# 6.200 - Lecture 24

Active “LC” Filters

- Successful LC Filters
- Active Filter Motivation
- Sallen-Key Topologies
- Examples

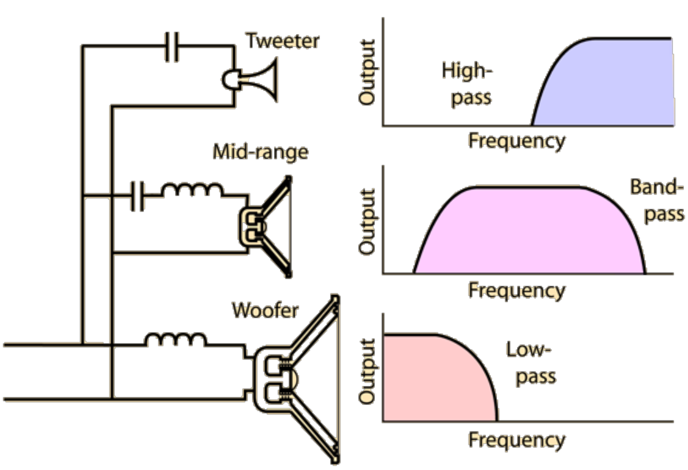
# Successful LC Filters

## Power System Inductor & Capacitor Banks

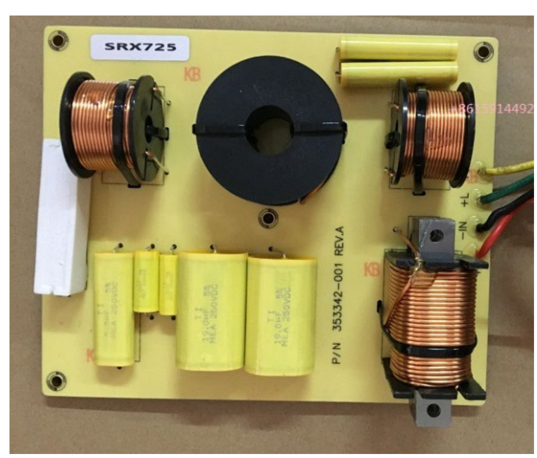
eaton.com

https://electrical-engineering-portal.com

## Audio Crossover Networks

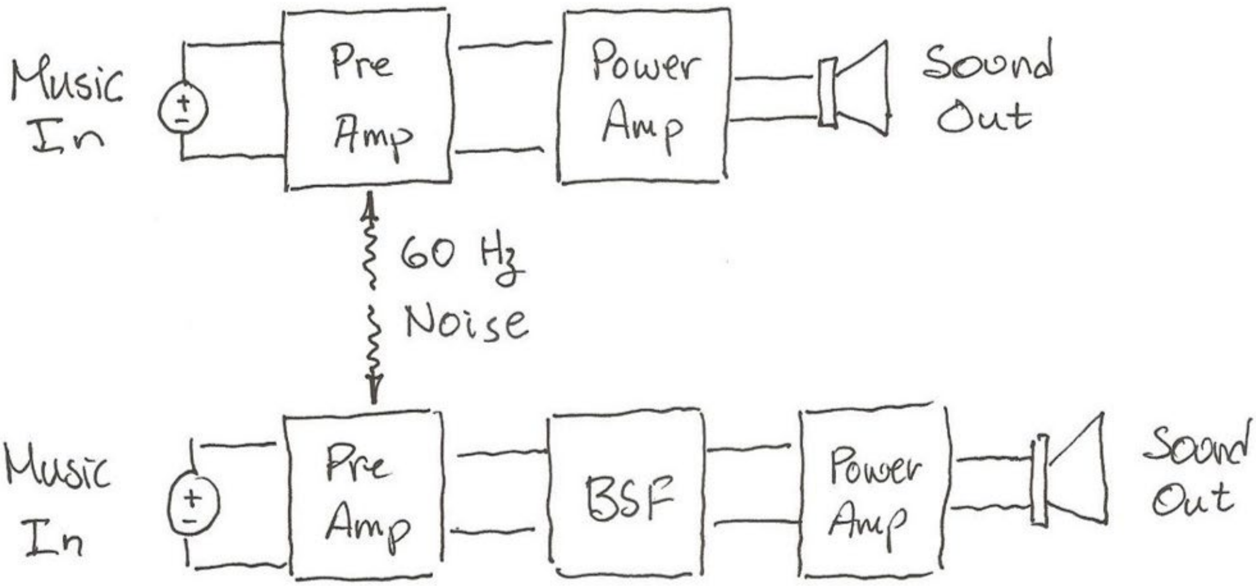



http://hyperphysics.phy-astr.gsu.edu

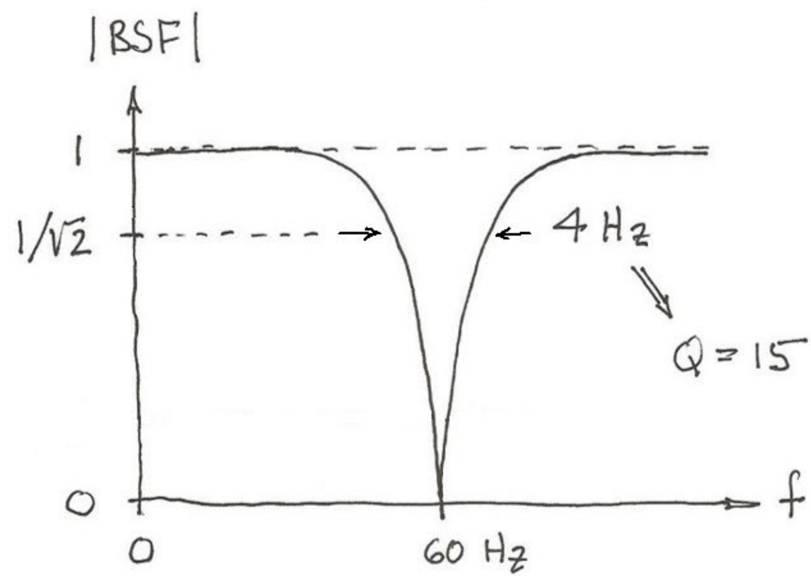



guangshou shengda audio

# Motivating Example

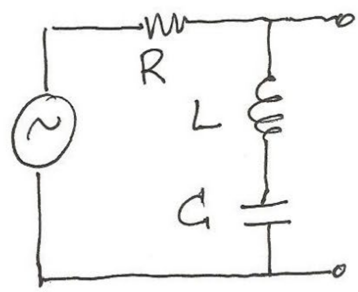



(Best is to eliminate the noise itself.
Next best is to employ a bandstop filter.)



# Passive Band-Stop Filter

Design




$\omega_0 = 1/\sqrt{LC}$

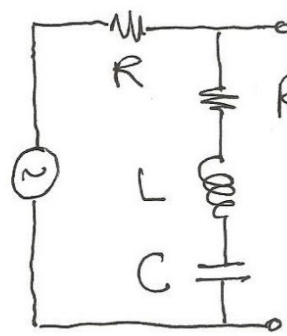
$Z_0 = \sqrt{L/C}$

$Q = Z_0/R$

$L = 1\,H$ $\quad C = 7\,\mu F$ $\quad R = 25\,\Omega$

$\omega_0 = 377\ \frac{rad}{s} = 60\ Hz$ $\quad Z_0 = 377\,\Omega$ $\quad Q = 15$

