Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.200 – Circuits & Electronics
Spring 2026

Final Exam

20 May 2026

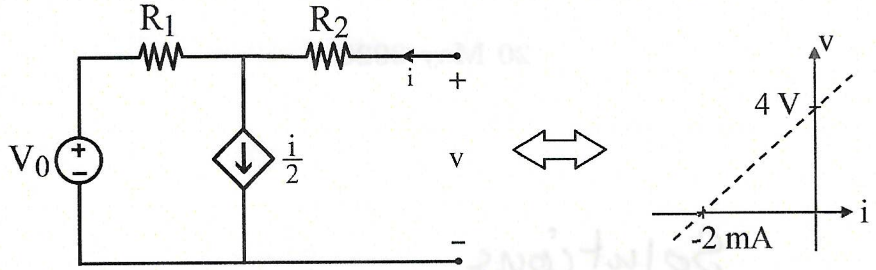
Name: Solutions

MIT EMail: ______________________ @MIT.EDU

- There are 19 pages in this quiz, including this cover page.
- Please put your name and MIT EMail ID in the spaces provided above.
- Please do not remove any pages from this quiz.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page.
- *This exam cannot be re-graded. Therefore, do your work in a neat and organized manner so that it can be easily understood. When finished with each part, clearly write your answer for that part into the corresponding answer box or graph.*
- *All numerical answers require proper units.*
- *In order to guarantee receipt of full credit, all answers should be justified by supporting math and/or explanations.*
- This is a closed-book closed-electronics quiz but a single two-sided page of notes is allowed.
- Good luck!

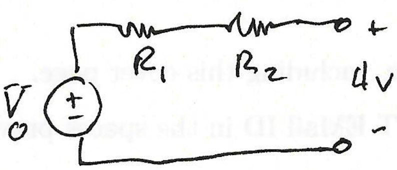
Problem 1: Unknown Parameters - 10%

The network shown below comprises an independent voltage source, a dependent current source, and two resistors. All devices but the dependent current source have unknown values. The network also has a port at which the i-v relation is as shown below. Finally, it is known that R_1 dissipates 2 mW when $v = 0$. Using the information given, determine the unknown values V, R_1 and R_2.



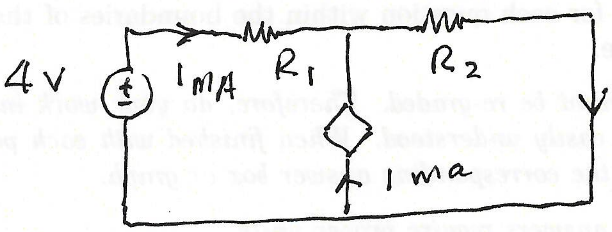
$V_0 = 4$ V	$R_1 = 2$ kΩ	$R_2 = 1$ kΩ

With $i = 0$:



$\Rightarrow V_0 = 4V$

With $v = 0$:



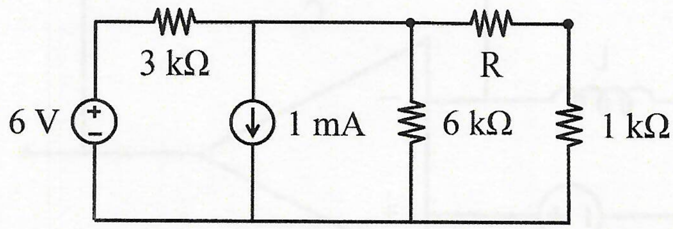
R_1 power $= (1\,mA)^2 R_1 = 2\,mW \Rightarrow R_1 = 2k\Omega$

and the R_1 voltage drop is 2 V. So

$R_2 = 2V / 2\,mA = 1\,K\Omega$

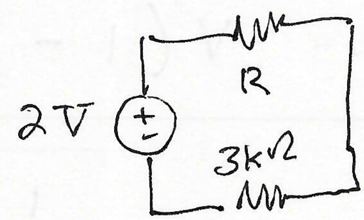
Problem 2: Maximum Power Transfer - 10%

The network shown below contains a resistor having an unknown resistance R. Determine the numerical value of R that maximizes the power dissipated in that resistor. Additionally, determine the maximized power.



