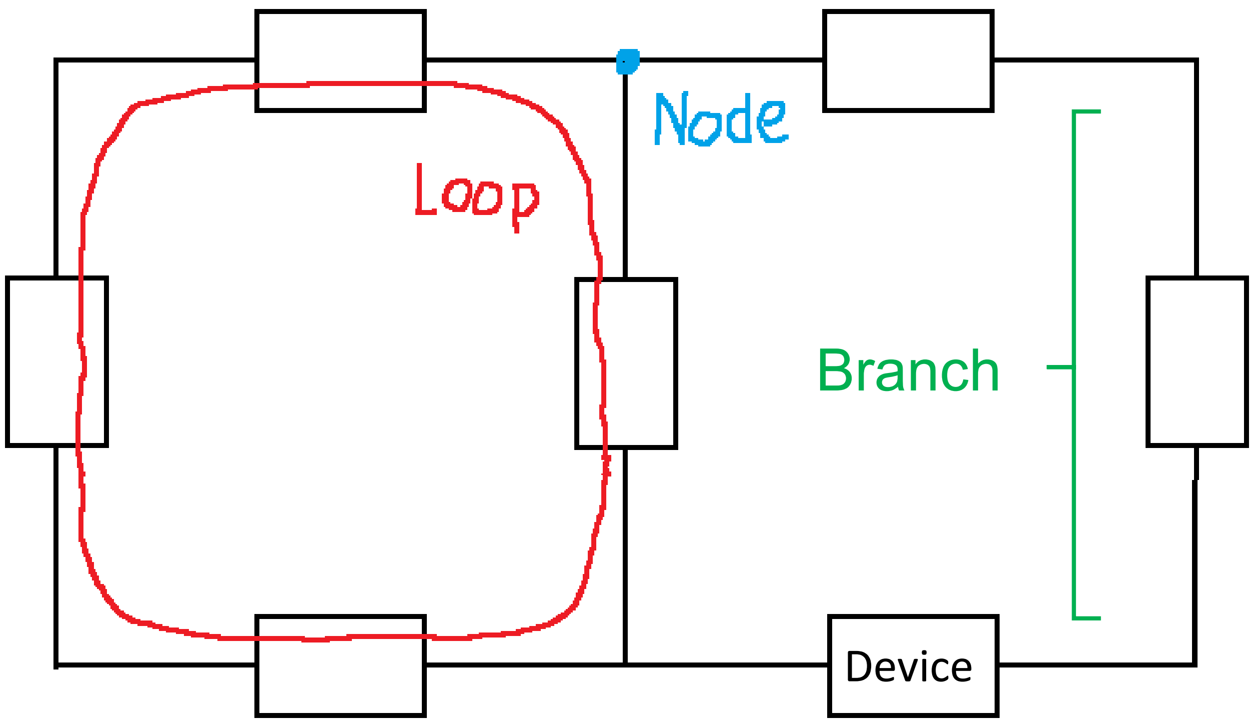
6.200 - Lecture 02

(Brute Force) Circuit Analysis

- Review: Labeling Devices
- Review: Nodes and Loops
- Brute Force Analysis
- Series and Parallel Resistors
- Voltage and Current Dividers
- Power

Circuit Terminology



Device: a multi-terminal lumped-parameter piece of electrical hardware having identifiable voltages across (between) its terminals and currents through its terminals. Most devices in 6.200 have only two terminals.

Circuit: a collection of devices joined together at their terminals, presumably designed for a useful purpose.

Node: the topological point at which one or more device terminals are joined together. Nodes can be spatially distributed, occupying more than a point in real space.

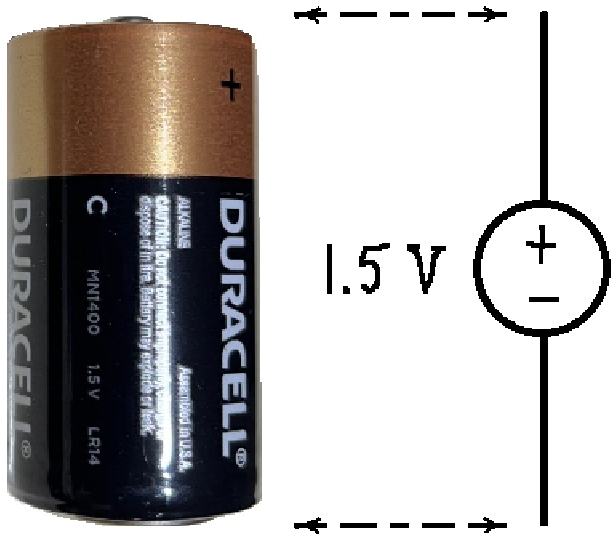
Branch: the topological pathway between two electrically neighboring nodes. Branches are occupied by devices.