Reality




$R_L = 43\,\Omega$

$$\text{Minimum Gain} = \frac{43}{43+25} = 0.63$$

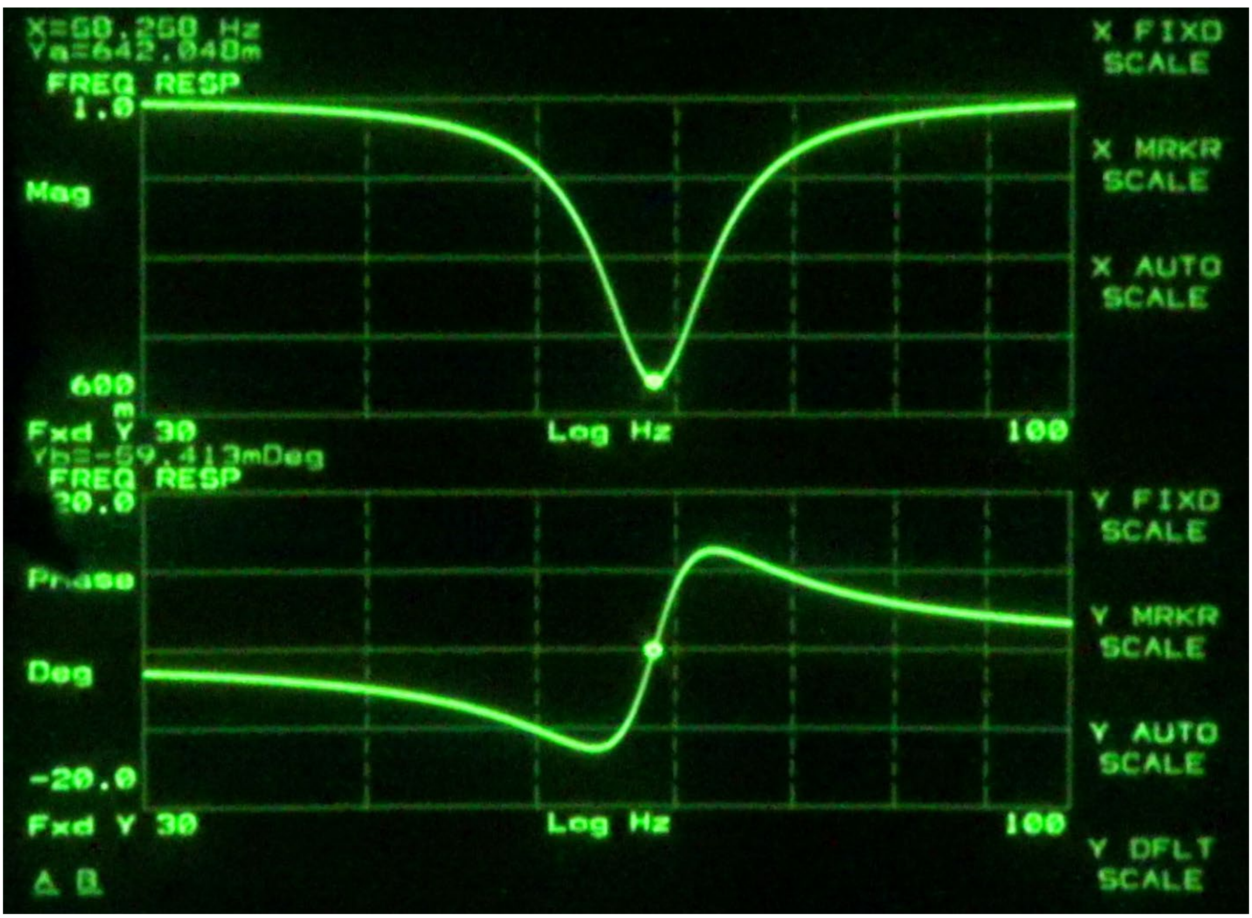
$$Q \approx Z_0/(R+R_L) = 5.5$$

# Demo

L = 1 H ; C = 7 μF ; R = 25 Ω (Hopefully)

$(2\pi\sqrt{LC})^{-1}$ = 60 Hz ; $\sqrt{L/C}$ = 377 Ω

Q = 15 (Hopefully)

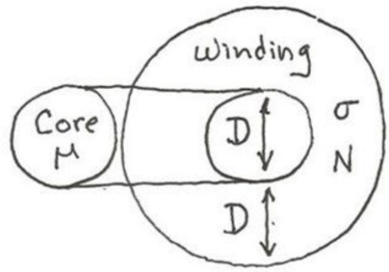



The DC resistance of the inductor is 43 Ω, so the stop band will drop only to 0.63 while Q drops to 5.5!

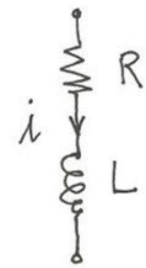
# Magnetics Scaling

Inductor


$\Rightarrow$




$\Rightarrow$




$$L = \frac{N^2 \mu \, \text{Area}}{\text{Length}} = \frac{N^2 \mu \, \pi D^2/4}{2 \pi D} = \frac{N^2 \mu D}{8}$$

$$R = \frac{N^2 \, \text{Length}}{\sigma \, \text{Area}} = \frac{N^2 \, 2 \pi D}{\sigma \, \pi D^2/4} = \frac{8 N^2}{\sigma D}$$

