6.200 Final Exam

Fall 2024

Name:

Kerberos/Athena Username:

6 questions   3 hours

- Please **do not open this exam or the pink envelope at your table** until we tell you to begin.
- Write your name and kerberos **ONLY** on the front page.
- This exam is closed-book, but you may use one 8.5″ × 11″ sheet of handwritten notes (both sides) as a reference. This sheet must be **handwritten** directly on the page (not printed).
- You may **NOT** use any electronic devices (including computers, calculators, phones, etc.).
- If you have questions, please **come to us at the front** to ask them.
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit provided you indicate (near the answer box) where that work can be found.
- You may remove sheets from the exam if you wish, but we must receive **all** sheets with QR codes back from you at the end of the exam.
- **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their exams.
- You may not discuss the details of the exam with anyone other than course staff until final exam grades have been assigned and released.

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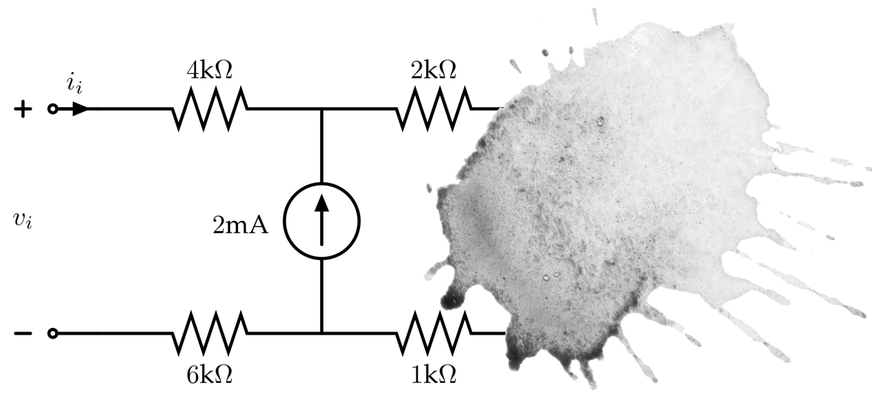
1 No Food or Drinks in the Lab

You're working late in the lab one night, designing a circuit consisting of only linear components, when you accidentally spill coffee on your desk, destroying part of your carefully-drawn schematic! Oh, no! Here is what you are able to recover:



Amidst the coffee-soaked mess, you are also able to recover two calculations you had previously performed:

- When $v_i = -3\text{V}$, $i_i = -1\text{mA}$.
- When $v_i = 23\text{V}$, $i_i = 1\text{mA}$.

Answer the following questions about this circuit.

1.1 Resistor

Could the obscured portion be a single constant resistor? Circle One: **Yes** or **No**

If yes, what is its resistance? If not, why not?

1.2 Voltage Source

Could the obscured portion be a single constant independent voltage source? Circle One: **Yes**or **No**

If yes, what is its voltage? If not, why not?

1.3 Current Source

Could the obscured portion be a single constant independent current source? Circle One: **Yes** or **No**

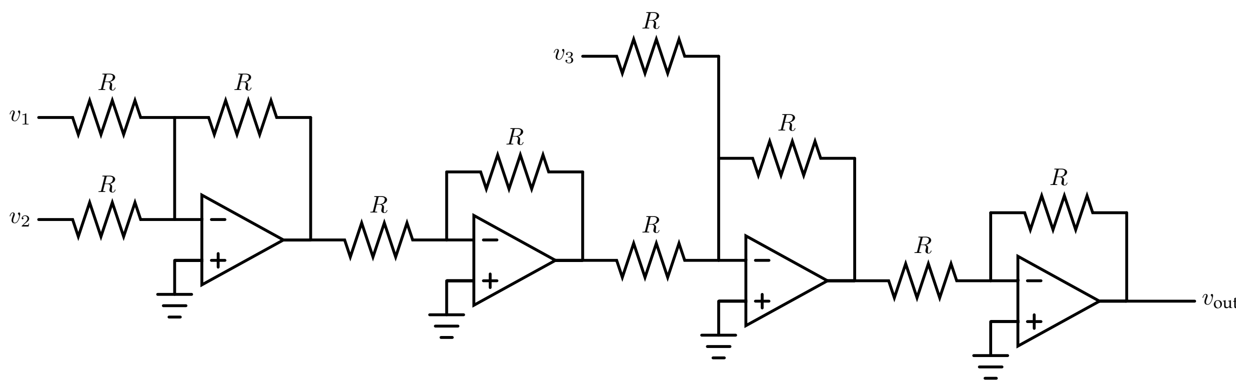
If yes, what is its current? If not, why not?