$R =$ 3 kΩ	Power = $\frac{1}{3}$ mW

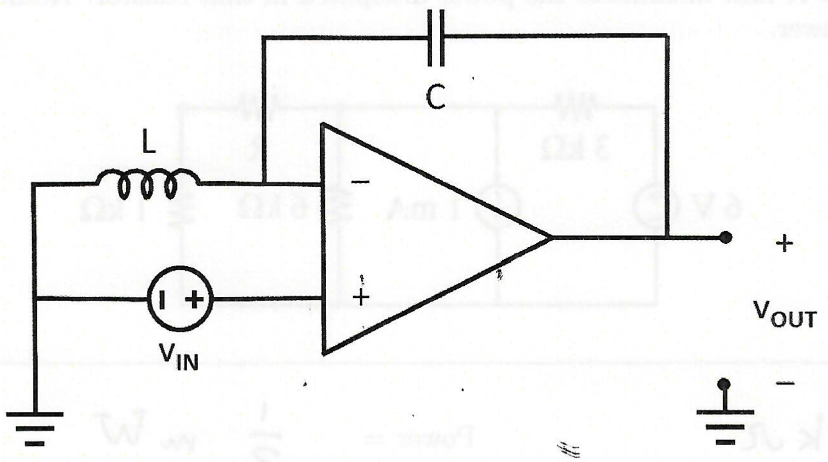
Thevenin equivalence =>



So, for maximum power, $R = 3\,k\Omega$ and the power is $\left(\frac{2V}{2}\right)^2 / 3\,k\Omega = \frac{1}{3}\,mW$.

Problem 3: Op Amp With LC Twice - 15%

Both parts of this problem concern the circuit shown below in which the op amp is ideal.



(3A) Let $v_{IN} = V\cos(\omega t)$. Further, assume that v_{OUT} has a zero-valued time average. Determine v_{OUT} in the sinusoidal steady state.

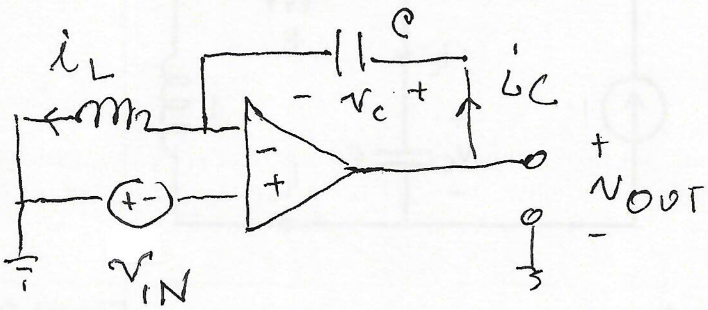
$v_{OUT} = V\left(1 - \frac{1}{\omega^2 LC}\right)\cos(\omega t)$

$$\tilde{V}_{out} = \frac{j\omega L + \frac{1}{j\omega C}}{j\omega L} V_{in} = \left(1 - \frac{1}{\omega^2 LC}\right) V_{in}$$

$$\Rightarrow v_{OUT} = V\left(1 - \frac{1}{\omega^2 LC}\right)\cos(\omega t)$$

(3B) Assume that for $t < 0$, $V_{IN} = 0$ so that the capacitor voltage and inductor current are both zero. Then, at $t = 0$, V_{IN} steps up to $V_{IN} = V$ where V is constant. Determine v_{OUT} for $t \geq 0$.

$$\boxed{v_{OUT} = V\left(1 + \frac{t^2}{2LC}\right)}$$



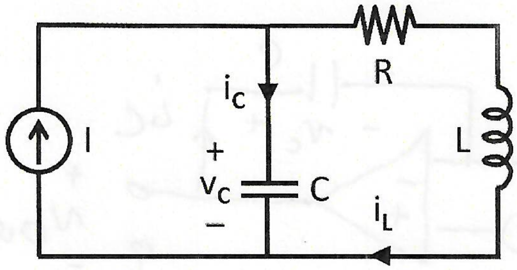
$$V = v_{IN} = v_+ = v_- = L\frac{di_L}{dt} \Rightarrow i_L = i_C = \frac{Vt}{L}$$