Loop: A closed-path through the branches that traverses no branch more than once.

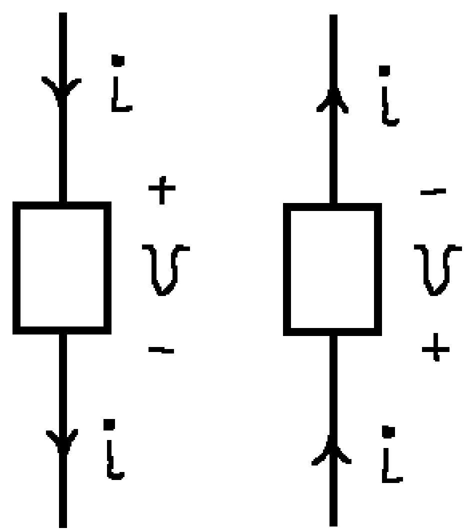
Circuit Analysis: determination of all branch (device) voltages and currents.

Labeling

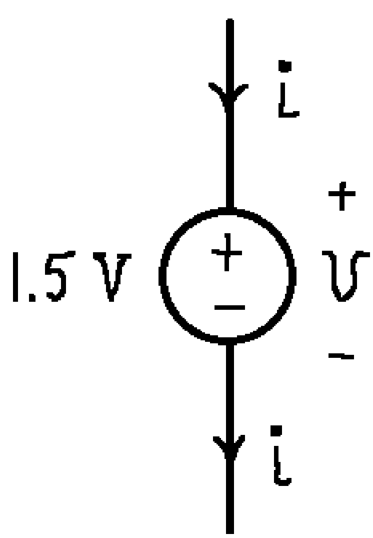
Intrinsic



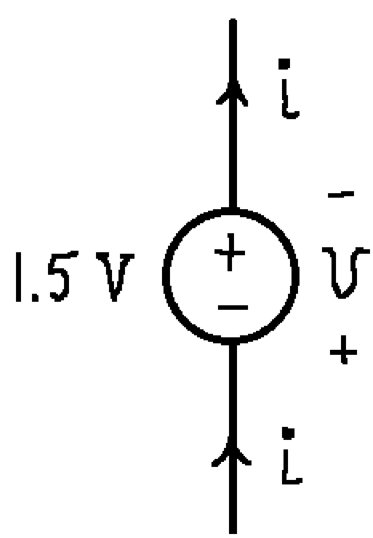
Extrinsic



$v = 1.5\text{ V}$

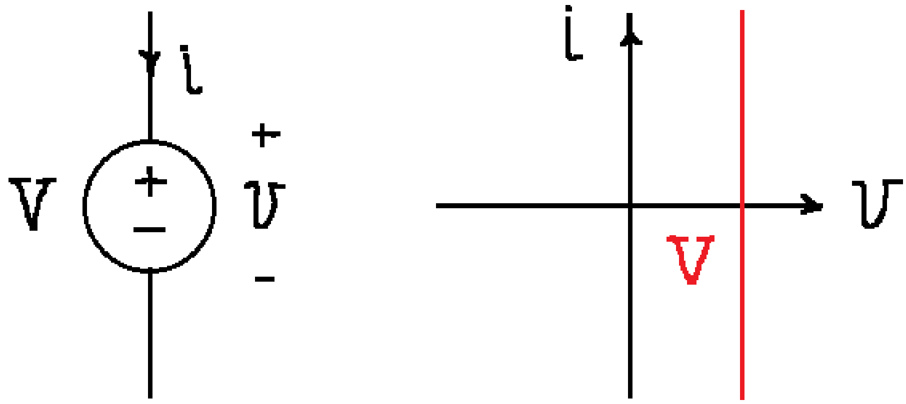


$v = -1.5\text{ V}$

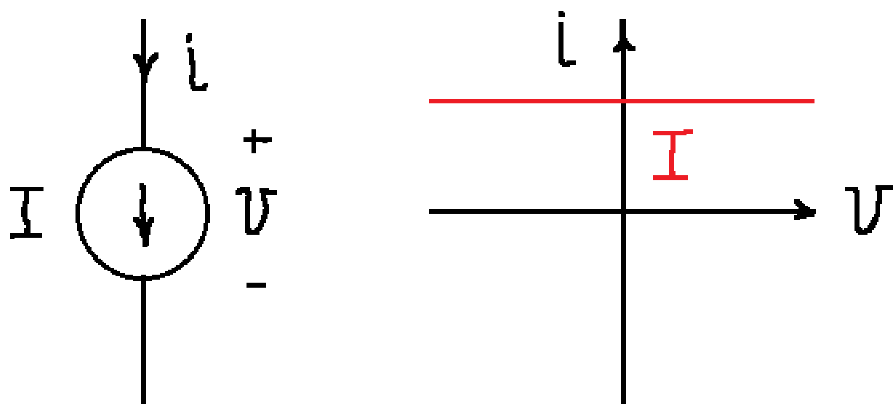


Devices