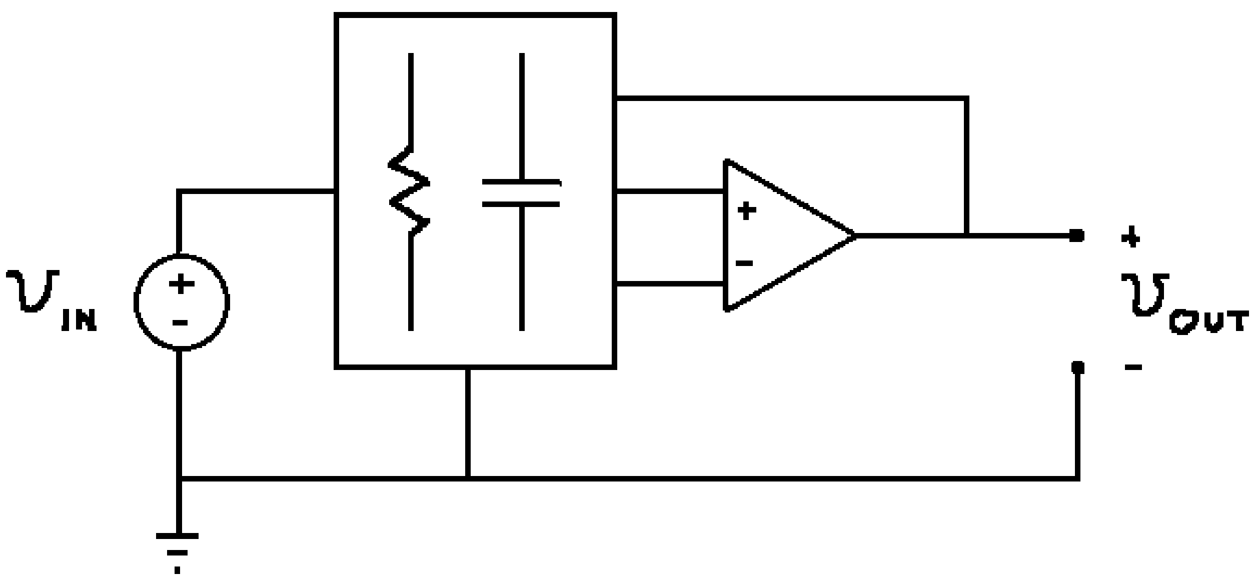
Sinusoidal Steady State $\Rightarrow$ $i = I \cos(\omega t)$

$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy Lost Per Cycle}} = 2\pi \frac{L I^2/2}{\frac{1}{2} I^2 R \frac{2\pi}{\omega}} = \frac{\omega L}{R} = \frac{\omega \mu \sigma D^2}{64}$$

$Q \sim D^2 \Rightarrow$ { Bigger is better / Smaller is worse

# Sallen-Key-Style Filters

R. P. Sallen and E. L. Key, “A Practical Method of Designing RC Active Filters”, *IRE Circuit Theory*, 2:1, 74-85, March 1955

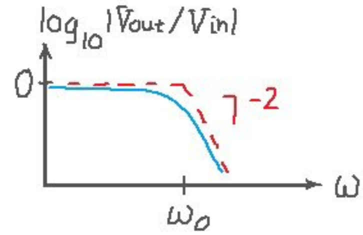



- Simple second-order filters
- Low cost and small size
- “Insensitive” to component variations
- Low power and power handling
- Difficult to tune

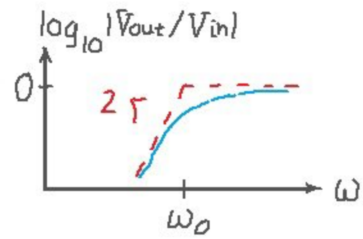
# Unity-Gain Second-Order Filters

$$s \equiv j\omega \quad \longleftrightarrow \quad d/dt$$

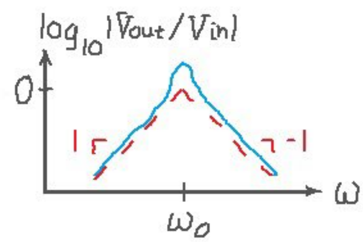
LPF: $$\frac{V_{out}}{V_{in}} = \frac{\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$



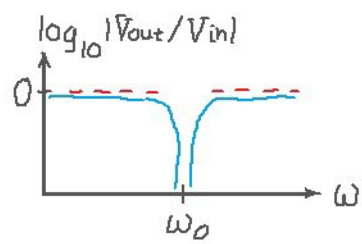

HPF: $$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + s\omega_0/Q + \omega_0^2}$$




BPF: $$\frac{V_{out}}{V_{in}} = \frac{s\omega_0/Q}{s^2 + s\omega_0/Q + \omega_0^2}$$



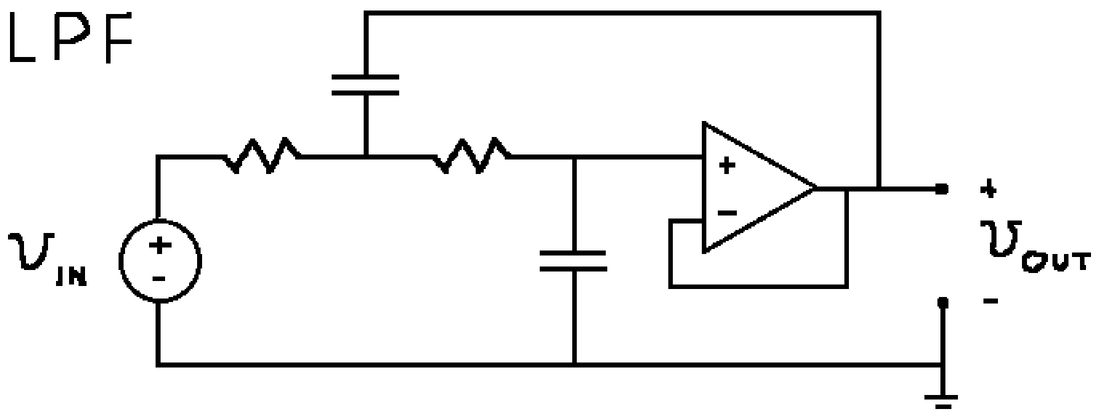

BSF: $$\frac{V_{out}}{V_{in}} = \frac{s^2 + \omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$