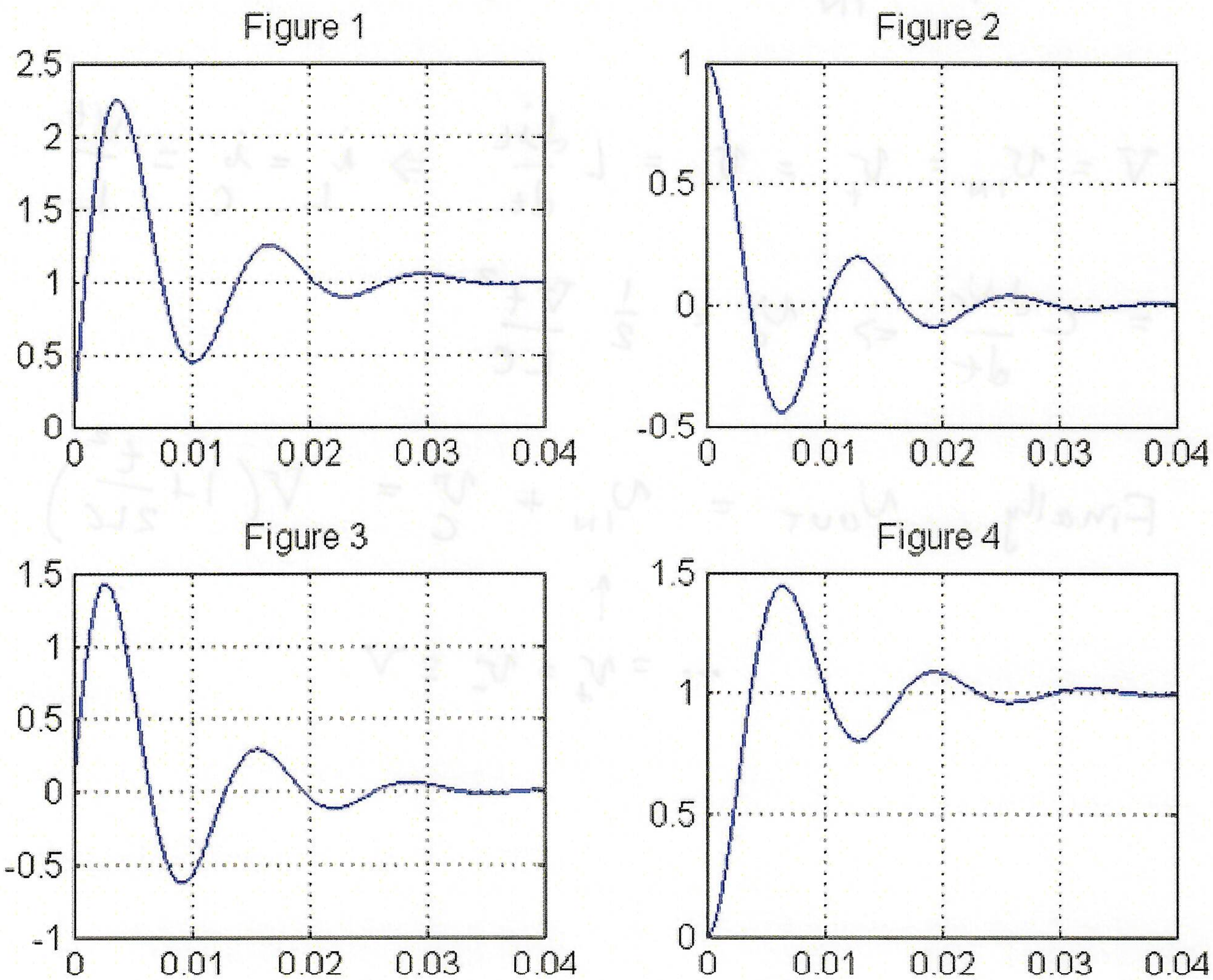
$$= C\frac{dv_C}{dt} \Rightarrow v_C = \frac{1}{2}\frac{Vt^2}{LC}$$

$$\text{Finally, } v_{OUT} = \underset{\substack{\uparrow \\ \ldots = v_+ = v_- \equiv V}}{v_{IN}} + v_C = V\left(1 + \frac{t^2}{2LC}\right)$$