2 Op-Amps

In this problem, we'll consider four different circuit designs that all share the same desired input/output relationship. For all parts of this problem, make the ideal op-amp assumption and ignore limitations imposed by the power supply.

2.1 Circuit 1

Ben Bitdiddle designed the following circuit, which uses four op-amps to compute an output voltage v_{out} based on three input voltages v_1, v_2, and v_3:

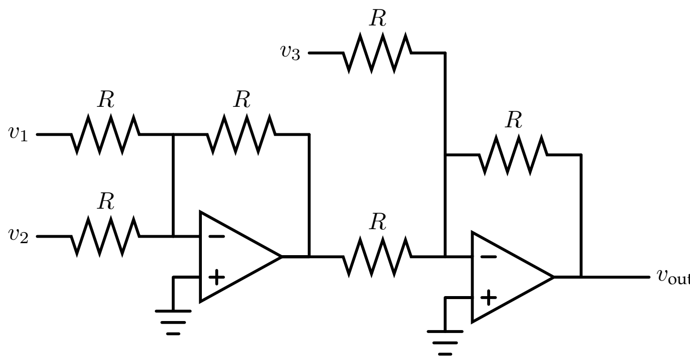


What calculation is this circuit performing? Enter your answer below by solving for v_{out} in terms of v_1, v_2, v_3, R, and/or any other constants you may need:

$v_{\text{out}} =$

2.2 Circuit 2

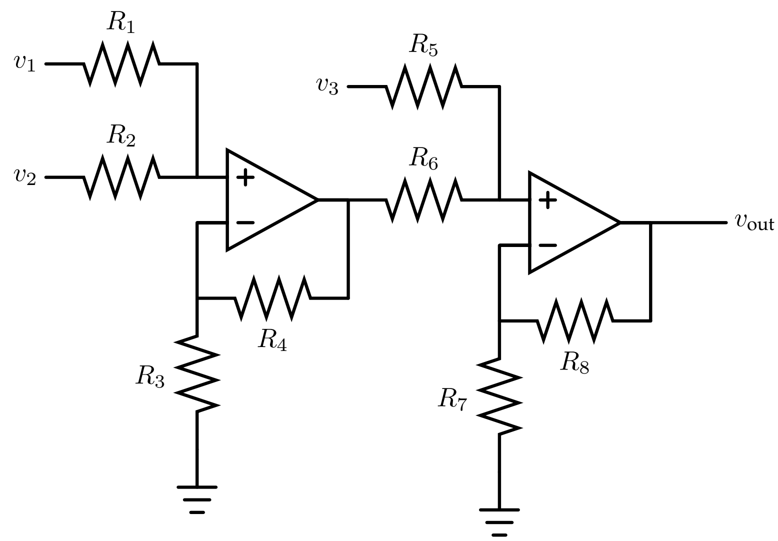
Ben's friend, Lem E. Tweakit, suggests that two of the op-amps in that design are unnecessary because their effects cancel each other out, so they propose the following design instead, claiming that it will compute the same result:



Does this circuit implement the same behavior as circuit 1? Include a brief justification for your answer.

2.3 Circuit 3

Their mutual acquaintance, Ivana de Bugyu, proposes a third alternative, which computes the same result as the design from part 1, just in a different way. Here is Ivana's design:



By choosing the right resistor values, this circuit can be made to have the same input/output relationship as the circuit in part 1. How would you set R_1 through R_8 to accomplish that?

