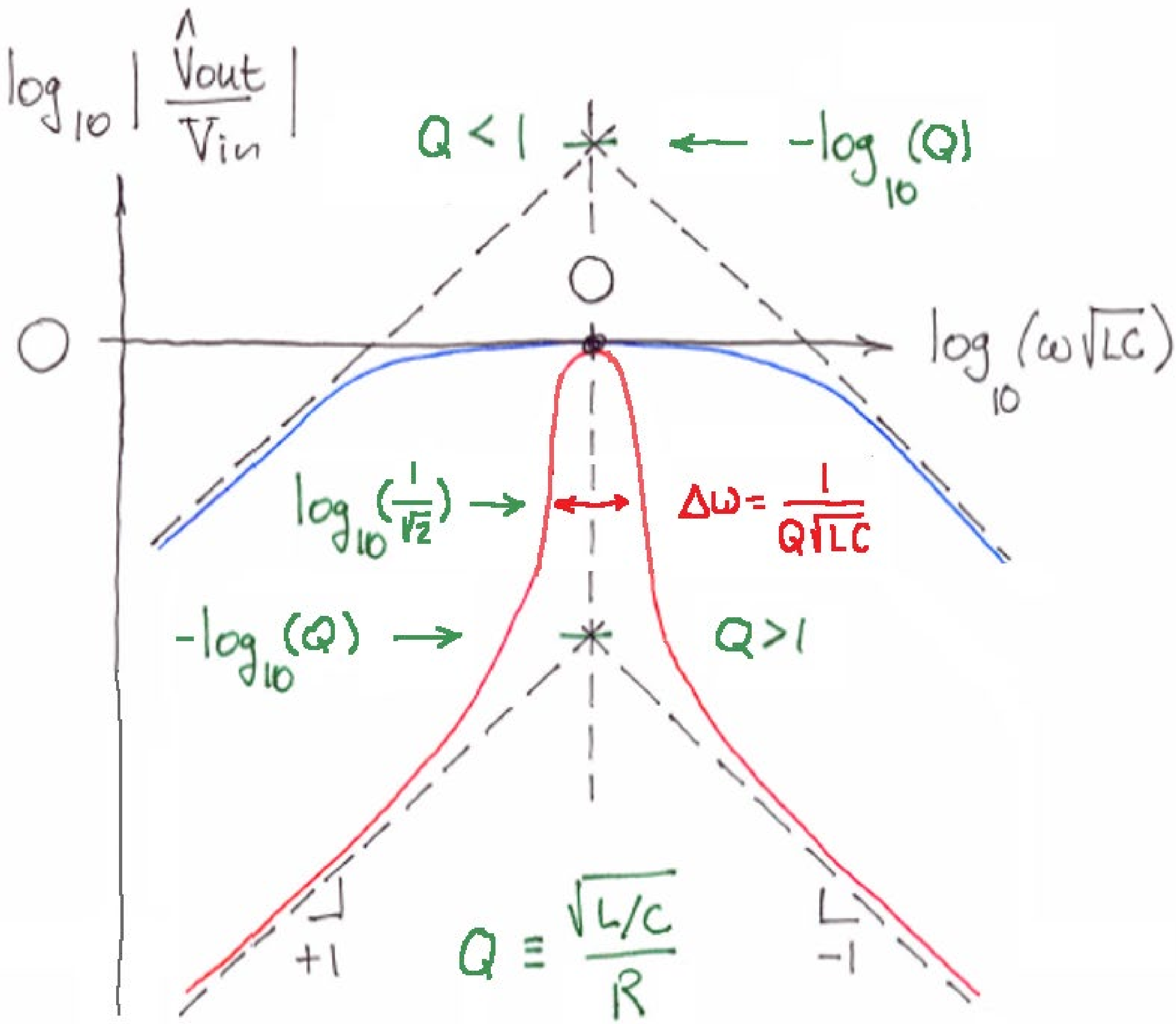


$$\hat{V}_{out} = \frac{R}{R + j\omega L + 1/j\omega C}\,\hat{V}_{in} = \frac{j\omega RC}{1-\omega^2 LC + j\omega RC}\,V_{in}$$

$$V_{out} = \left|\hat{V}_{out}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2+(\omega RC)^2}}\,V_{in}$$

$$\phi = \angle \hat{V}_{out} = \tan^{-1}\left(\frac{1-\omega^2 LC}{\omega RC}\right)$$