Problem 4: RLC Transient - 25%

The circuit shown below is initially at rest such that $v_C = 0$ and $i_L = 0$ for $t < 0$. At $t = 0$, the current source steps up to $I = 1$ A. The figures shown below plot four waveforms within the circuit following the current step. Their horizontal axes all show time measured in seconds [s]. Their vertical axes show either voltage measured in Volts [V] or current measured in Amperes [A], as appropriate. *Note that the initial slopes of the waveforms in Figures 1 and 3 are positive, while the initial slopes of the waveforms in Figures 2 and 4 are zero.*





(4A) For each variable listed below, identify the corresponding figure by circling the figure number. *Briefly explain your reasoning in the space below.*

v_C	$\rightarrow$	Figure	1	2	3	4
i_C	$\rightarrow$	Figure	1	2	3	4
i_L	$\rightarrow$	Figure	1	2	3	4

Explain:

At $t=0$, the source current goes entirely through the capacitor. Only Figure 2 shows this behavior. At $t=0$, therefore, the capacitor voltage increases linearly. As $t \to \infty$ it settles to $R \cdot 1A$. Only Figure 1 shows this behavior. At $t=0$, the inductor current is proportional to the integral of the capacitor voltage, and hence grows quadratically. It settles to $1A$ as $t \to \infty$. Only Figure 4 shows this behavior. Figure 3 is the inductor voltage.

(4B) Estimate the value of R.

$R = 1\,\Omega$

$v_c(\infty) = R \cdot 1A$. From Figure 1, $R = 1\Omega$

(4C) Estimate the value of C.

$C = 0.001$ F

The figures show that about 1.5 cycles of oscillation take approximately 20 ms. Therefore

$2\pi\sqrt{LC} \times 1.5 \approx 20$ ms or $\sqrt{LC} \approx 2$ ms.

Combining 4B and 4E, $\sqrt{\frac{L}{C}} \approx 2\,\Omega$. Thus,

$C \approx \frac{2\,ms}{2\,\Omega} = 0.001$ F

(4D) Estimate the value of L.

$L =$ 0,004 H

Following 4C, $L \approx 2\,\text{ms} \cdot 2\,\Omega \approx 0.004$ H

(4E) Estimate the quality factor Q of the circuit.

$Q =$ 2

It takes approximately Q cycles of oscillation for a signal to drop to 4 % of its original amplitude. Thus, it takes about $Q/2$ cycles to drop to 20 %. From the figures all signals drop to about 20 % of their original amplitude in just one cycle. Therefore, $Q \approx 2$. Note that $Q = \sqrt{L/C}/R$ for a series resonator.

(4F) For each statement below, circle the correct completion. *Briefly explain your reasoning in the space below.*

- If the resistance R is increased, the quality factor Q will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

- If the capacitance C is increased, the quality factor Q will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

- If the inductance L is increased, the quality factor Q will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

Explain: $Q = \frac{\sqrt{L/C}}{R}$

(4G) For each statement below, circle the correct completion. *Briefly explain your reasoning in the space below.*

- If the resistance R is increased, the period of oscillation will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

- If the capacitance C is increased, the period of oscillation will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

- If the inductance L is increased, the period of oscillation will ⋯

⋯ increase.	⋯ decrease.	⋯ remain unchanged.

Explain:

The oscillations come from the natural (homogeneous) response of the network.

$$KVL \Rightarrow 0 = v_C + R\left(C\frac{dv_C}{dt}\right) + L\frac{d}{dt}\left(C\frac{dv_C}{dt}\right)$$

$$\Rightarrow \ddot{v}_C + \frac{R}{L}\dot{v}_C + \frac{1}{LC}v = 0$$

$$v_C \sim e^{st} \Rightarrow s = -\frac{R}{2L} \pm j\underbrace{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}_{\omega_d}$$

$$\text{Period} = \frac{2\pi}{\omega_D}$$

(4H) After a long time T, the current source steps down to turn off. For each statement below concerning the subsequent decay of stored energy, circle the correct completion. *Briefly explain your reasoning in the space below.*

- If the resistance R is increased, the time at which the stored energy falls to half its value at time T will ···

··· increase.	··· decrease.	··· remain unchanged.