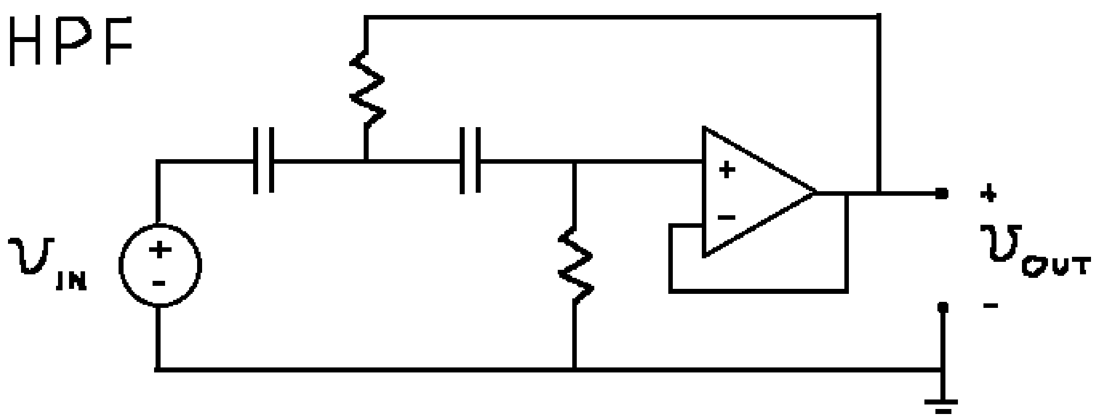

$$\ddot{V}_{OUT} + \frac{\omega_0}{Q}\dot{V}_{OUT} + \omega_0^2 V_{OUT} = \underbrace{\ddot{V}_{IN} + \frac{\omega_0}{Q}\dot{V}_{IN} + \omega_0^2 V_{IN}}_{\text{Selected by filtering type}}$$

