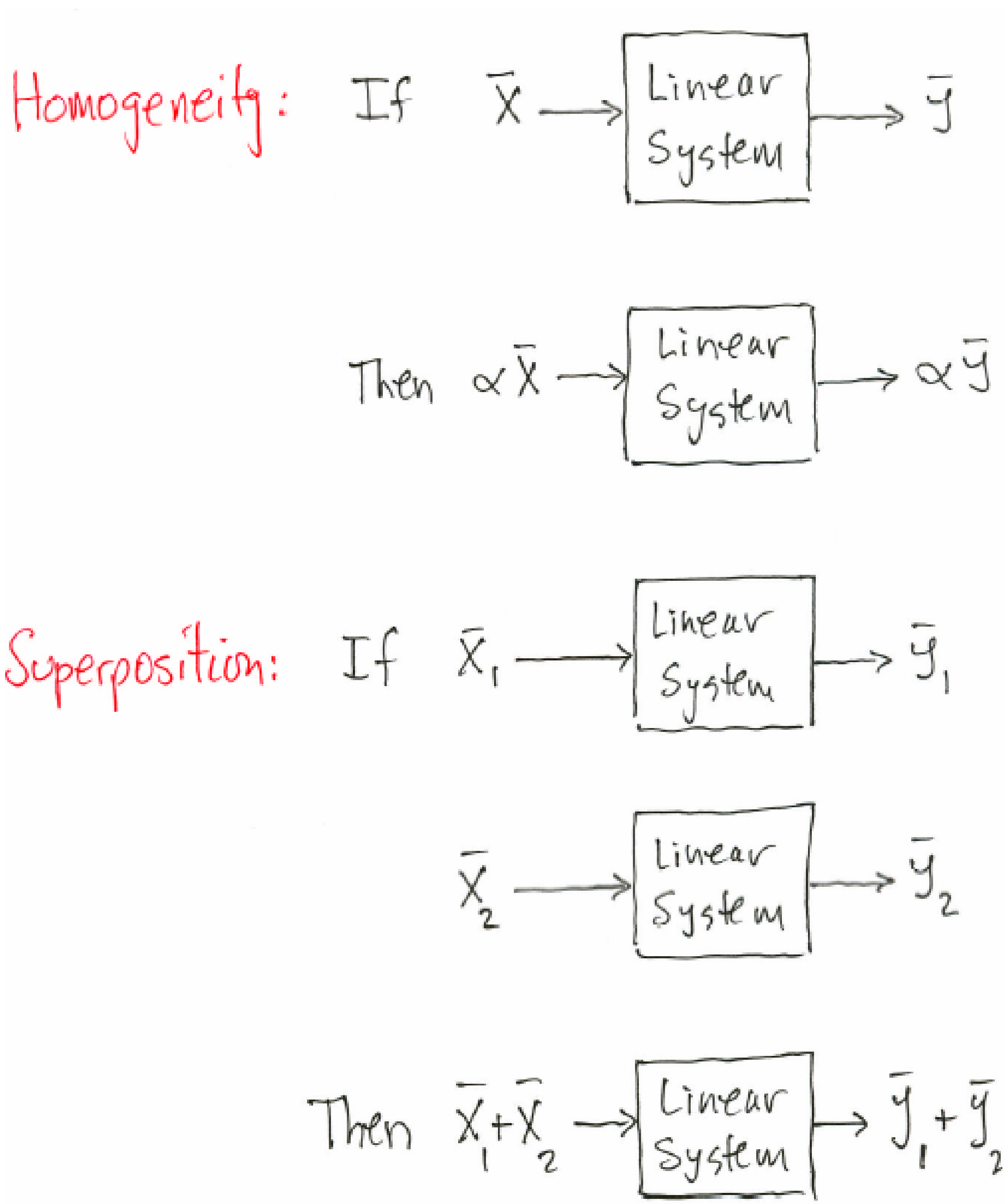
6.200 - Lecture 05

Circuit Analysis Simplifications

- Linearity Review
- Superposition Review
- Thevenin & Norton Equivalence
- Five-Lecture Summary