- If the capacitance C is increased, the time at which the stored energy falls to half its value at time T will ···

··· increase.	··· decrease.	··· remain unchanged.

- If the inductance L is increased, the time at which the stored energy falls to half its value at time T will ···

··· increase.	··· decrease.	··· remain unchanged.

Explain:

From 4G, $s = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

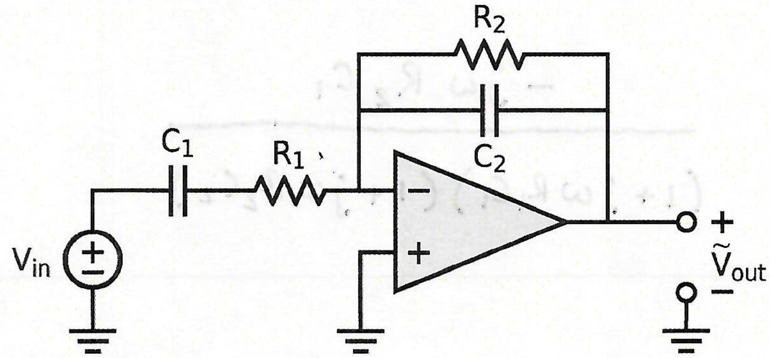
$\uparrow$

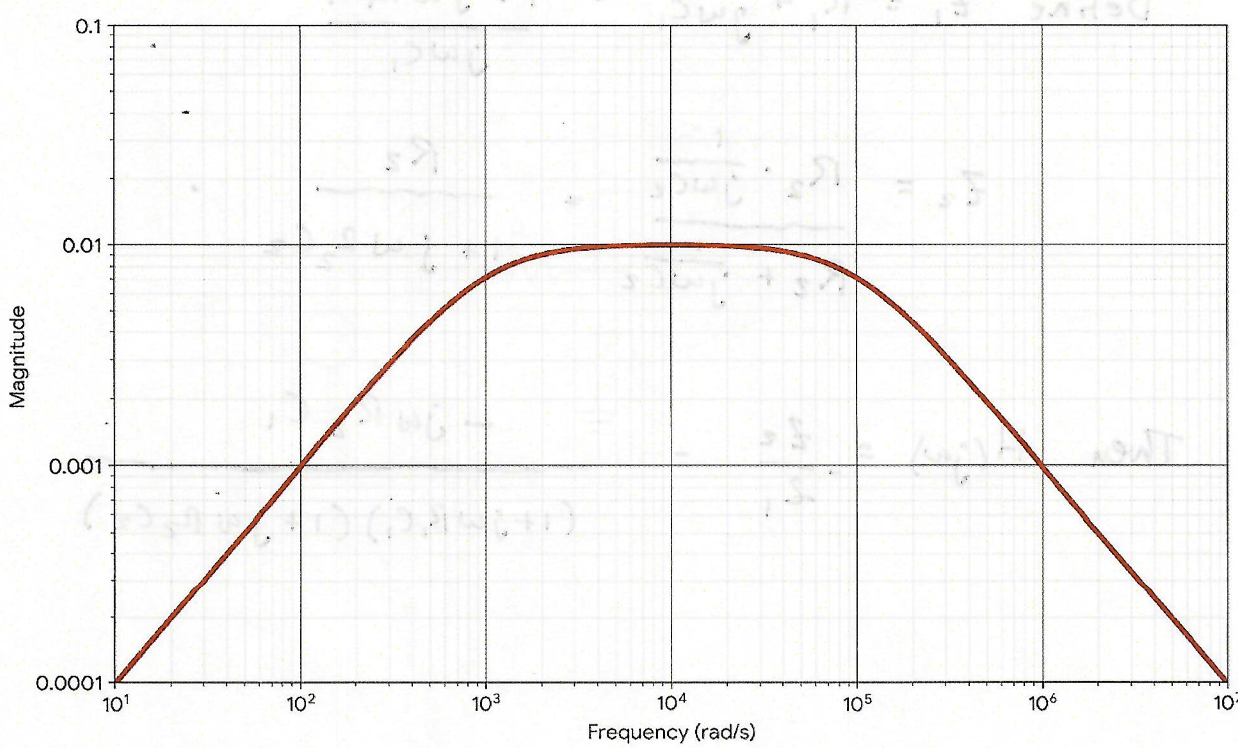
Decay rate

Decay time $\sim \frac{L}{R}$

Problem 5: RC Filter - 20%

This problem concerns the filter shown below together with a measurement of its transfer function.





(5A) Determine the symbolic transfer function $H(j\omega) \equiv \tilde{V}_{\text{out}}/\tilde{V}_{\text{in}}$ in terms of R_1, R_2, C_1, and C_2 where $\tilde{V}_{\text{in}}$ and $\tilde{V}_{\text{out}}$ are the complex amplitudes of $v_{\text{IN}}(t)$ and $v_{\text{OUT}}(t)$, respectively, when operating in the sinusoidal steady state.

$$\boxed{H(j\omega) = \frac{-j\omega R_2 C_1}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}}$$

Define $Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1+j\omega R_1 C_1}{j\omega C_1}$

$$Z_2 = \frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1+j\omega R_2 C_2}.$$