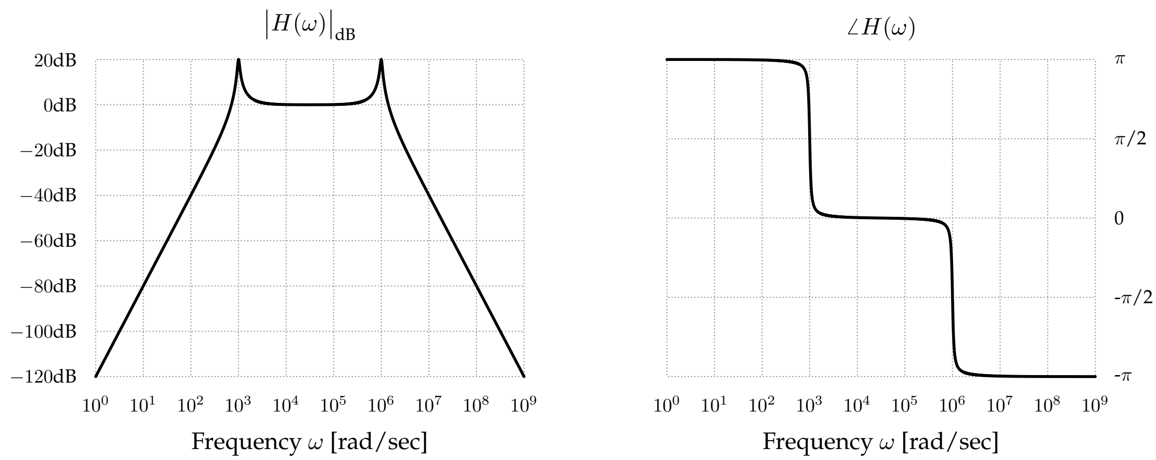
2.4 Circuit 4

It turns out that it is possible to implement the same behavior as circuit 1 by using only a single op-amp. In the box below, draw a schematic for such a circuit. Clearly label the inputs (v_1, v_2, and v_3), the output (v_{out}), and the values of any components in the circuit.

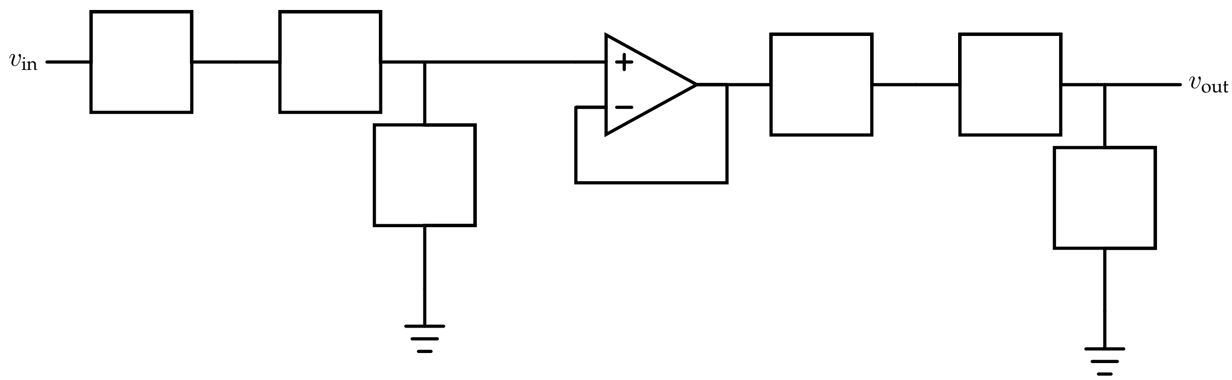
3 Meow

Playing around with ideas from the past few weeks of 6.200, it's possible to implement a kind of band-pass filter whose magnitude plot looks vaguely like the silhouette of a cat (as seen below). Note that this plot attenuates some frequencies, keeps some frequencies' amplitudes roughly the same, and amplifies some frequencies.

Whether this particular kind of frequency response is useful in practice is debatable, but it's kind of cute, so let's go ahead and think about how we would implement it. Here is the Bode plot of the filter we would like to implement:

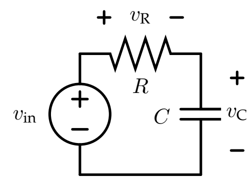


Show an implementation of this filter by drawing a single component in each box on the facing page. Clearly label the values associated with each component, *including units*.


v_{in}
v_{out}
+
−

4 RC Circuits

In this problem, we will consider the following familiar circuit:



On the last page of this exam (page 29, which you may remove if you wish) are four graphs of this circuit's sinusoidal steady state behavior, each depicting the circuit's behavior at a different frequency of input wave. Each graph shows the input wave v_{in} with a peak centered on the scope's display, as well as another curve v_{out} that resulted from measuring either v_R or v_C. Both of the curves are plotted on the same vertical scale.