Theory: Series-LC BPF



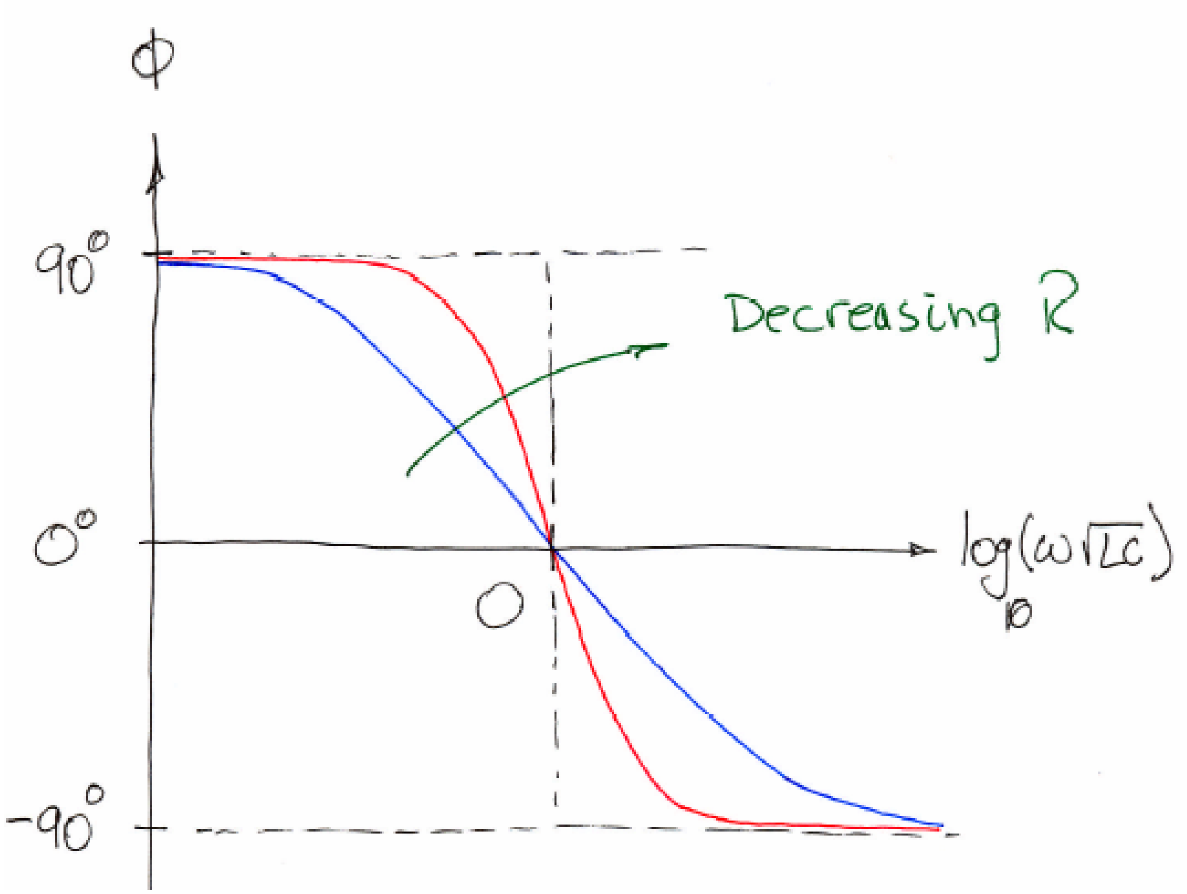
Low Frequency

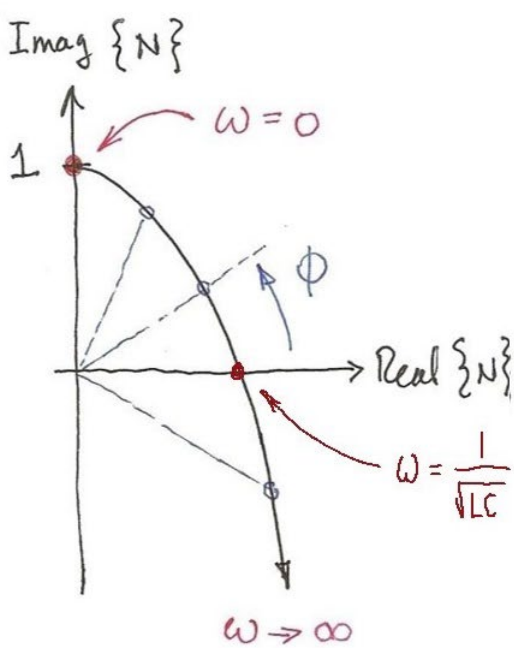
$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \omega RC = \frac{\omega\sqrt{LC}}{Q}$$

High Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \frac{R}{\omega L} = \frac{1}{\omega\sqrt{LC}\,Q}$$

Theory: Series-LC BPF





$$\phi \equiv \measuredangle \left| \frac{\hat{V}_{out}}{\hat{V}_{in}} \right|$$

$$= \measuredangle \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

$$= \measuredangle \frac{j\omega RC\,(1 - \omega^2 LC - j\omega RC)}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

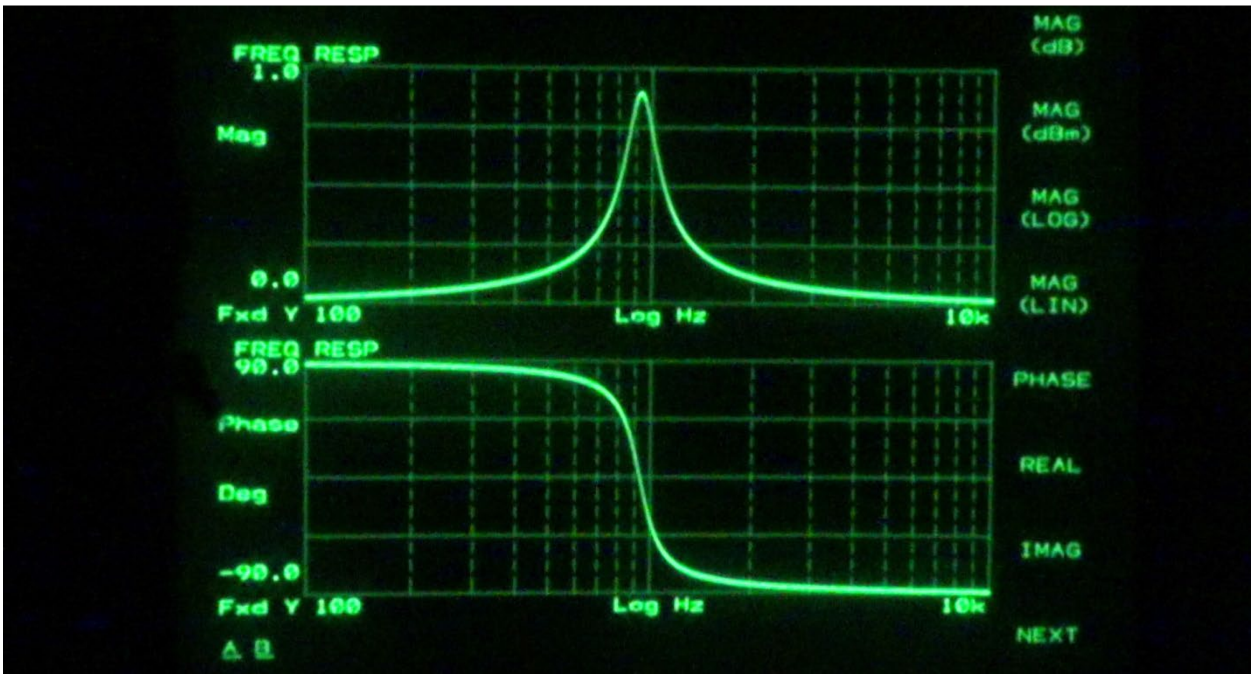
$$= \measuredangle\, j(1-\omega^2 LC) + \omega RC$$