# Filter Examples

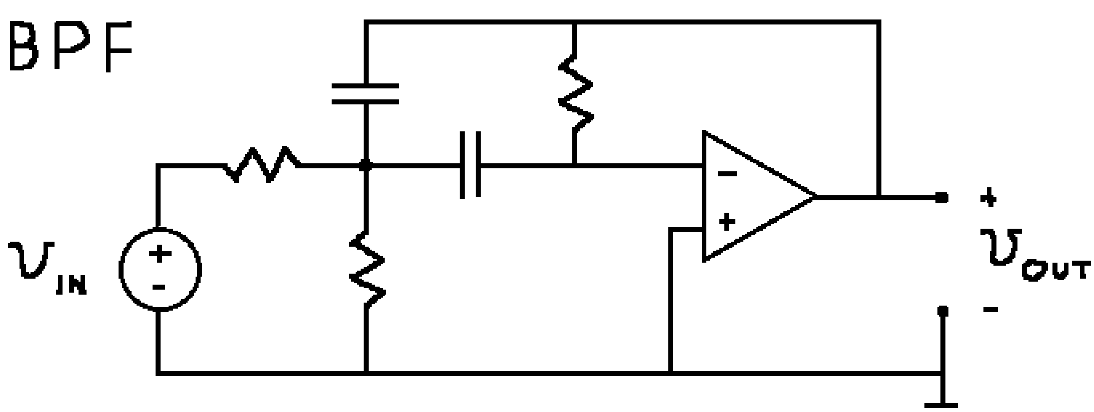
LPF



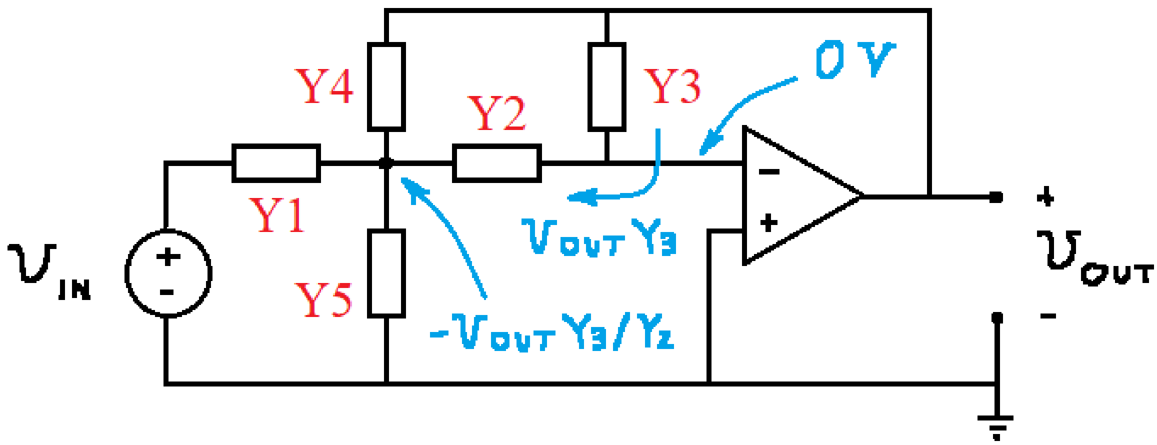

HPF




BPF



# Inverting BPF Design

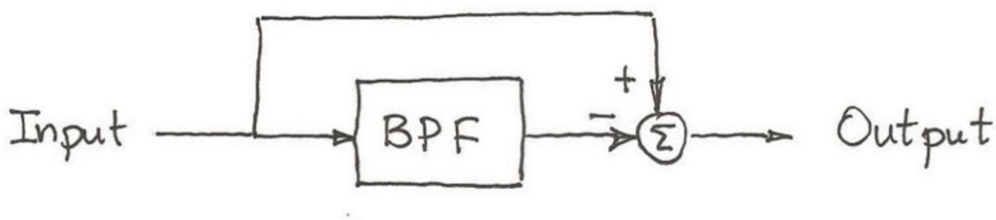


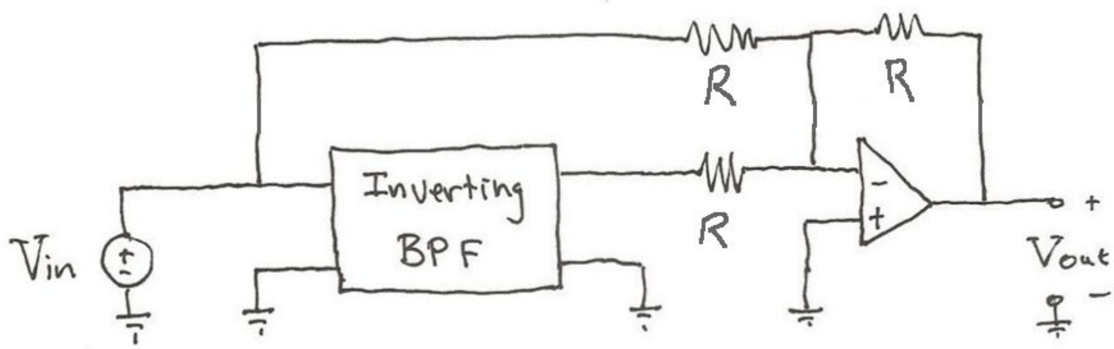

$$\text{KCL} \Rightarrow 0 = \left(V_{IN} + V_{OUT} Y_3/Y_2\right) Y_1 + \left(V_{OUT} Y_3/Y_2\right) Y_5$$
$$+ \left(V_{OUT} + V_{OUT} Y_3/Y_2\right) Y_4 + \left(V_{OUT}\right) Y_3$$

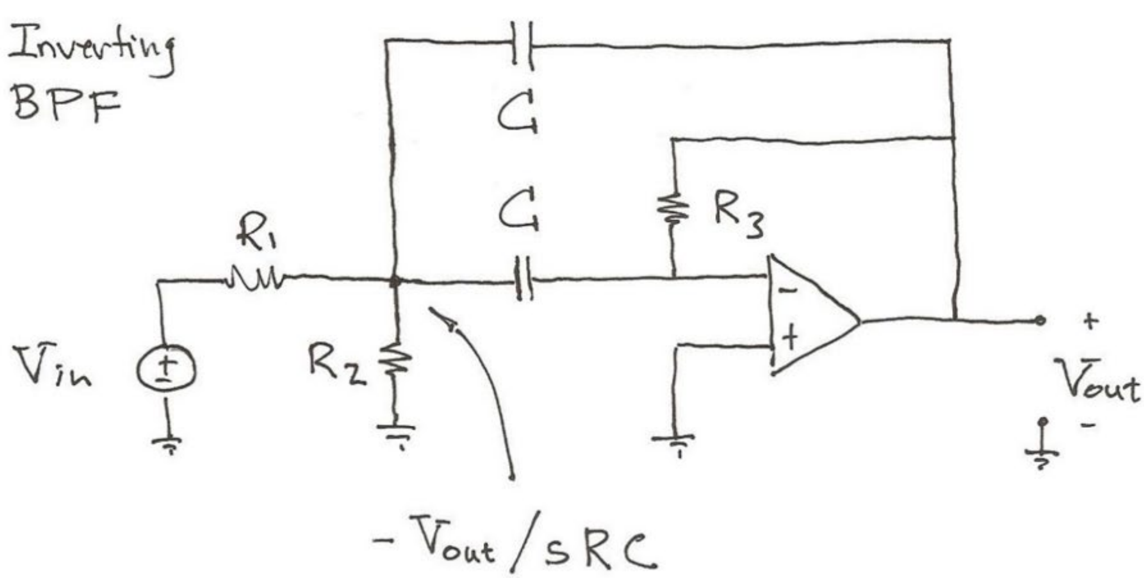
$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{-Y_1 Y_2}{Y_1 Y_3 + Y_2 Y_3 + Y_2 Y_4 + Y_3 Y_4 + Y_3 Y_5}$$