Note that the input frequency is changing by a factor of 2 between each successive graph, but the time scale has been adjusted so that the input wave looks the same in each. Note also that the input frequency is explicitly specified for one of the graphs but not for the others.

Answer the following questions about this circuit, and provide a brief justification for your answer. For any of the questions, if there is not enough information to determine an answer, indicate that by writing "Not Enough Information" in the box, along with a justification for that answer.

4.1 Graph Ordering

Looking at the graphs on the last page of the exam, are they sorted in order of **increasing** or **decreasing** frequency of the input waveform? Enter your answer (or "Not Enough Information") in the box below, along with your justification.

4.2 Values

What are the values of R and C that were used in this circuit? Enter your answer (or "Not Enough Information") in the box below, along with your justification.

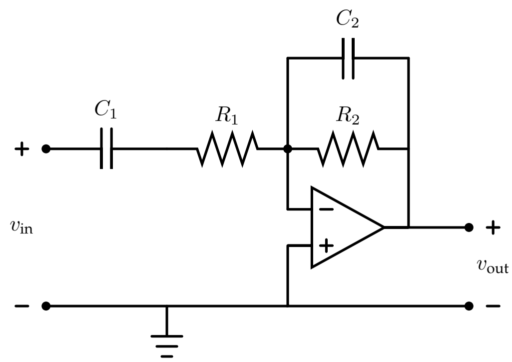
4.3 DC Response

Now consider the case where instead of sinusoidal input, v_{in} is held constant at 0 for a long time, until a time t_0, at which point v_{in} changes to be a constant 2 Volts.

Find approximate expressions for $v_{\text{R}}(t)$ and $v_{\text{C}}(t)$ that are valid for $t > t_0$. Enter your answer (or "Not Enough Information") in the box below, along with your justification.

5 Completeness

The circuit shown below is used as one stage in an AM radio receiver. For purposes of this problem, you may treat the op-amp as acting ideally (e.g., with infinite open-loop gain, infinite bandwidth, and no power supply limitations).



Answer the questions below about this circuit.

5.1 Transfer Function

Find the transfer function $H(s)$ from the input voltage to the output voltage, in terms of the component values above. You do not need to simplify your answer fully.

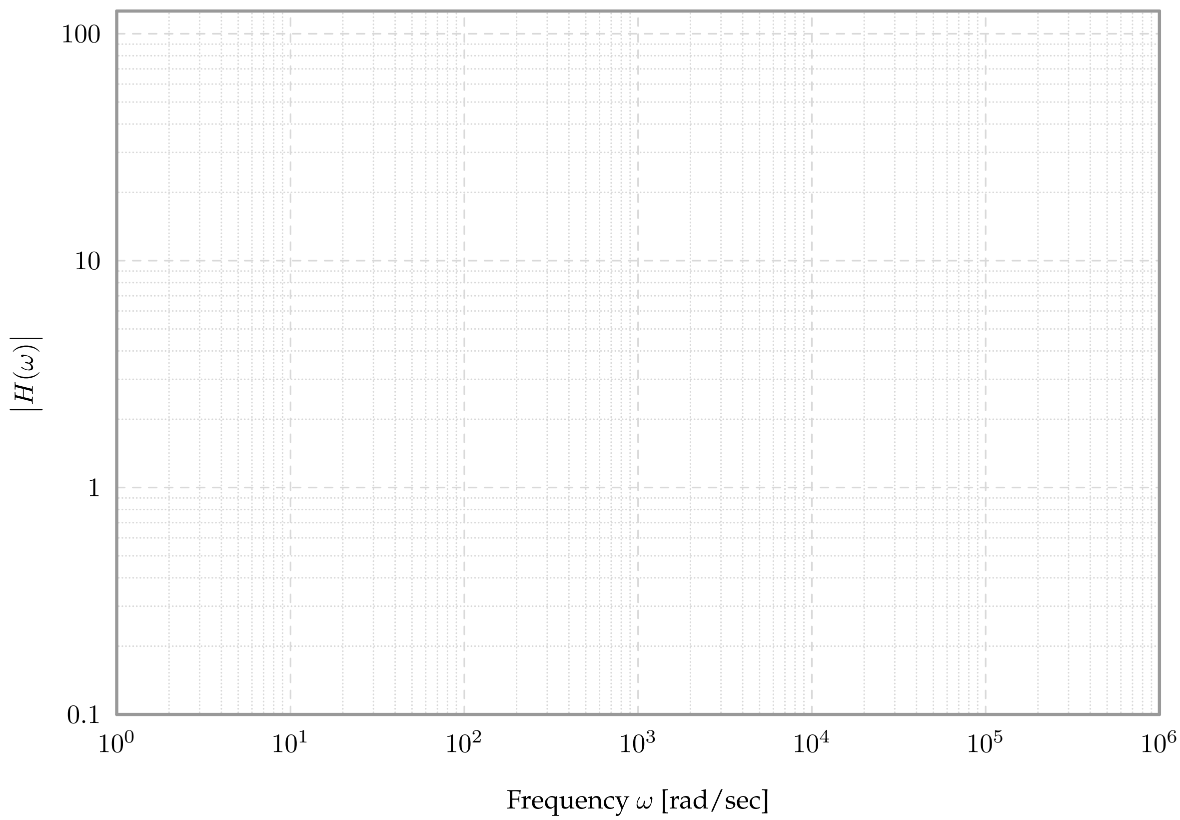
$$H(s) = \frac{v_{\text{out}}(s)}{v_{\text{in}}(s)} =$$