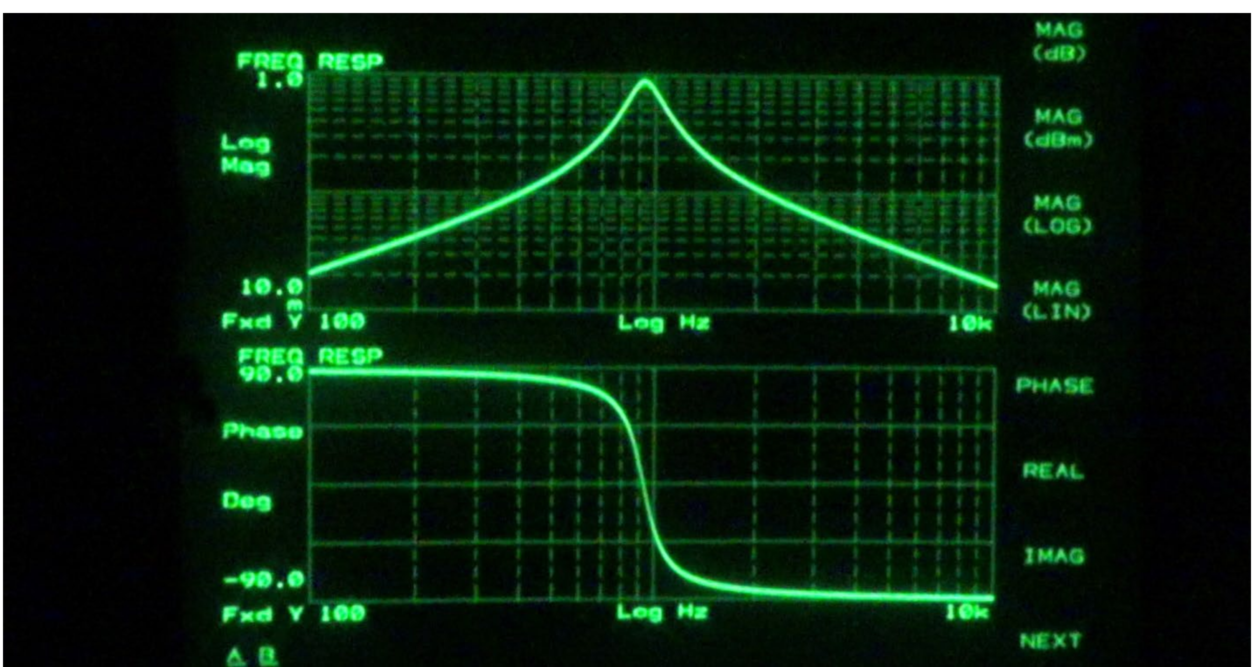
$$\equiv \measuredangle N$$

Demo: Series-LC BPF

L = 0.1 H ; C = 0.25 μF ; R = 100 Ω

$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ = 632 Ω

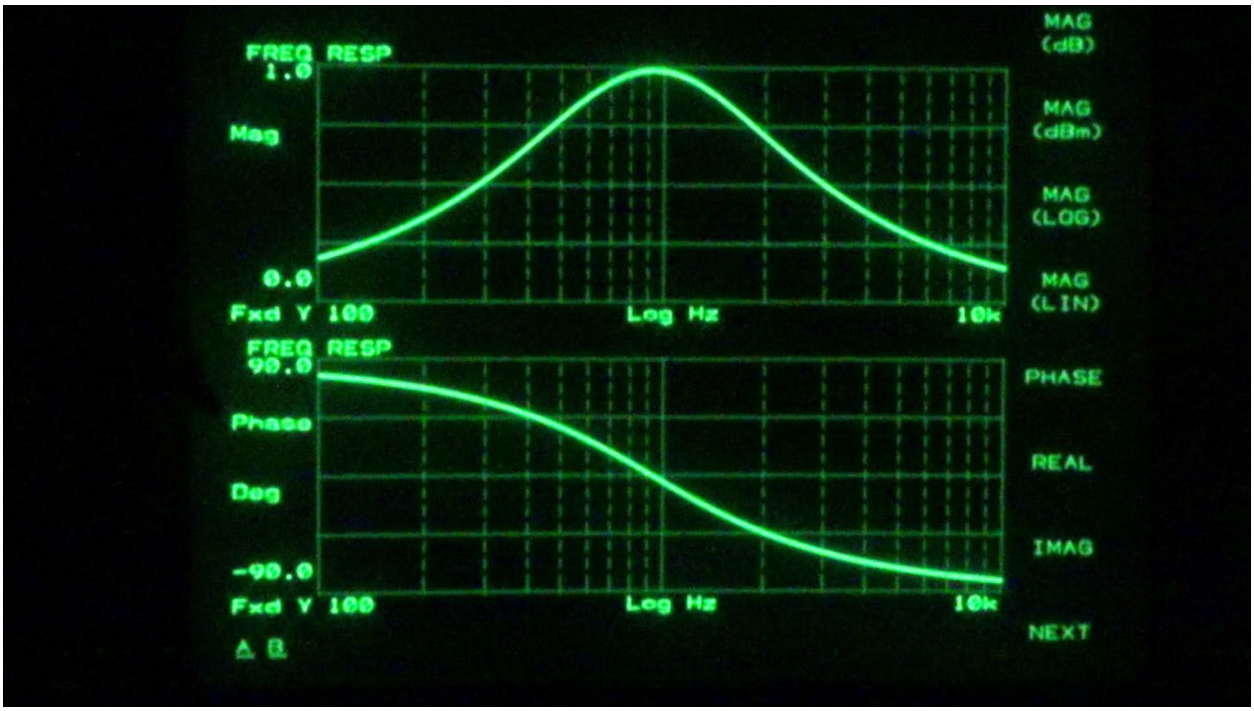




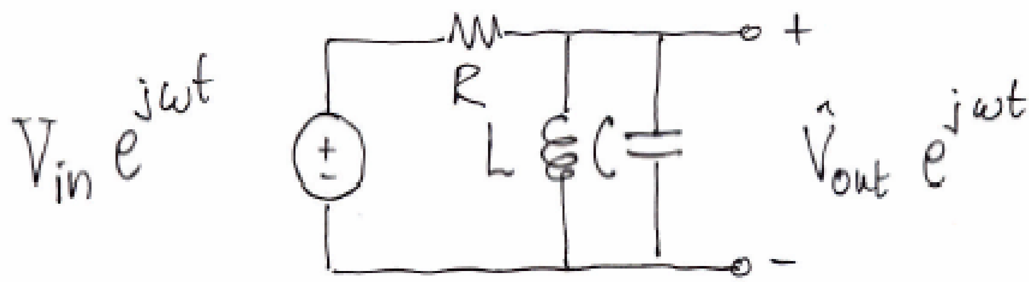
Demo: Series-LC BPF

L = 0.1 H ; C = 0.25 μF ; R = 1 kΩ

$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ = 632 Ω

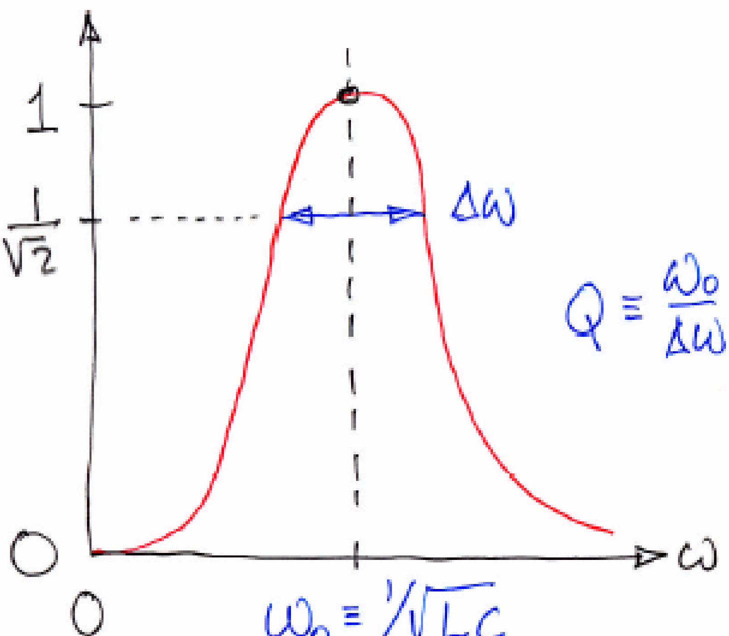


Theory: Parallel-LC BPF



$$\frac{\hat{V}_{out}}{V_{in}} = \frac{1/R}{1/R + 1/j\omega L + j\omega C} = \frac{j\omega L/R}{1-\omega^2 LC + j\omega L/R}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{(\omega L/R)}{\sqrt{(1-\omega^2 LC)^2 + (\omega L/R)^2}}$$



$$Q \equiv \frac{\omega_0}{\Delta\omega} = \frac{R}{\sqrt{L/C}}$$

Numbers:

$L = 0.1$ H

$C = 0.25\ \mu F$

$R = 10\ k\Omega$

$\frac{1}{2\pi\sqrt{LC}} = 1$ kHz

$\sqrt{L/C} = 632\ \Omega$

$Q = 16$

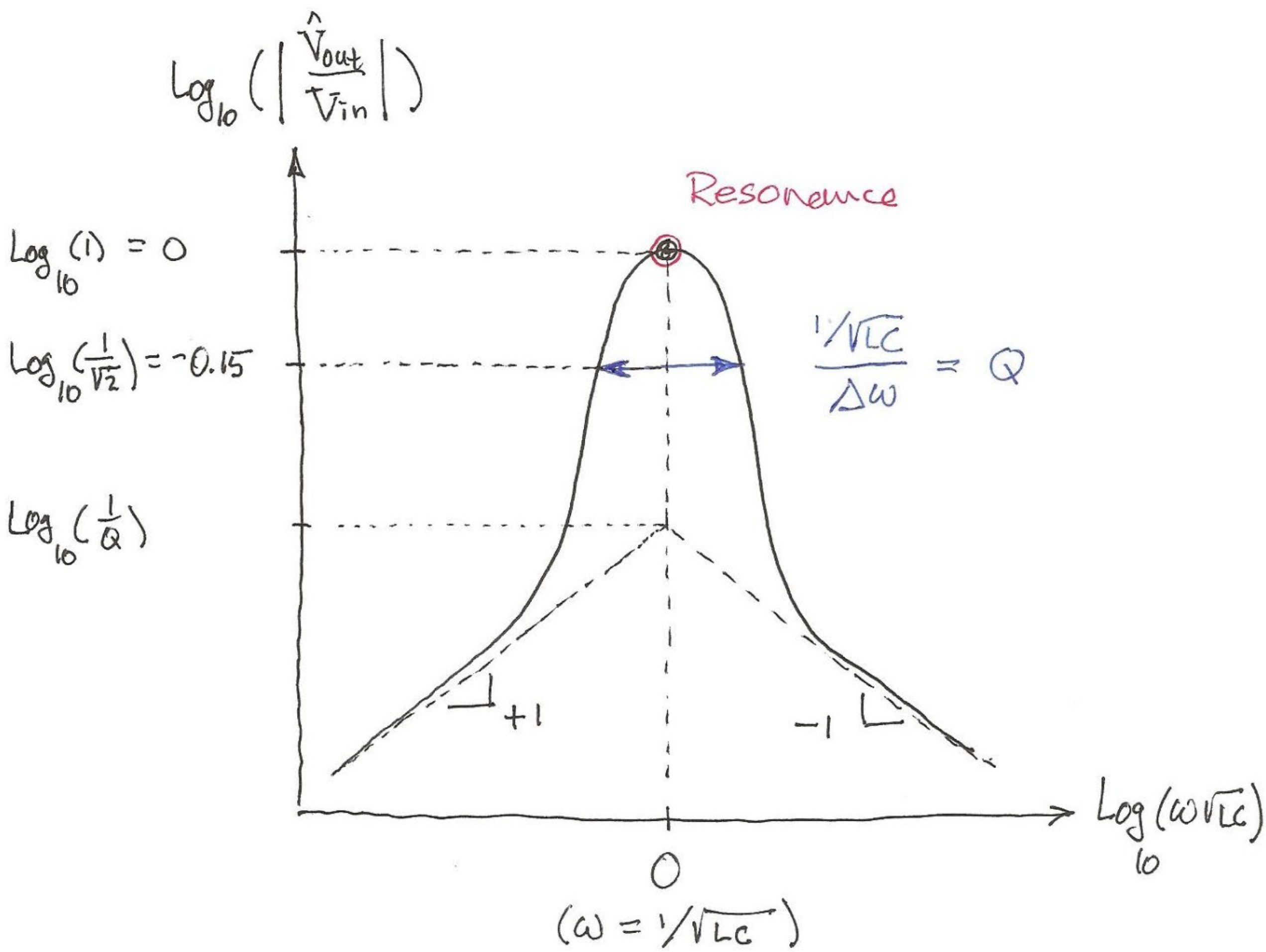
Theory: Parallel-LC BPF

Low ω: $\left|\frac{\hat{V}_{out}}{\hat{V}_{in}}\right| \approx \frac{\omega L}{R} = \frac{\omega\sqrt{LC}}{Q}$

High ω: $\left|\frac{\hat{V}_{out}}{\hat{V}_{in}}\right| \approx \frac{1}{\omega RC} = \frac{1}{\omega\sqrt{LC}\,Q}$

} Assymptotes

Assymptotes cross at $\omega\sqrt{LC} = 1$ with value $1/Q$.



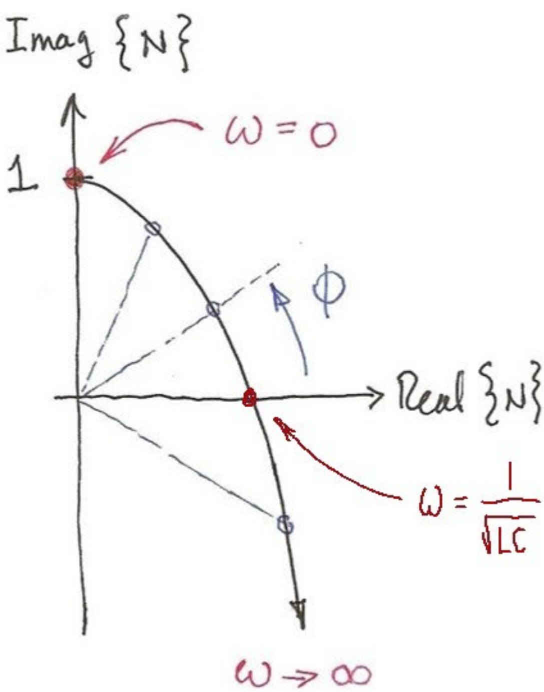
Theory: Parallel-LC BPF

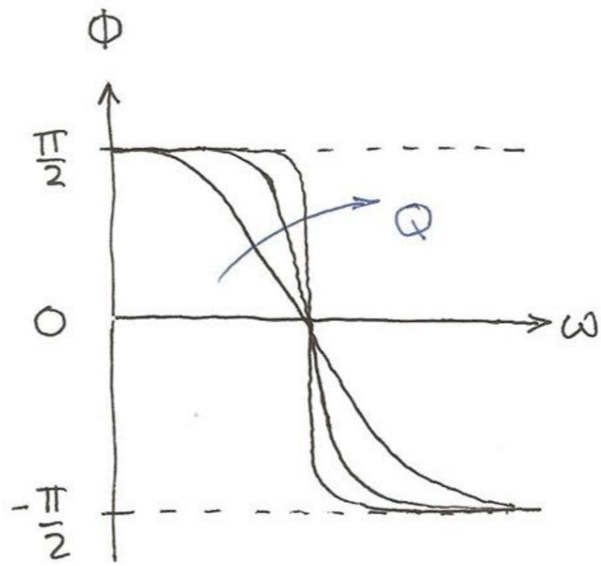
$$\measuredangle \left| \frac{\hat{V}_{out}}{V_{in}} \right| \equiv \phi = \measuredangle \frac{j\omega L/R}{1-\omega^2 LC + j\omega L/R}$$

$$= \measuredangle \frac{j\omega L/R \left(1-\omega^2 LC - j\omega L/R\right)}{\left(1-\omega^2 LC\right)^2 + \left(\omega L/R\right)^2}$$

