

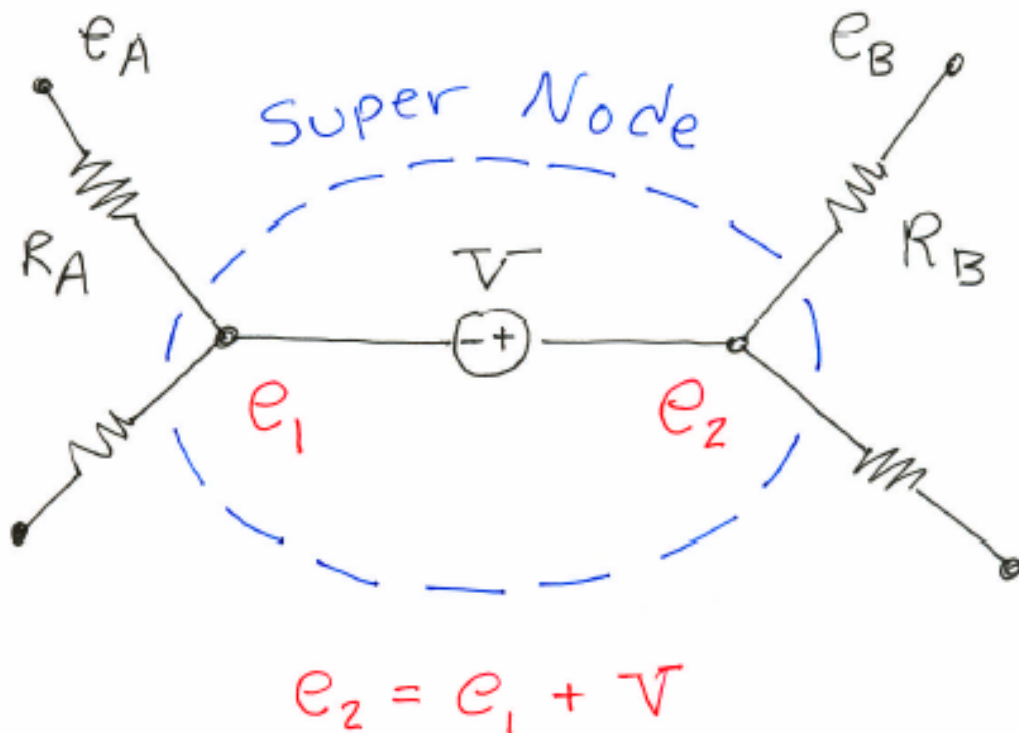
6.200 - Lecture 04

Circuit Analysis Simplifications

- (Floating Voltage Sources)
- Linearity
- Superposition

Super Nodes

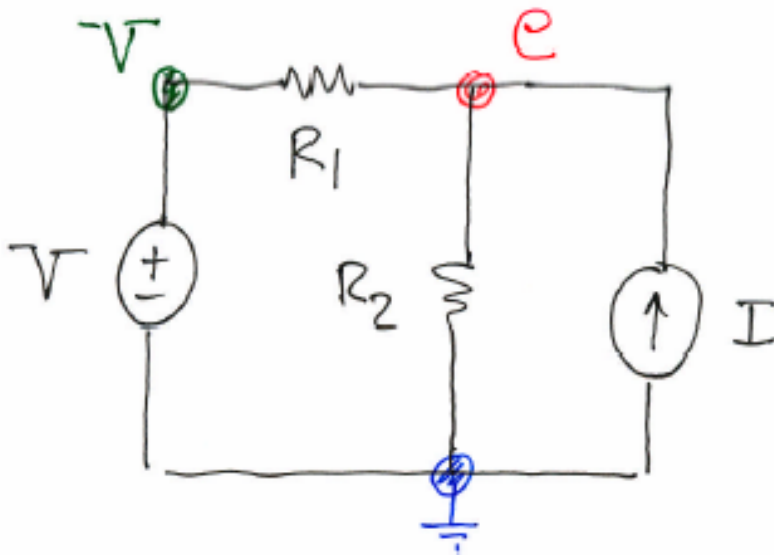
Super nodes treat floating voltage sources.