Then $H(j\omega) = -\frac{Z_2}{Z_1} = \frac{-j\omega R_2 C_1}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$

(5B) Assume that $C_1 = C_2 = 1\mu F$. Determine numerical values for R_1 and R_2 from the measured transfer function.

$R_1 =$ 1 KΩ	$R_2 =$ 10 Ω

Low frequencies $\Rightarrow$ $|H| = 10^{-5} \frac{s}{rad} \cdot \omega = \omega R_2 C_1$.

with $C_1 = 10^{-6}$ F, $R_2 = 10\,\Omega$.

High frequencies $\Rightarrow$ $|H| = 10^{3} \frac{rad}{s} / \omega = \frac{1}{R_1 C_2 \omega}$.

with $C_2 = 10^{-6}$ F, $R_1 = 1000\,\Omega$

(5C) Suppose the filter is excited with the input $v_{IN}(t) = \tilde{V}_{in}\cos(\omega t + \phi)$, where $\tilde{V}_{in} = 5$ V, $\omega = 10^4$ rad/s, and $\phi = \pi/4$ rad. Determine a numerical expression for $v_{OUT}(t)$.

$$V_{out} = -\frac{5\text{V}}{101}\cos\left(10^4 \frac{\text{rad}}{\text{s}} t + \frac{\pi}{4}\right)$$

At $\omega = 10^4 \frac{\text{rad}}{\text{s}}$, $\quad H(j\omega) = \dfrac{-j\,10^4\,10\,10^{-6}}{(1 + j\,10^4\,10^3\,10^{-6})(1 + j\,10^4\,10\,10^{-6})}$

$$= \frac{-0.1j}{(1+\frac{j}{10})(1+j10)} = -\frac{1}{101}$$

Problem 6: Filter Design - 20%

This objective of this problem is to design an op-amp-based band-pass filter that implements the transfer function $H(j\omega)$ given by

$$H(j\omega) = \frac{\tilde{V}_{\text{out}}}{\tilde{V}_{\text{in}}} = \frac{\pm G(j\omega)\omega_o/Q}{(j\omega)^2 + (j\omega)\omega_o/Q + \omega_o^2}$$