- To get $s^1$ in the numerator choose Y1 = 1/R1 and Y2 = s·C1
- To bias the op amp, choose Y3 = 1/R3
- Now Y1·Y3 gives $s^0$, and Y2·Y3 gives $s^1$, in the denominator
- To get $s^2$ in the denominator choose Y4 = s·C2
- Arbitrarily choose Y5 = 1/R2
- Could repeat with Y1 = s·C and Y2 = 1/R

# BSF Design


Input
BPF
+
−
Σ
Output


R
R
Inverting
BPF
R
−
+
Vin
Vout
+
−


Inverting
BPF
C
C
R3
R1
Vin
R2
−
+
+
Vout
−
− Vout / sRC

# Inverting BPF Analysis

$$\text{KCL} \Rightarrow \quad \frac{\bar{V}_{in} + \dfrac{\bar{V}_{out}}{sR_3C}}{R_1} + \frac{\dfrac{\bar{V}_{out}}{sR_3C}}{R_2} + \frac{\bar{V}_{out}}{sR_3C} sC + \left[ \bar{V}_{out} + \frac{\bar{V}_{out}}{sR_3C} \right] sC = 0$$

$$\bar{V}_{in} \left[ \frac{s}{R_1C} \right] + \bar{V}_{out} \left[ s^2 + \frac{s}{R_3C} + \frac{s}{R_3C} + \frac{1}{R_1R_3C^2} + \frac{1}{R_2R_3C^2} \right] = 0$$

$$\bar{V}_{out} \left[ s^2 + \underbrace{\frac{2}{R_3C}}_{\omega_o/Q} s + \underbrace{\frac{R_1+R_2}{R_1R_2R_3C}}_{\omega_o^2} \right] = -\bar{V}_{in} \left[ \underbrace{\frac{1}{R_1C}}_{G\,\omega_o/Q} s \right]$$

$$\omega_o^2 = \frac{R_1+R_2}{R_1R_2R_3C^2} \qquad R_1 = \frac{Q}{G\,\omega_o C}$$

$$Q = \frac{1}{2}\sqrt{\frac{(R_1+R_2)R_3}{R_1R_2}} \qquad R_2 = \frac{Q}{[2Q^2 - Q]\,\omega_o C}$$