5.2 Magnitude Graph

Consider the circuit from the facing page with the following specific component values:

- $R_1 = 1\text{k}\Omega$
- $R_2 = 10\text{k}\Omega$
- $C_1 = 10\mu\text{F}$
- $C_2 = 10\text{nF}$

On the axes below, sketch a plot of the magnitude of the frequency response of this system. Recall that the frequency response $H(\omega) = H(s)|_{s=j\omega}$. Note that both axes are on a log scale. Please label all crossover frequencies, and indicate and provide expressions for all asymptotes.



5.3 Input/Output Pairs

Suppose the circuit from the previous part is driven by an input voltage $v_{\text{in}} = 1\text{Volt} \times \cos\left(1000t - \frac{\pi}{2}\right)$ for all time t. Under this input, what is the sinusoidal steady-state response $v_{\text{out}}(t)$?

Your answer does not need to be exact; you should feel free to make reasonable approximations based on asymptotic behaviors of the frequency response, but if you do, please clearly indicate where such approximations are being made in your work.

$v_{\text{out}}(t) =$

Now suppose instead that the circuit is driven by an input voltage $v_{\text{in}} = 1\text{Volt} \times \cos\left(1000t - \frac{\pi}{2}\right) u(t)$, i.e., a sinusoidal input voltage is applied starting at $t = 0$, after the circuit has been at rest for a long time). Under this input, what is the response $v_{\text{out}}(t)$ for $t \geq 0$?

Your answer does not need to be exact; you should feel free to make reasonable approximations based on asymptotic behaviors of the frequency response, but if you do, please clearly indicate where such approximations are being made in your work.

$v_{\text{out}}(t) =$

6 Let There Be Light

In the folder you were given, you will find a circuit and a multimeter. This circuit was designed to be a brightness controller for a light. Similar to streetlights, we want our circuit to turn a light on where there is little ambient light falling on a sensor, and turn it off when there is a lot of ambient light.

The circuit is designed to have two stages:

- a voltage divider (consisting of one fixed resistor and one photoresistor) that produces a voltage that goes up as ambient light goes down, and
- an amplifier, designed to scale this voltage up by a factor of 12.

The voltage output from the amplifier is then applied across an LED similar to those we used in lab this semester.

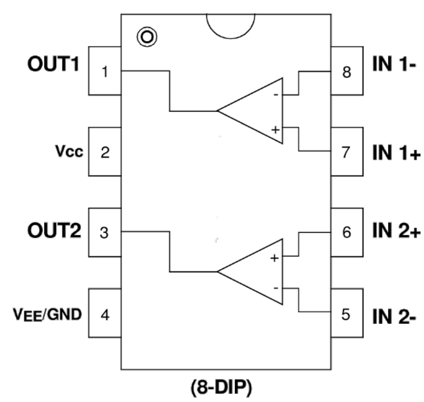
The circuit you have been given was intended to have these properties, but it was not properly assembled. Ultimately, your goal in this problem is to fix the circuit you were given; but we will proceed in several steps.