Write only one KCL for each super node.

$$\frac{e_1 - e_A}{R_A} + \frac{\overbrace{e_1 + V}^{e_2} - e_B}{R_B} + \dots = 0$$

Simple Nodal Analysis



Linear in $\bar{e}$ and $\bar{I}$.

$$\frac{e - V}{R_1} + \frac{e}{R_2} - I = 0$$

$$e \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V}{R_1} + I$$

$$e = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

Conductance
Matrix



$$G \bar{e} = H \bar{I}$$



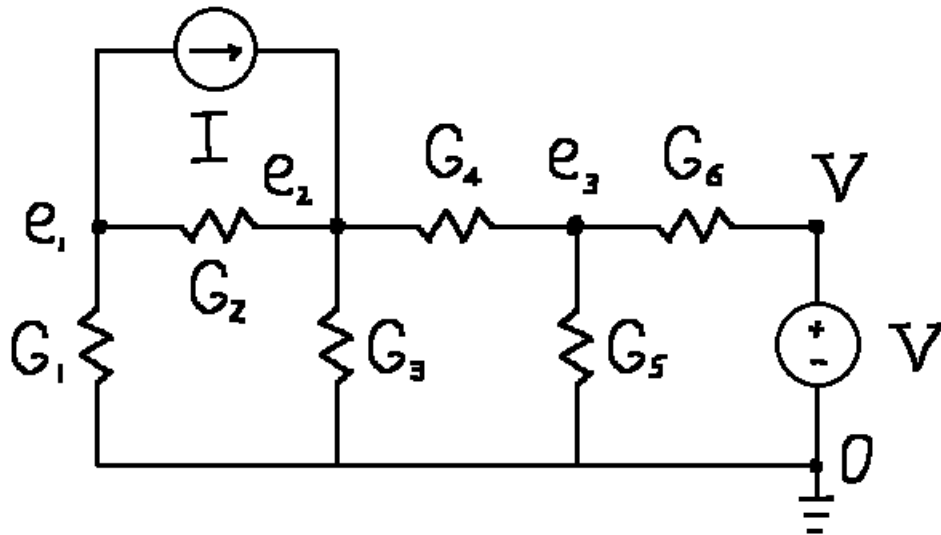
Node
Voltage
Vector

Coefficient
Matrix



Source
Vector

Another Nodal Analysis



$$e_1 \text{ KCL: } G_1(e_1 - 0) + G_2(e_1 - e_2) + I = 0$$

$$e_2 \text{ KCL: } G_3(e_2 - 0) + G_2(e_2 - e_1) + G_4(e_2 - e_3) - I = 0$$

$$e_3 \text{ KCL: } G_5(e_3 - 0) + G_4(e_3 - e_2) + G_6(e_3 - V) = 0$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 & 0 \\ -G_2 & G_2 + G_3 + G_4 & -G_4 \\ 0 & -G_4 & G_4 + G_5 + G_6 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \\ 0 \end{bmatrix} I + \begin{bmatrix} 0 \\ 0 \\ G_6 \end{bmatrix} V$$

Homogeneity & Superposition

Homogeneity and superposition are properties of linear systems.

Homogeneity: If $\bar{x} \rightarrow \boxed{\text{Linear System}} \rightarrow \bar{y}$

Then $\alpha \bar{x} \rightarrow \boxed{\text{Linear System}} \rightarrow \alpha \bar{y}$

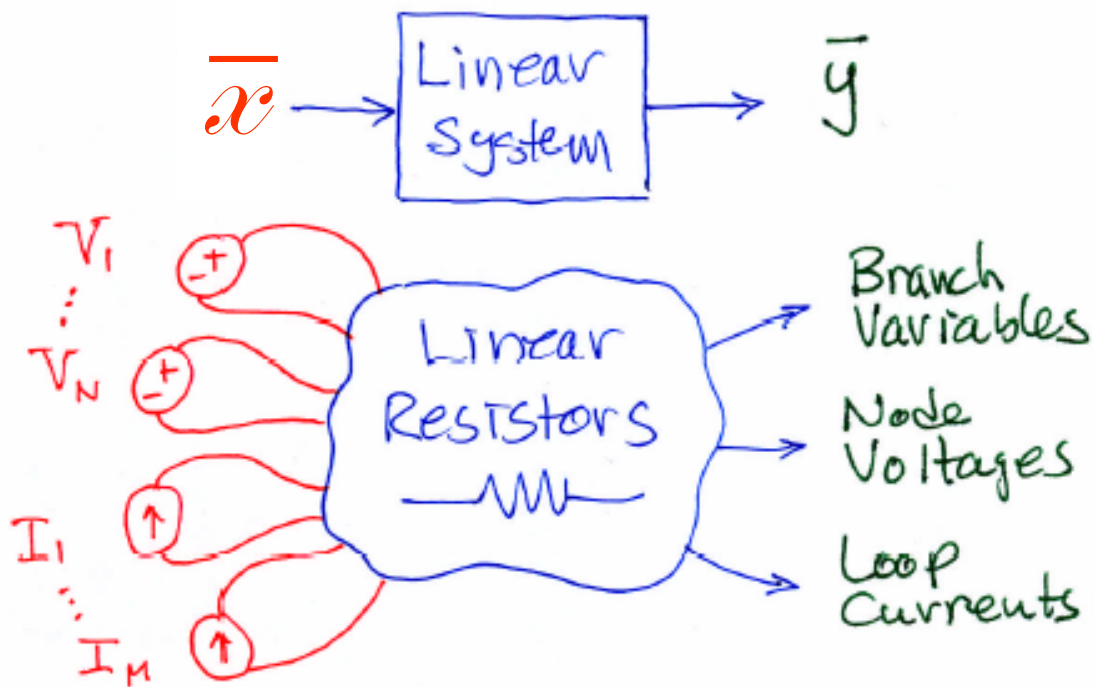
Superposition: If $\bar{x}_1 \rightarrow \boxed{\text{Linear System}} \rightarrow \bar{y}_1$