$$= \measuredangle\, j\left(1-\omega^2 LC\right) + \omega L/R$$

$$\equiv \measuredangle\, N$$

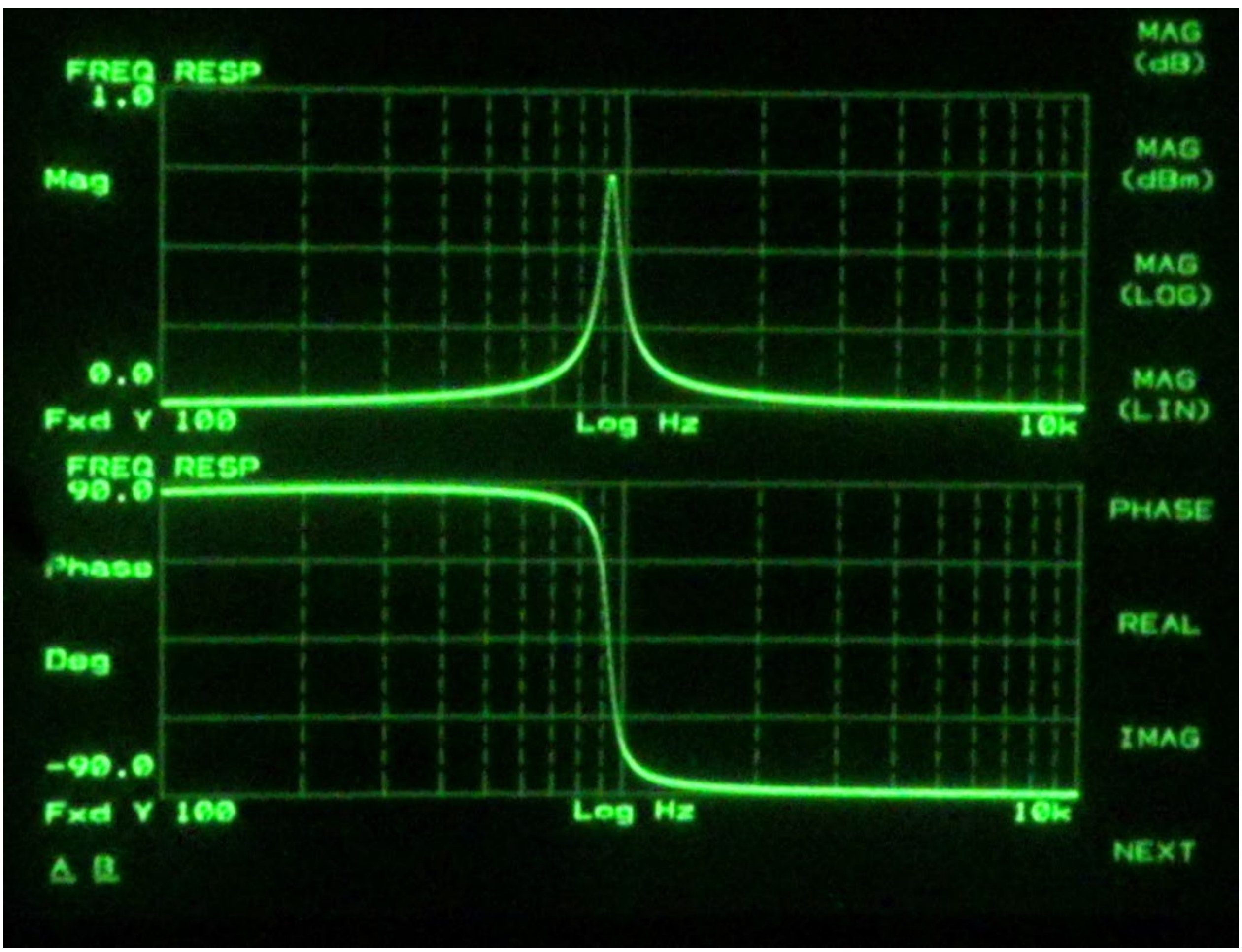




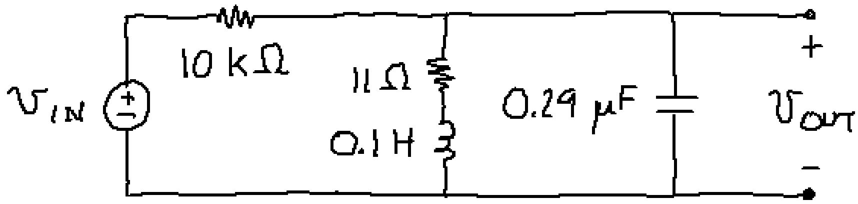
Demo: Parallel-LC BPF

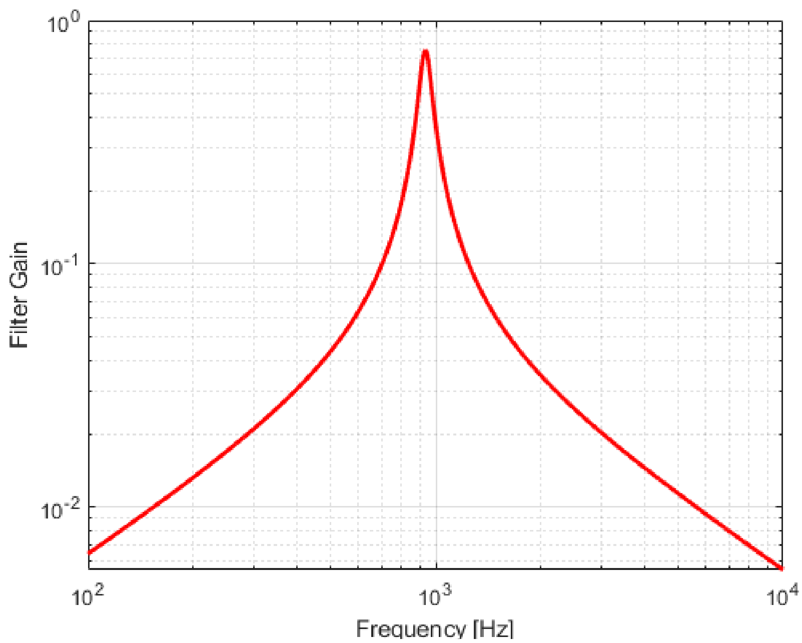
L = 0.1 H ; C = 0.25 μF ; R = 10 kΩ

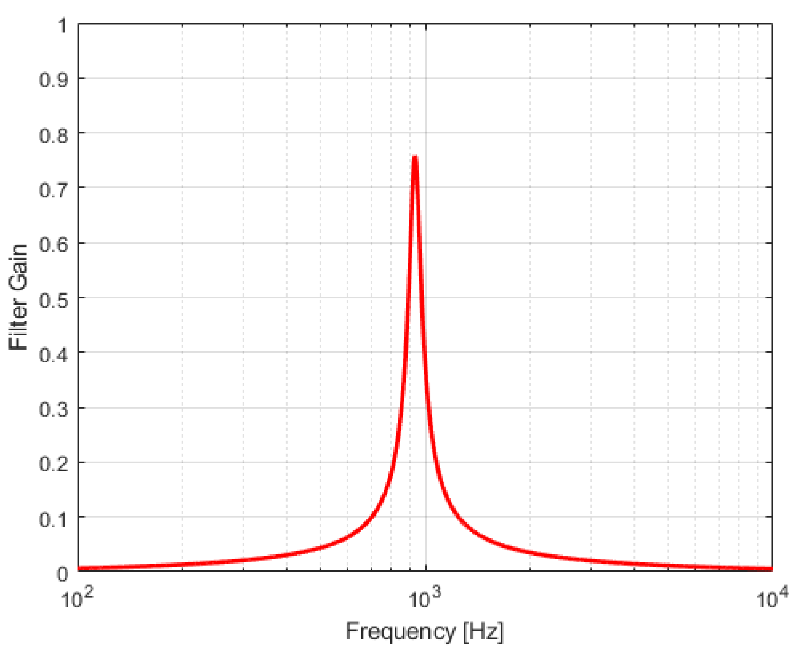
$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $R/\sqrt{L/C}$ = 16