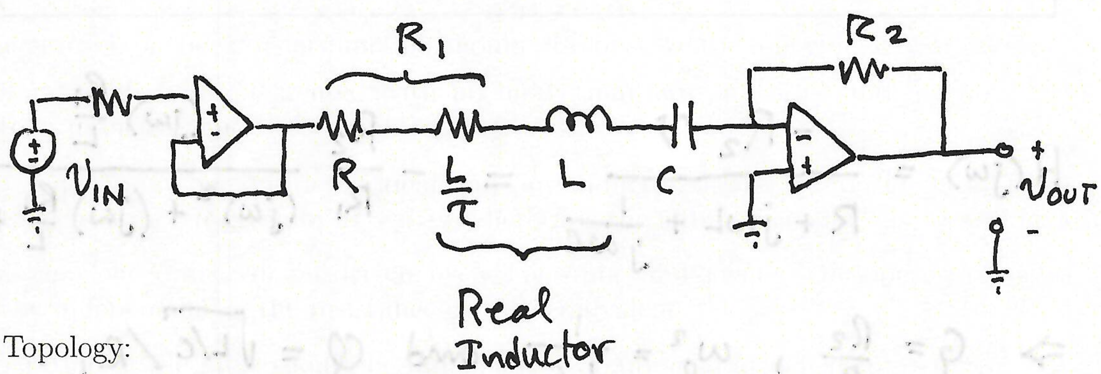
where $\tilde{V}_{\text{in}}$ and $\tilde{V}_{\text{out}}$ are the complex amplitudes of the sinusoidal-steady-state input and output voltages, respectively, and G, ω_o, and Q are pre-specified constants. The "$\pm$" in the transfer function indicates that both signs are acceptable.

The implementation of the transfer function should also observe the following constraints.

- The filter should be implemented with no more than two op amps, and no more than four capacitors, inductors and resistors in total.
- The op-amp(s) may be considered ideal, but any inductor should be considered imperfect due to a series resistance R given by $R = L/\tau$ where L is the inductance and τ is a known constant.
- Assume that the filter will be driven by a Thevenin equivalent. The operation of the filter should be independent of the resistance of that equivalent.
- The operation of the filter should be independent of any load attached to its output.

(6A) Draw the filter topology in space below, clearly labeling the symbolic component values.

Topology:



Note that the imperfect inductor precludes the use of a band-pass filter based on the parallel resonator.

(6B) Derive expressions for the component (capacitor, inductor, resistor) values in terms of G, ω_o, Q and τ. If the component values are under-specified, then pick one (or more) as being pre-specified until the remaining component values are uniquely specified. Be clear which component value is being specified below.

Component #1:	C	Value =	$1/L\omega_0^2$
Component #2:	R	Value =	$L\left(\frac{\omega_0}{Q} - \frac{1}{\tau}\right)$
Component #3:	R_2	Value =	$G\omega_0 L/Q$
Component #4:		Value =	
Under-specified component(s):	L		

$$H(j\omega) = \frac{-R_2}{R + j\omega L + \frac{1}{j\omega C}} = -\frac{R_2}{R_1} \frac{(j\omega)\frac{R_1}{L}}{(j\omega)^2 + (j\omega)\frac{R_1}{L} + \frac{1}{LC}}$$

$\Rightarrow$ $G = \frac{R_2}{R_1}$, $\omega_0^2 = \frac{1}{LC}$ and $Q = \sqrt{L/C}/R_1$.

R_1, R_2, L and C are underspecified. Choose L as the free variable so R becomes $R_1 - L/\tau$, or $R_1 = R + L/\tau$.

Then, $C = 1/L\omega_0^2$, $R_1 = \sqrt{L/C}/Q = \omega_0 L/Q = R + L/\tau$

$\Rightarrow$ $R = L\left(\frac{\omega_0}{Q} - \frac{1}{\tau}\right)$, and $R_2 = G R_1 = G\left(R + \frac{L}{\tau}\right)$

$\Rightarrow$ $R_2 = \frac{G\omega_0 L}{Q}$

(6C) Is the achievable Q limited given practical engineering constraints? If so, to what range? Circle the appropriate answer and provide a range if "Yes".

Yes	No	Range:	$Q < \omega_0 \tau$

$R = L\left(\frac{\omega_0}{Q} - \frac{1}{\tau}\right)$ must be positive to be a practical resistance. So, $\frac{\omega_0}{Q} - \frac{1}{\tau} > 0$

or $Q < \omega_0 \tau$