Some notes about the circuit you've been given:

- There are four resistor values on the board, which can be identified by their color codes:
  - 1kΩ: Brown, Black, Red
  - 3kΩ: Orange, Black, Red
  - 11kΩ: Brown, Brown, Orange
  - 1MΩ: Brown, Black, Green

  There are also two other resistors in the envelope:
  - 12kΩ: Brown, Red, Orange
  - 13kΩ: Brown, Orange, Orange

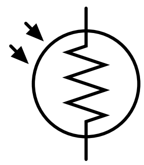
  If you are having trouble recognizing the color codes, note that you can use your meter to determine which resistor is which.
- The op-amp in the board is an L272, which we've seen several times in lab as well. Its pinout is shown here (where V_{CC} refers to the positive supply voltage and V_{EE} refers to the negative supply voltage):

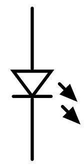


6.1 Schematic

In the box below, draw a schematic diagram representing **exactly what was on the board when it was given to you.** Clearly label the values of all constant resistors. You can model the battery as a 9-Volt voltage source.

For reference, the schematic symbols for a photoresistor and an LED, respectively, are shown here:


photoresistor


LED

6.2 What Is Wrong?

Connect your circuit to the included battery. When doing so, **do not look directly into the LED.** Using circuit theory and/or measurements from the board, determine what is wrong with this circuit with respect to the design goals from the initial description of the problem.

Note that you can test the circuit's behavior in darkness by covering the photoresistor from all directions with your hands.

You may assume that nothing is wrong with the load (the combination of the 3kΩ resistor and the LED); you should not modify that part of the circuit.

In the box below, briefly explain the error/errors in the circuit you were given, with respect to the design goals mentioned earlier, as well as what changes you would need to make to fix it/them. Justify your plan using circuit theory.

6.3 Fix It!

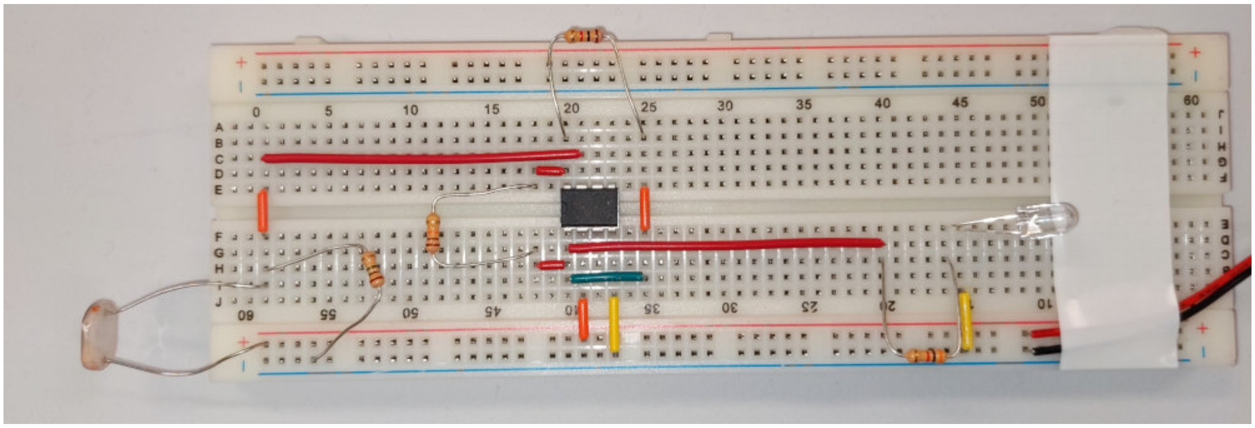
Finally, make modifications to the physical circuit so that it meets the original design criteria. It should work by using the two stages described in the first part of this problem, and the LED should be bright when the photoresistor is covered up, and the LED should be off (or barely on) when the photoresistor is exposed to the ambient light in the room.

You are welcome to use any of the additional resistors, and/or the other op-amp in the L272 package.