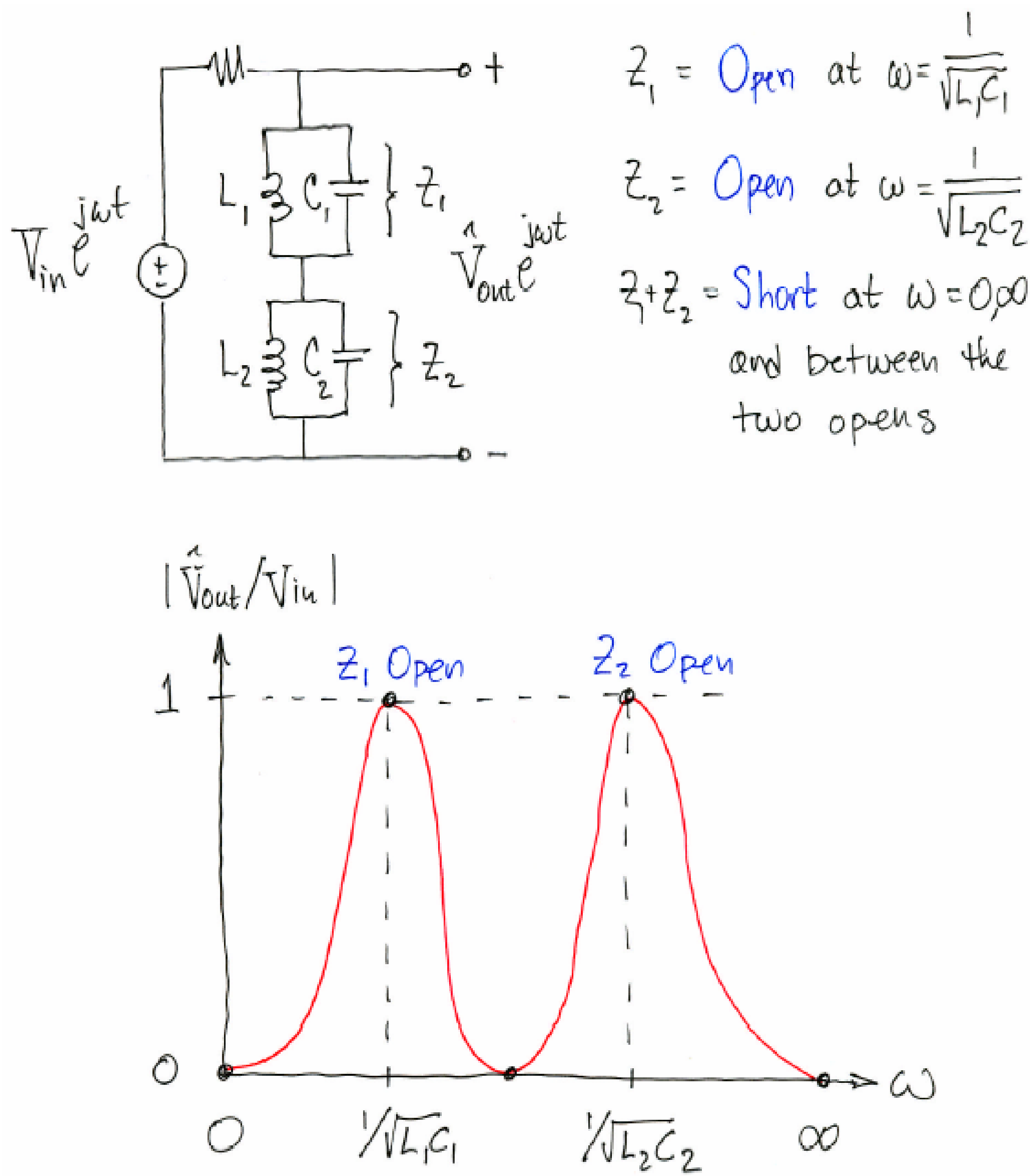
Numerical: Parallel-LC BPF





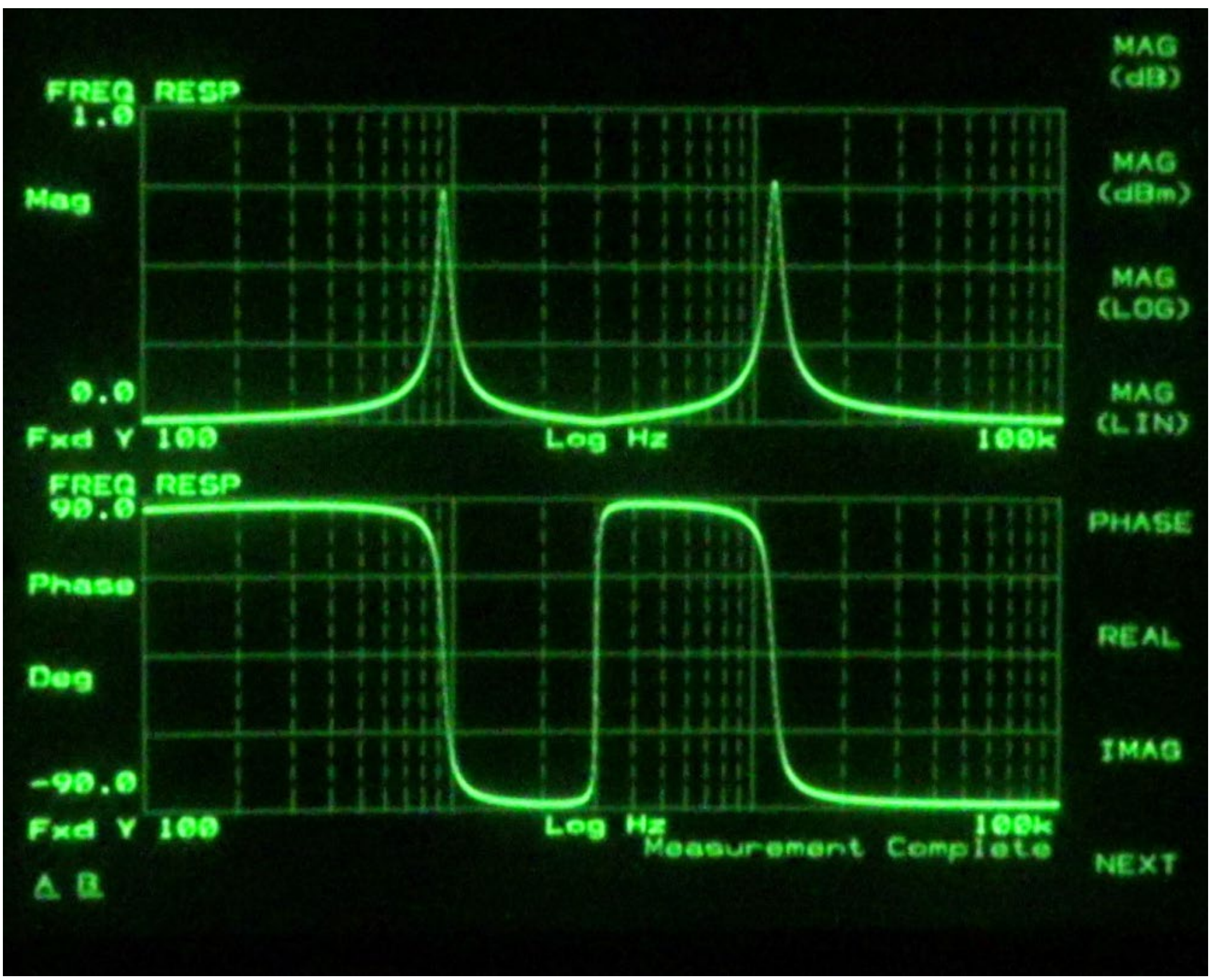


LC Comb Filter



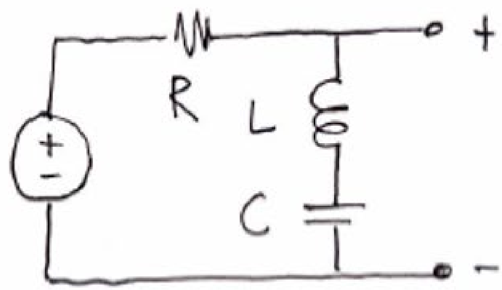
Demo: LC Comb Filter

L_1 = 0.1 H ; C_1 = 0.25 μF ; f_1= 1 kHz
L_2 = 0.01 H ; C_2 = 0.022 μF ; f_2= 10 kHz
R = 10 kΩ ; Q1 = Q2 = 16