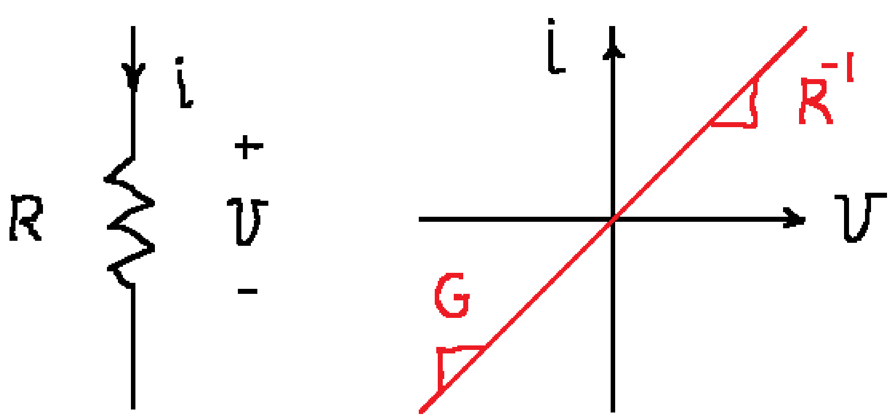
Voltage Source



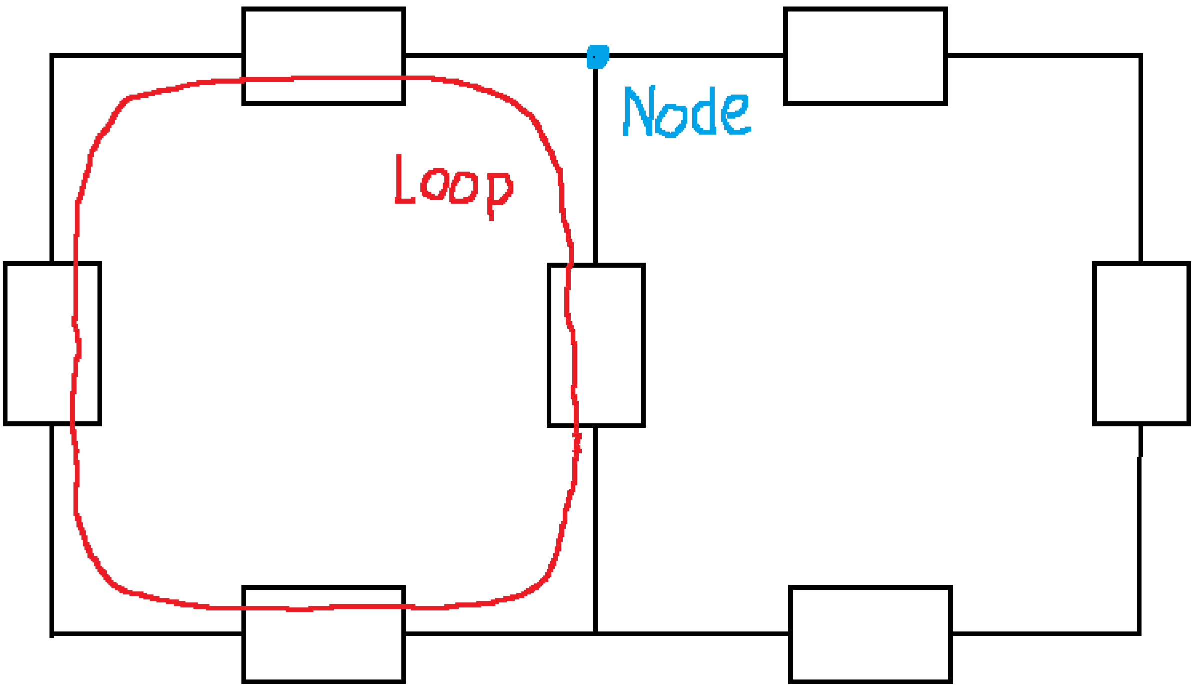
Current Source



Linear Resistor



Connections



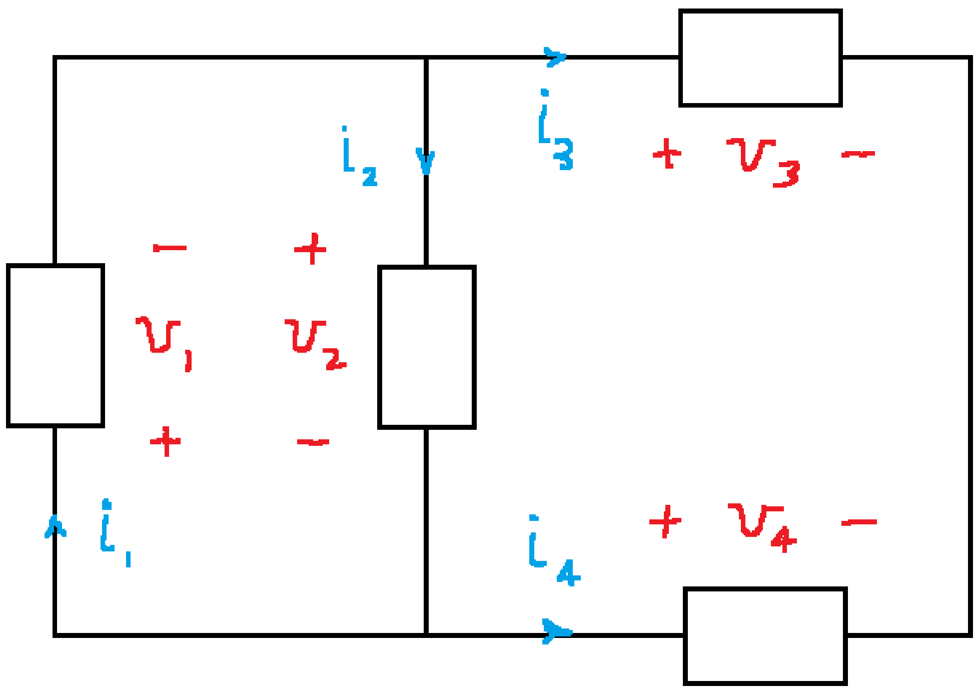
KVL: $\Sigma v = 0$

Around Loop ← Either Direction