$\bar{x}_2 \rightarrow \boxed{\text{Linear System}} \rightarrow \bar{y}_2$

Then $\bar{x}_1 + \bar{x}_2 \rightarrow \boxed{\text{Linear System}} \rightarrow \bar{y}_1 + \bar{y}_2$

Linear Networks and Systems

Linear system $\Rightarrow$ all stimuli are external inputs. For example, $y = x_1 + x_2$ is linear from $\bar{x} \rightarrow y$, but $y = x_1 + x_2 + 1$ is not.

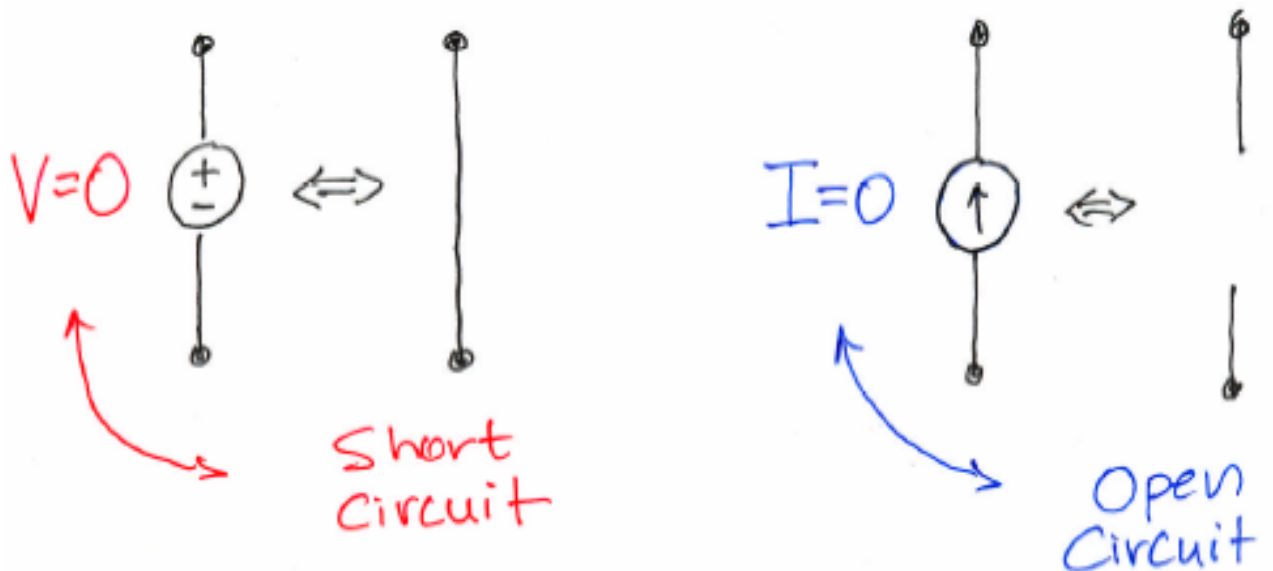


No source should be left within the network for the purposes of superposition. Superposition is carried out over all sources.