When you are finished, **unplug the battery and put the plastic cap back on it**, then carefully slide your circuit back into its envelope and leave it at your table; we'll pick it up after the exam has ended.

Original Circuit

If you're using this as a way to study for a future semester's exam, here is an image of the original circuit that was given to students during the exam:



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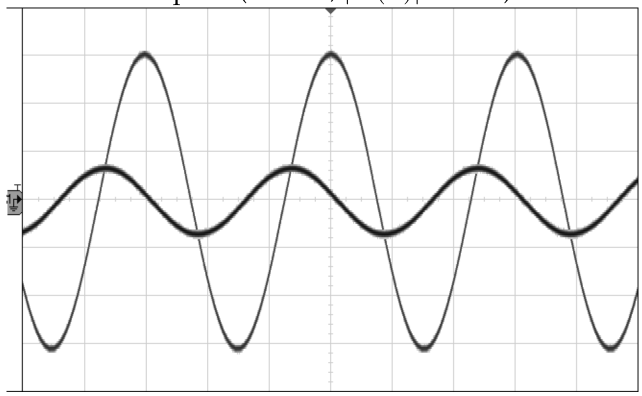
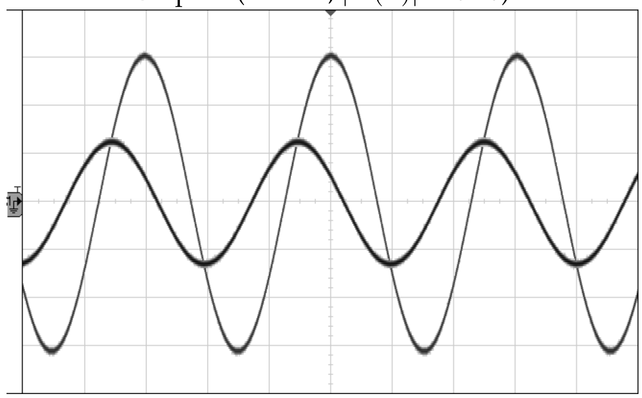
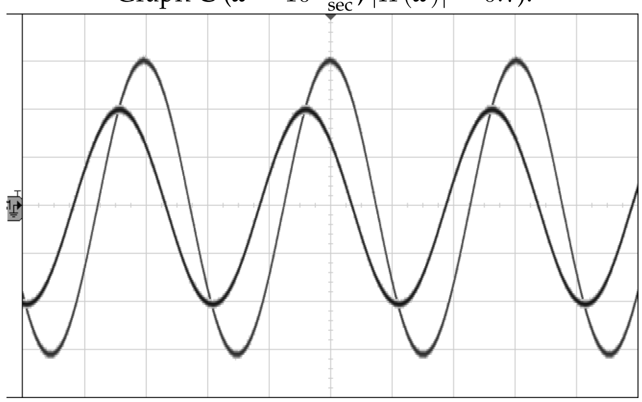
Graph A ($\omega =???$, $|H(\omega)| \approx 0.24$):



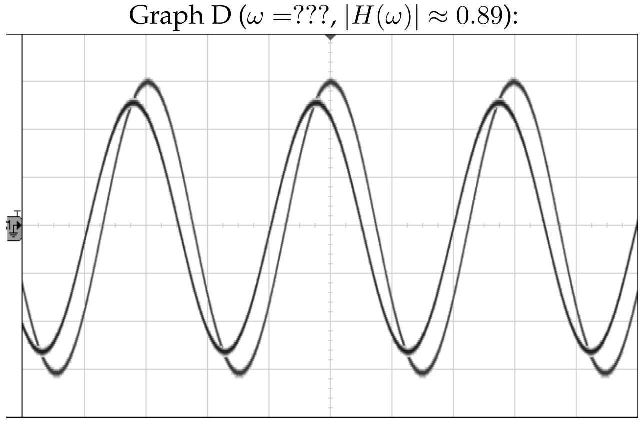
Graph B ($\omega =???$, $|H(\omega)| \approx 0.45$):



Graph C ($\omega = 10^4 \frac{\text{rad}}{\text{sec}}$, $|H(\omega)| \approx 0.7$):



Graph D ($\omega =???$, $|H(\omega)| \approx 0.89$):



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