KCL: $\Sigma i = 0$

Into Node ← Or Out From

KVL & KCL



KVL: $-v_1 - v_2 = 0$ or $v_1 = -v_2$

$v_4 + v_2 - v_3 = 0$ or $v_4 + v_2 = v_3$

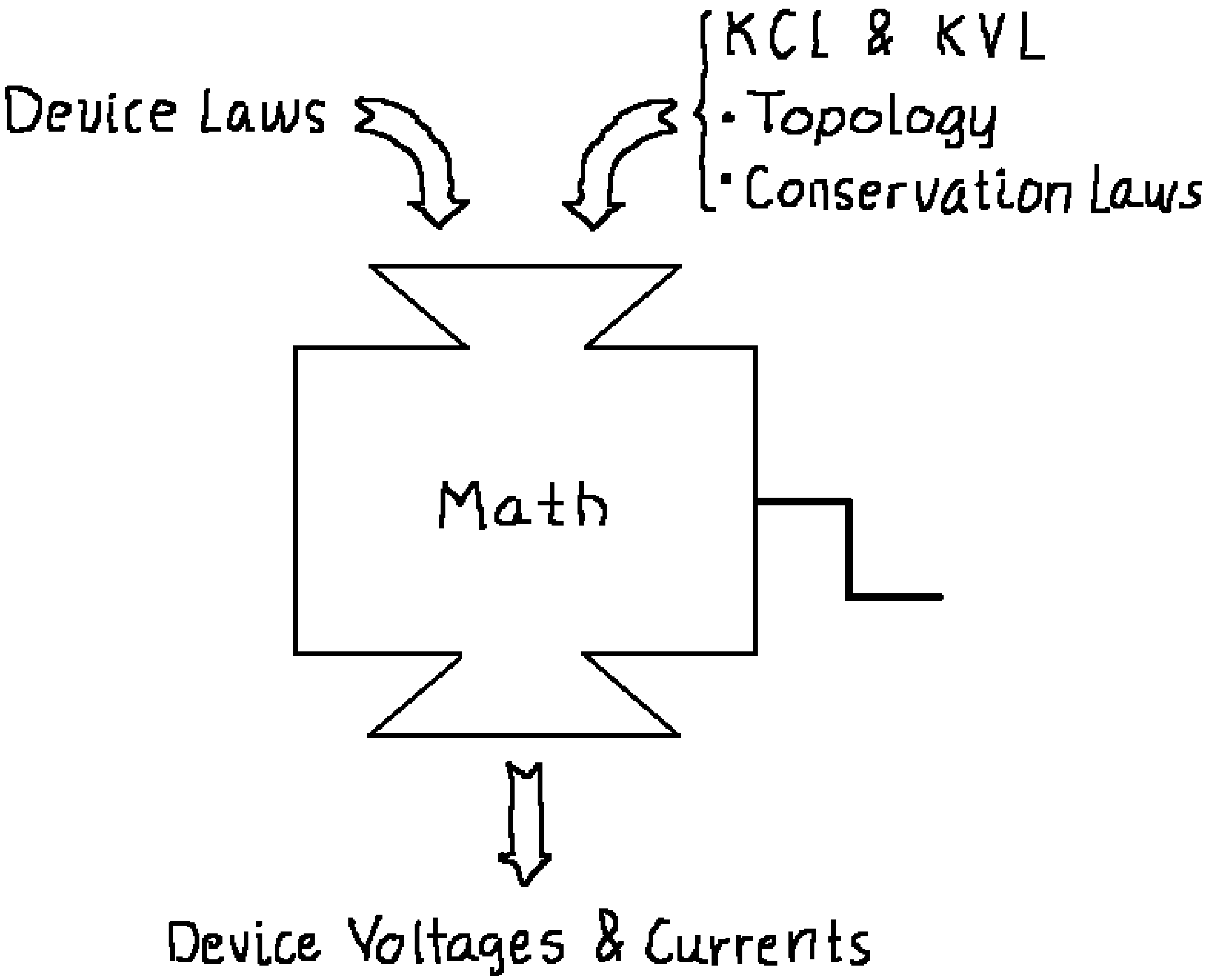
$-v_1 - v_3 + v_4 = 0$ or $v_4 = v_1 + v_3$