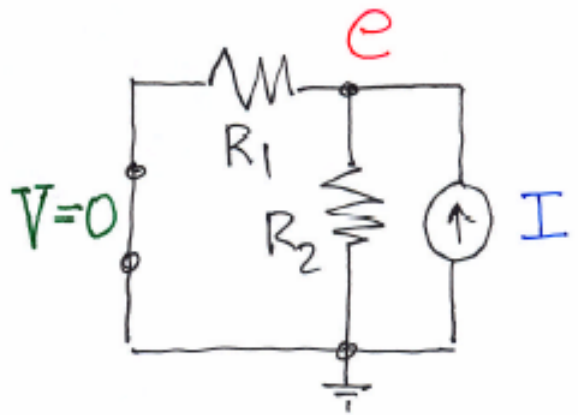
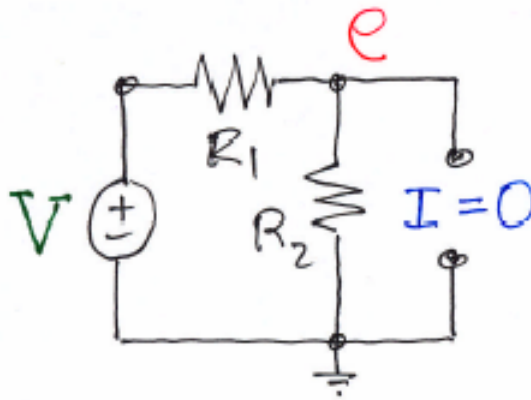
Divide & Conquer

Use superposition to analyze a linear circuit one source at a time. (All sources except one are set to zero each time.)

How to set a source to zero?