$$G = \frac{R_3}{2R_1} \qquad R_3 = \frac{2Q}{\omega_o C}$$

# Demo Design

$\omega_0 = 2\pi \times 60$ Hz

$Q = 15$

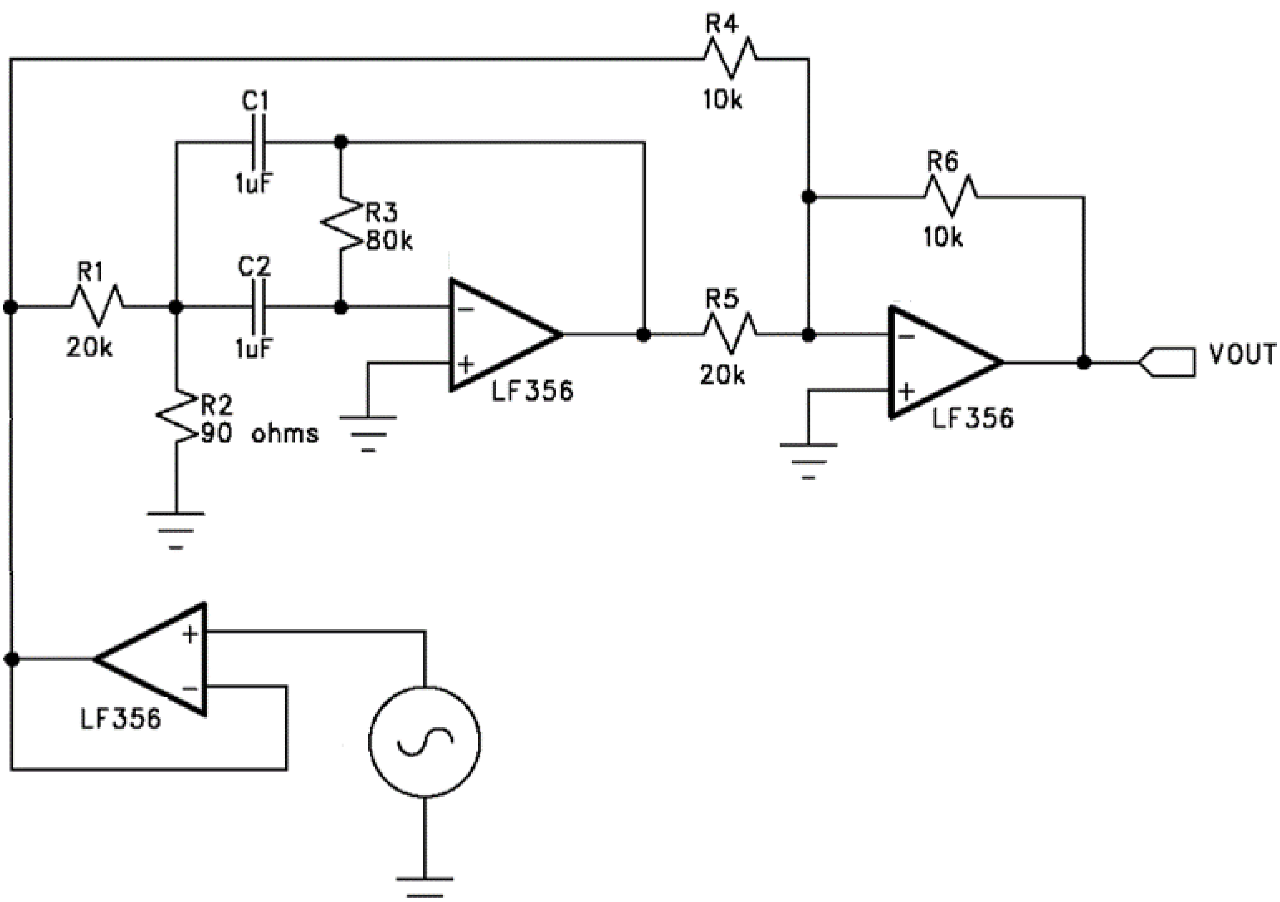
$G = 2$

$C = 1\ \mu F$

R1 = 20 kΩ

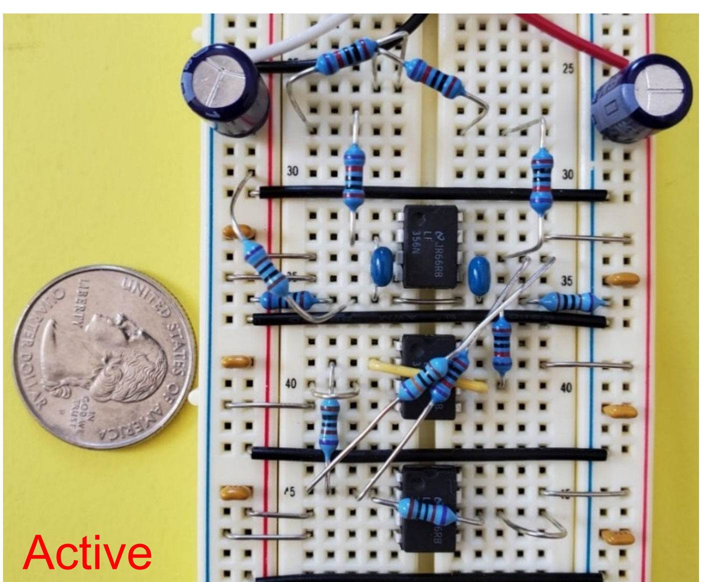
R2 = 90 Ω

R3 = 80 kΩ



# Demo

X=56.1778 Hz
Ya=-25.361 dB
FREQ RESP
0.0
dB
-30.0
0.054
Fxd Y 40
Log Hz
80
Yb=157.672 Deg
FREQ RESP
260
Phase
Deg
100
Fxd Y 40
Log Hz
80
Measurement Complete
A B
X FIXD SCALE
X MRKR SCALE
X AUTO SCALE
Y FIXD SCALE
Y MRKR SCALE
Y AUTO SCALE
Y DFLT SCALE

Active


D9301 X
100 M.H.PER STEP
DEC.INDUCTOR
300 M.A.MAX
Passive