KCL: $i_1 - i_2 - i_3 = 0$ or $i_1 = i_2 + i_3$

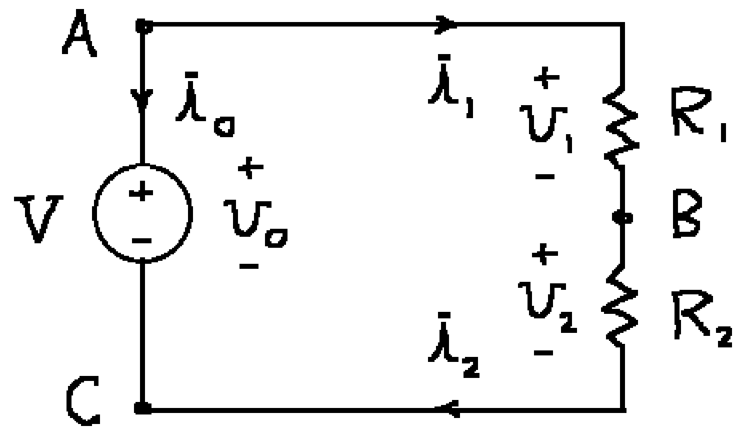
$i_3 + i_4 = 0$ or $i_3 = -i_4$

$-i_1 + i_2 - i_4 = 0$ or $i_2 = i_1 + i_4$

Brute Force Analysis



Brute Force Analysis #1



Devices	KCL	KVL
$v_0 = V$	A: $i_0 + i_1 = 0$	$v_0 - v_1 - v_2 = 0$
$v_1 = R_1 i_1$	B: $i_2 - i_1 = 0$	
$v_2 = R_2 i_2$	C: $-i_0 - i_2 = 0$	
	One redundant KCL	

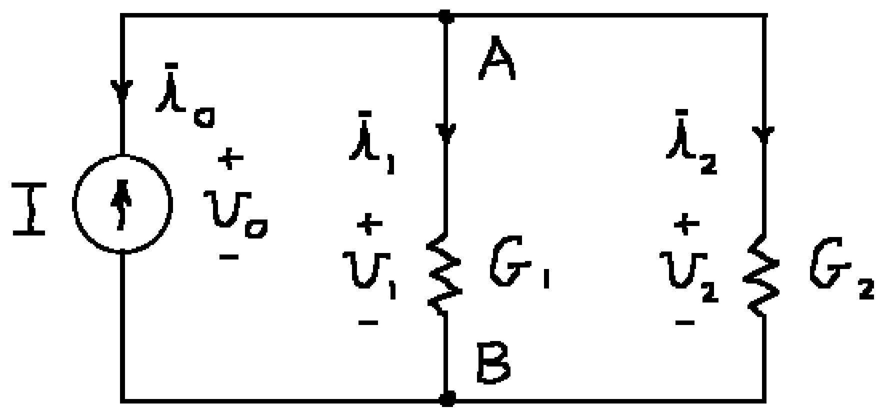
Solution

$i_1 = i_2 = -i_0 = V/(R_1+R_2) \equiv i$

$v_0 = V$; $v_1 = R_1 i = V \frac{R_1}{R_1+R_2}$; $v_2 = R_2 i = V \frac{R_2}{R_1+R_2}$

Note: $v_0 = v_1 + v_2 = i(R_1+R_2) \Rightarrow \frac{v_0}{i} = \underbrace{R_1 + R_2}_{\text{Effective } R}$

Brute Force Analysis #2



Devices	KCL	KVL
$i_0 = -I$	A: $i_0 + i_1 + i_2 = 0$	$v_0 - v_1 = 0$
$i_1 = G_1 v_1$	B: $-i_0 - i_1 - i_2 = 0$	$v_0 - v_2 = 0$
$i_2 = G_2 v_2$	One redundant KCL	$v_1 - v_2 = 0$
		One redundant KVL

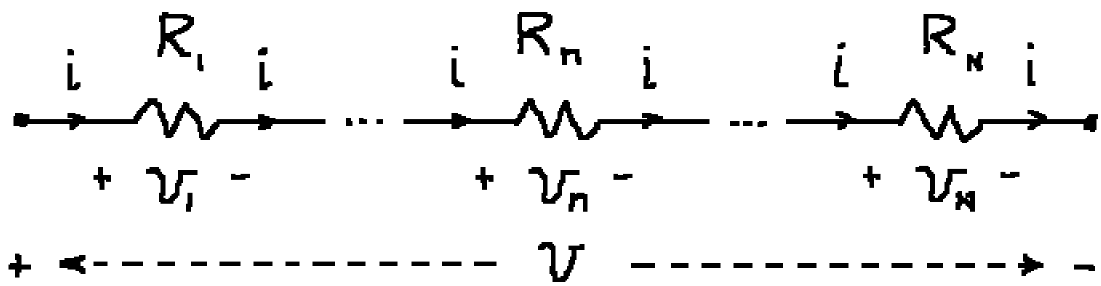
Solution

$v_0 = v_1 = v_2 = I/(G_1+G_2) = v$

$i_0 = -I$; $i_1 = G_1 v = I \dfrac{G_1}{G_1+G_2}$; $i_2 = G v_2 = I \dfrac{G_2}{G_1+G_2}$