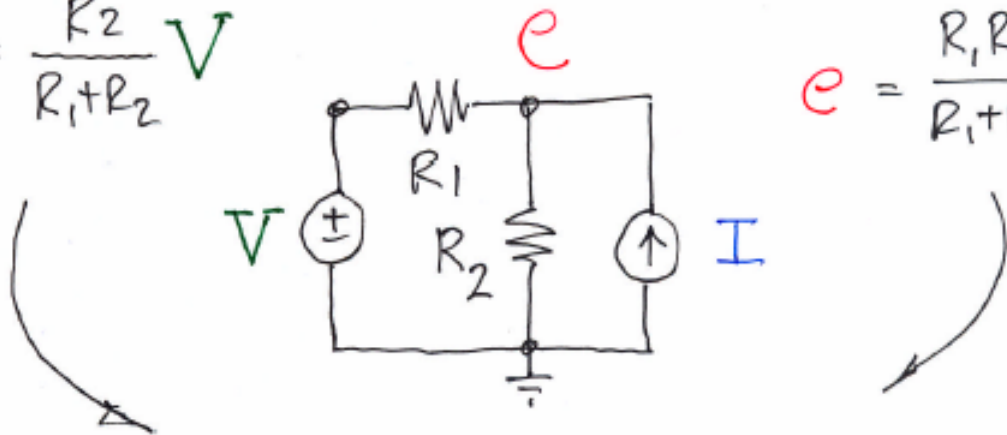


Superposition Examples

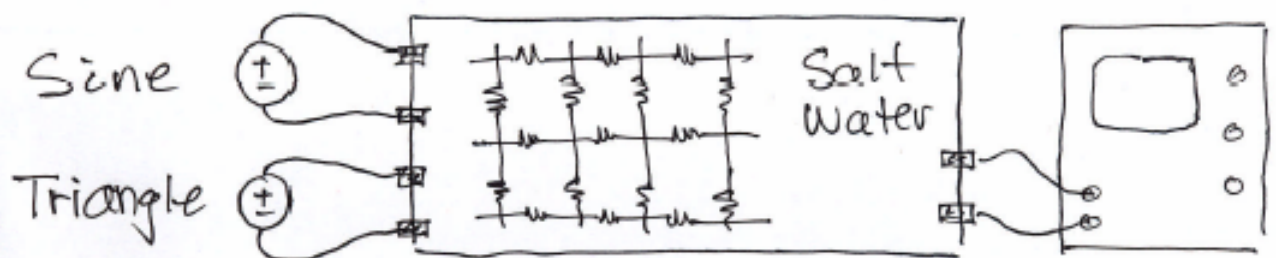


$$e = \frac{R_2}{R_1 + R_2} V$$

$$e = \frac{R_1 R_2}{R_1 + R_2} I$$

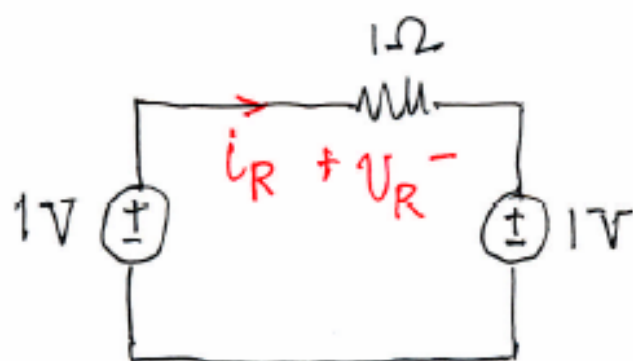


$$e = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$



No Superposition For Power

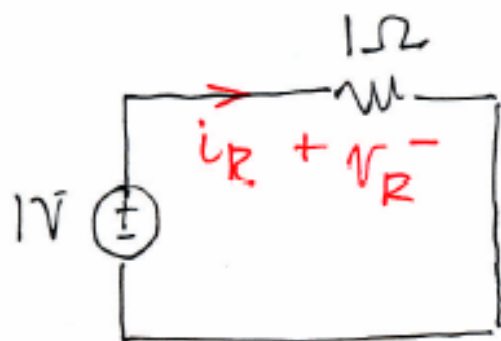
Power is a **nonlinear** network property, and so the superposition of power is not possible.



$$v_R = 0\text{ V}$$

$$i_R = 0\text{ A}$$

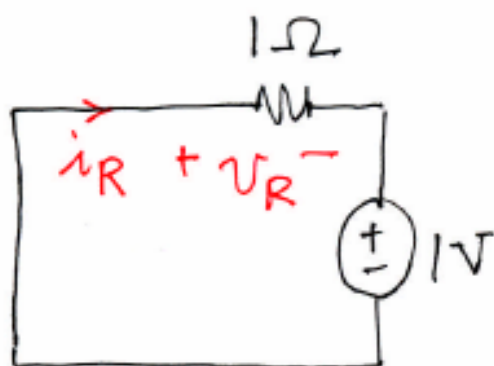
$$v_R i_R = 0\text{ W}$$



$$v_R = 1\text{ V}$$

$$i_R = 1\text{ A}$$

$$v_R i_R = 1\text{ W}$$

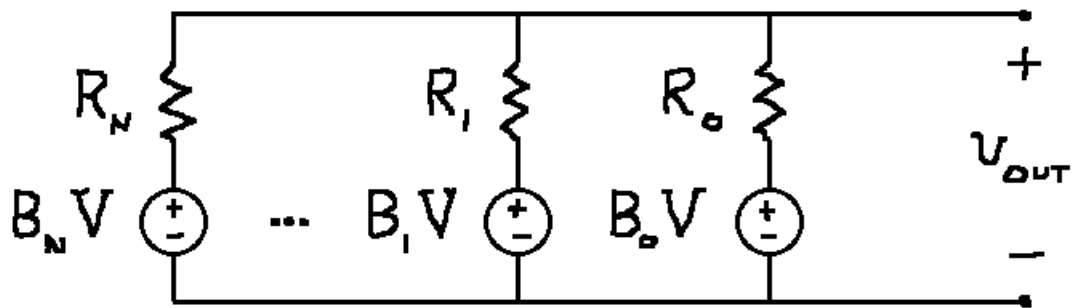


$$v_R = -1\text{ V}$$

$$i_R = -1\text{ A}$$

$$v_R i_R = 1\text{ W}$$

DAC Example



DAC $\Rightarrow$ Digital Data = $\boxed{B_N} \dots \boxed{B_1} \boxed{B_0}$

$$V_{OUT} = KV \sum_n B_n 2^n ; B_n = \{0, 1\}$$

Use superposition and voltage division:

$$\begin{aligned} V_{OUT,m} &= \frac{(\sum_{n \neq m} G_n)^{-1} B_m V}{R_m + (\sum_{n \neq m} G_n)^{-1}} = \frac{B_m V}{1 + R_m \sum_{n \neq m} G_n} \\ &= \frac{B_m V}{R_m G_m + R_m \sum_{n \neq m} G_n} = \frac{G_m B_m V}{\sum_n G_n} \end{aligned}$$

$$V_{OUT} = V(\sum_n G_n B_n) / (\sum_n G_n) \dots \text{by superposition}$$

$$\text{Design} \Rightarrow G_n = G^* 2^n \Rightarrow K = 1 / (\sum_m 2^m)$$