Theory: Series-LC BSF

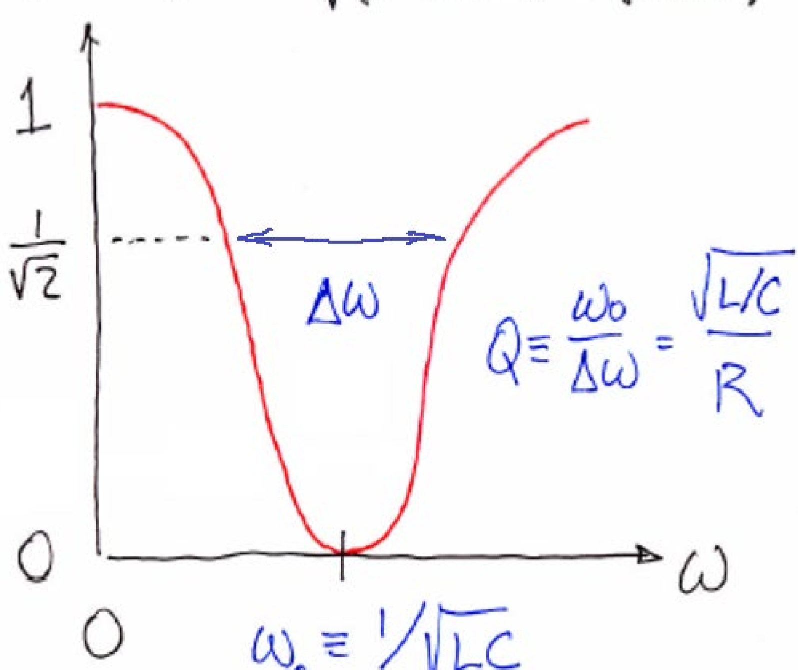
$V_{in} e^{j\omega t}$



$\hat{V}_{out} e^{j\omega t}$

$$\frac{\hat{V}_{out}}{V_{in}} = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1-\omega^2 LC}{1-\omega^2 LC + j\omega RC}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{1-\omega^2 LC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$



Numbers:

$L = 0.1\ H$

$C = 0.25\ \mu F$

$R = 100\ \Omega$

$\frac{1}{2\pi\sqrt{LC}} = 1\ kHz$

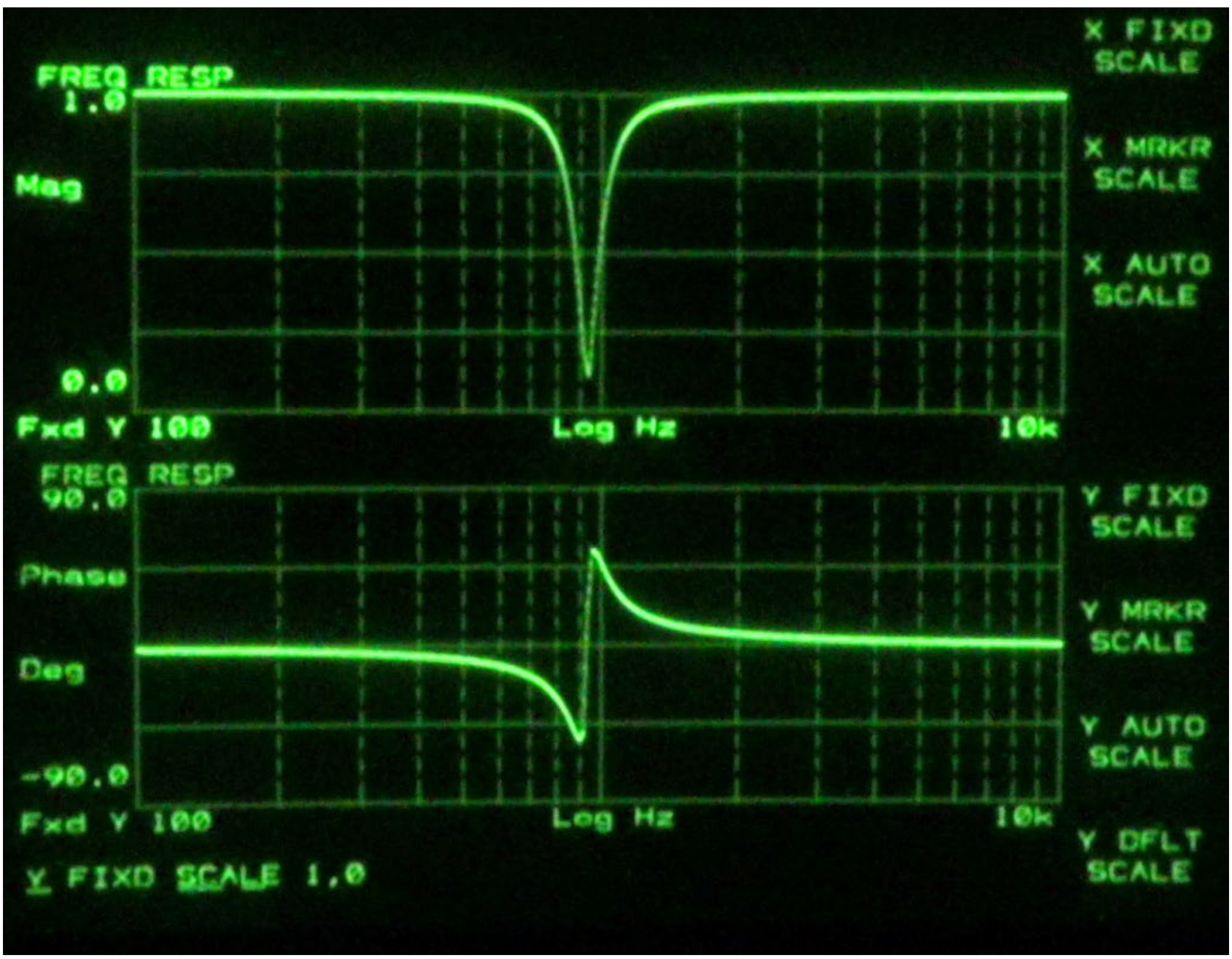
$\sqrt{L/C} = 632\ \Omega$

$Q = 6.3$

Demo: Series-LC BSF

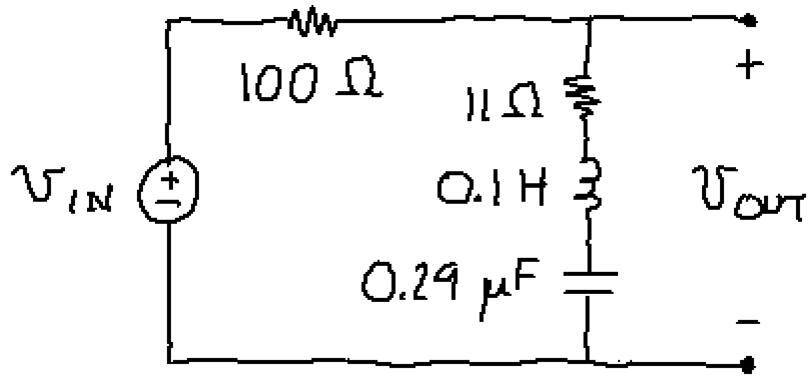
L = 0.1 H ; C = 0.25 μF ; R = 100 Ω

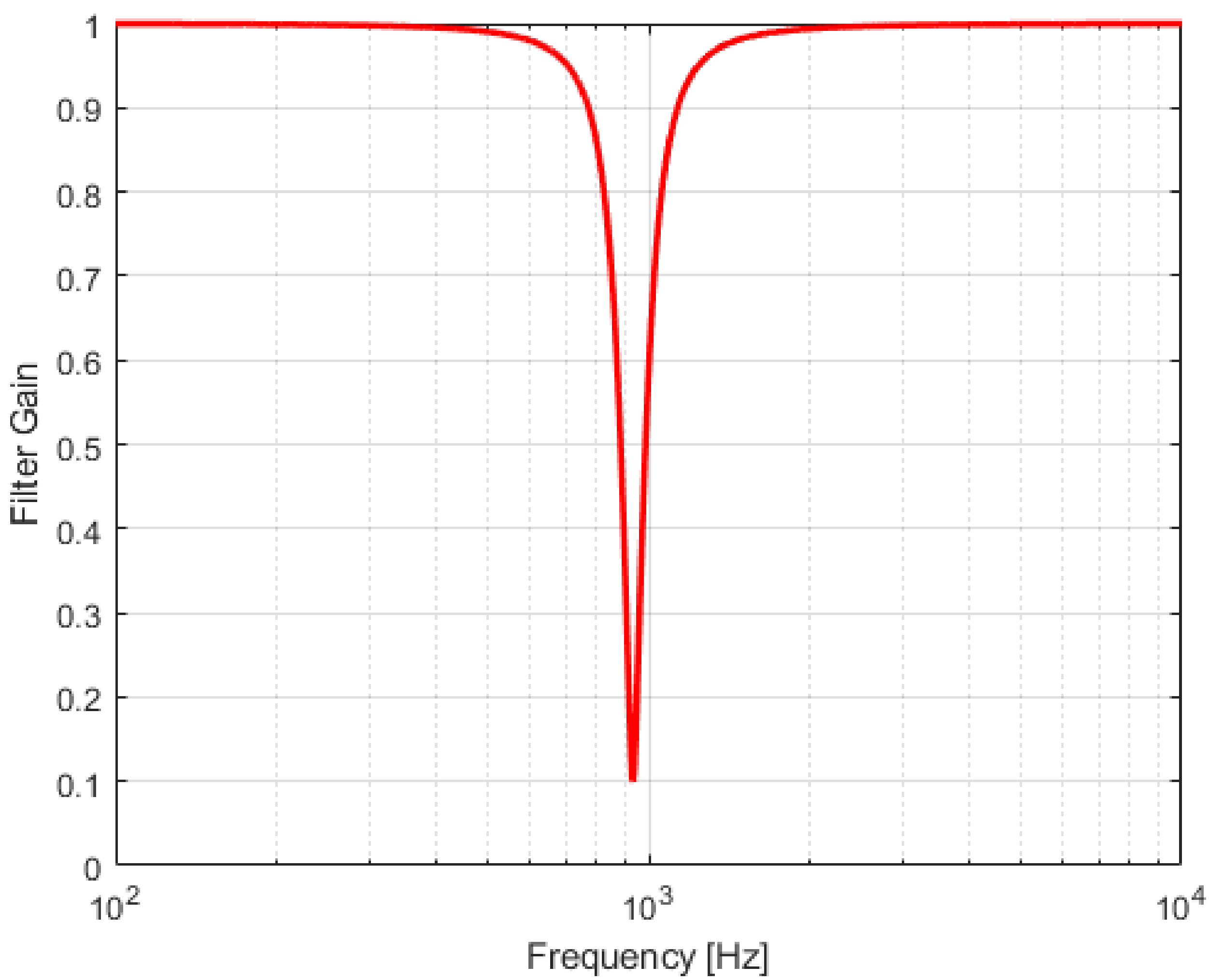
$(2\pi\sqrt{LC})^{-1}$ = 1 kHz ; $\sqrt{L/C}$ / R = 6.3



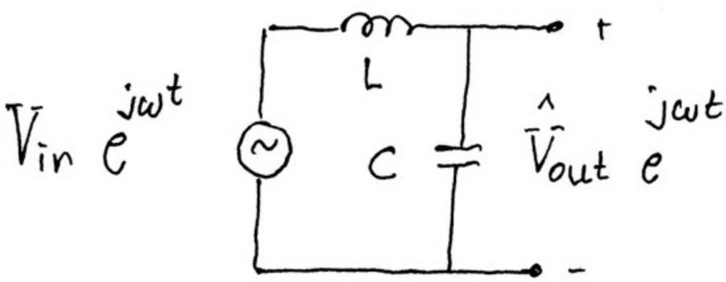
Note that the notch does not go to zero due to the parasitic series resistance of the inductor.

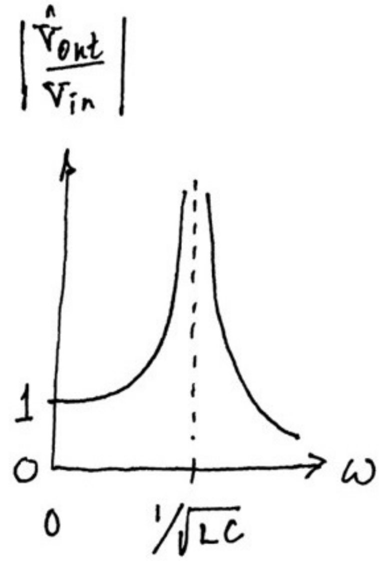
Numerical: Series-LC BSF





Resonators





$$\frac{\hat{\bar{V}}_{out}}{\bar{V}_{in}} = \frac{1/j\omega C}{j\omega L + 1/j\omega C} = \frac{1}{1-\omega^2 LC}$$

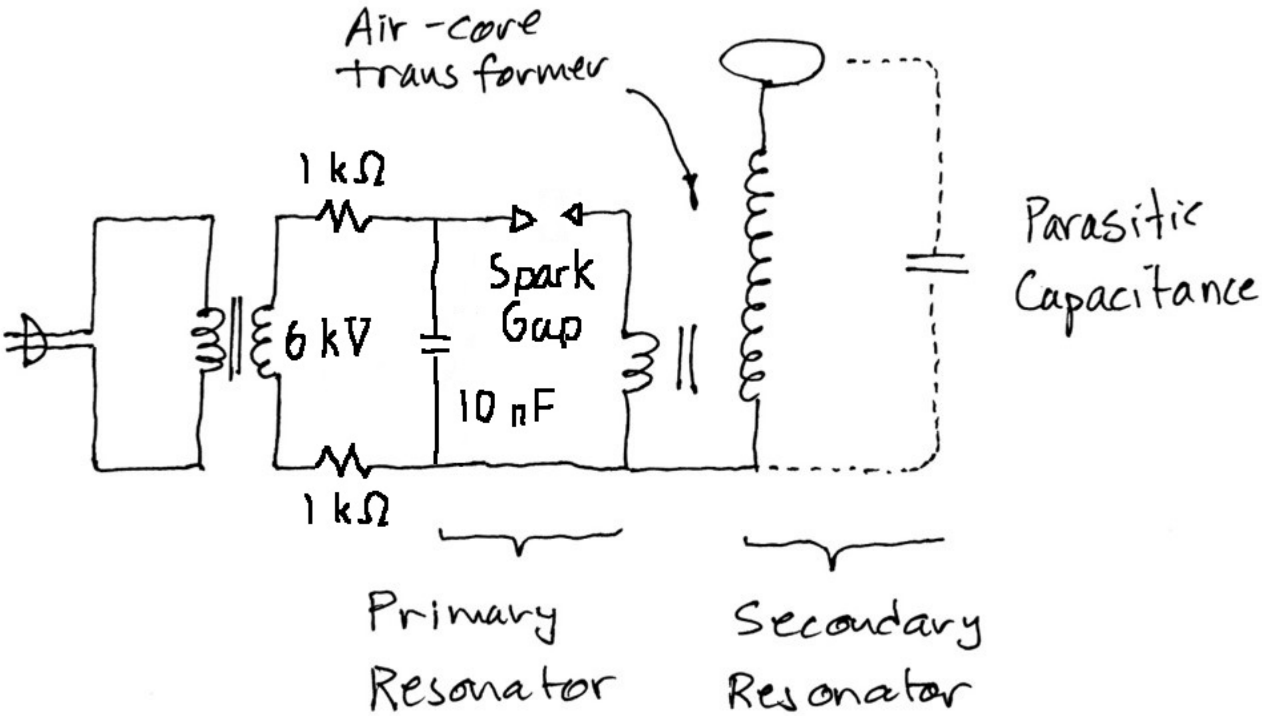
Peak voltage limited by parasitic resistances.

Applications include:

- capacitive shunts along inductive power-grid transmission lines;
- Tesla coil (dual tuned resonator).

Demo: Tesla Coil

The Tesla Coil employs two coupled resonators, with both resonators tuned to the same frequency.



The resistors protect the transformer, which charges the capacitor (+/-) at 120 Hz. When the capacitor voltage gets high enough, the spark gap sparks, and becomes a short allowing the LC resonator to oscillate, driving the rest of the Tesla coil.

Demo: Tesla Coil