Note: $-i_0 = i_1 + i_2 = v(G_1+G_2) \Rightarrow \dfrac{v}{-i_0} = \underbrace{G_1+G_2}_{\text{Effective } G}$

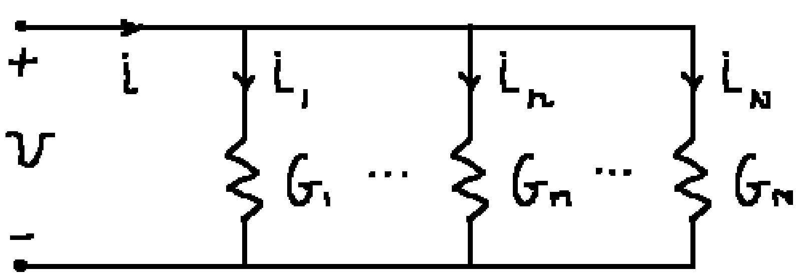
Resistor Combinations



$$v = \sum_n v_n = \sum_n R_n i = i \sum_n R_n \Rightarrow v/i \equiv R = \sum_n R_n$$

Series resistances add: $R = \sum_n R_n$

Voltage divider: $v_n = R_n i = (R_n / R)\, v$

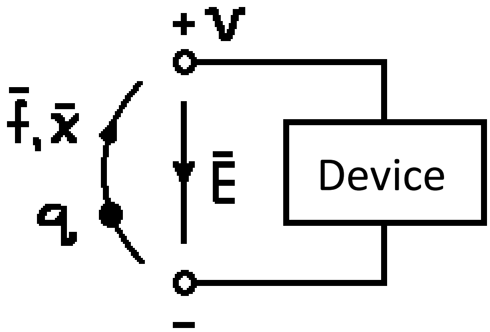


$$i = \sum_n i_n = \sum_n G_n v = v \sum_n G_n \Rightarrow i/v \equiv G = \sum_n G_n$$

Parallel conductances add: $G = \sum_n G_n$; $R^{-1} = \sum_n R_n^{-1}$

Current divider: $i_n = G_n v = (G_n / G)\, i$

Power

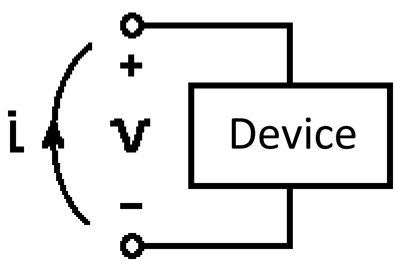


Lifting charge q across the terminals of the device requires energy W

$$W = \int_{-}^{+} \bar{f} \cdot \mathrm{d}\bar{x} = \int_{-}^{+} -q\bar{E} \cdot \mathrm{d}\bar{x}$$

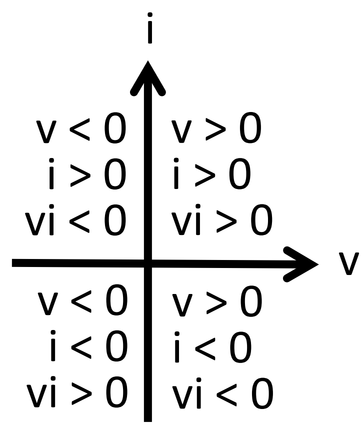
$$= q \int_{-}^{+} -\bar{E} \cdot \mathrm{d}\bar{x} = qv$$

which is delivered to the device.



Charge q is delivered to the device at the rate i so energy W is delivered at the rate vi. Thus, the power delivered to the device is vi.

The sign of power varies with v-i quadrant.



Tellegen's Theorem states that any circuit, linear or nonlinear, that obeys both KCL and KVL conserves energy and power at all times such that $\sum_n v_n i_n = 0$ where n indexes the devices in the circuit.

Summary

- Intrinsic and extrinsic labels
- Brute force analysis
- Series resistances add
- Parallel conductances add
- Voltage dividers
- Current dividers
- Power