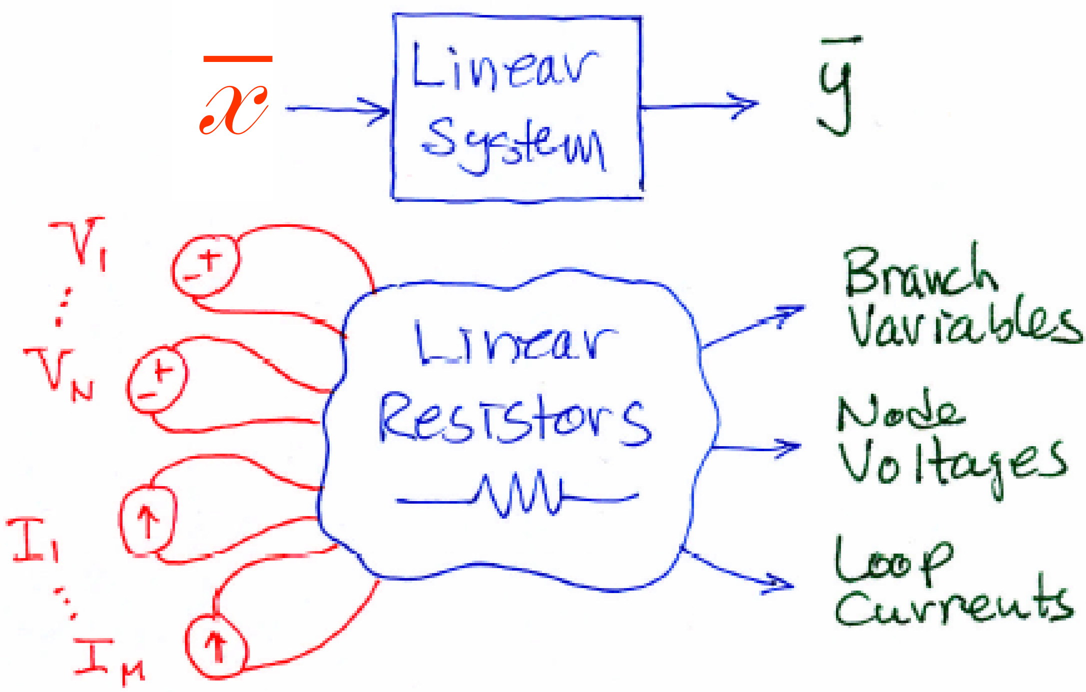
Homogeneity & Superposition

Homogeneity and superposition
are properties of linear systems.



Linear Networks and Systems

Linear system ⇒ all stimuli are external inputs. For example, $y = x_1 + x_2$ is linear from $\bar{x} \to y$, but $y = x_1 + x_2 + 1$ is not.

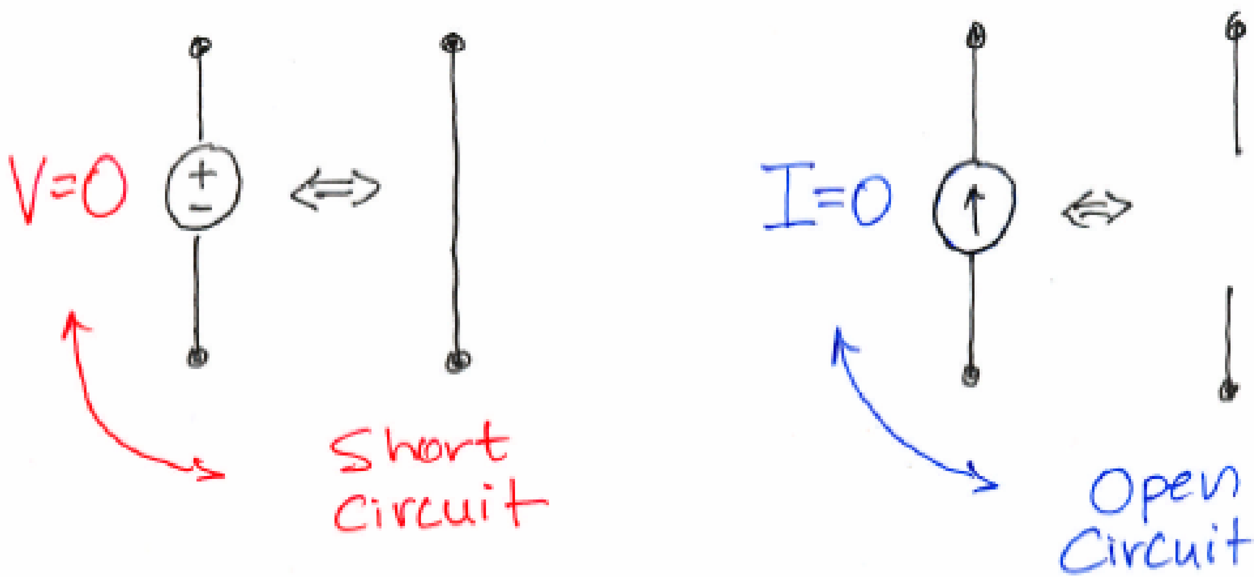


No source should be left within the network for the purposes of superposition. Superposition is carried out over all sources.

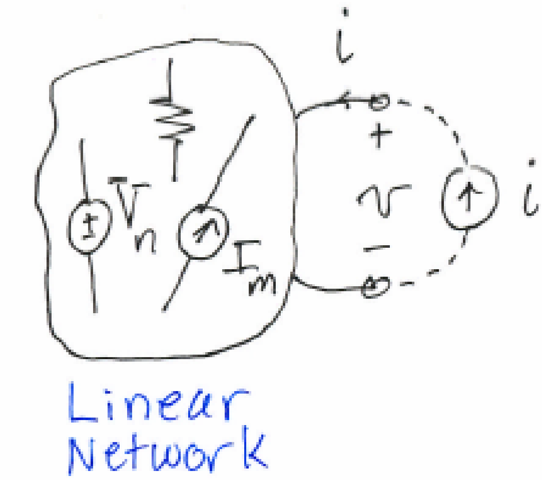
Divide & Conquer

Use superposition to analyze a linear circuit one source at a time. (All sources except one are set to zero each time.)

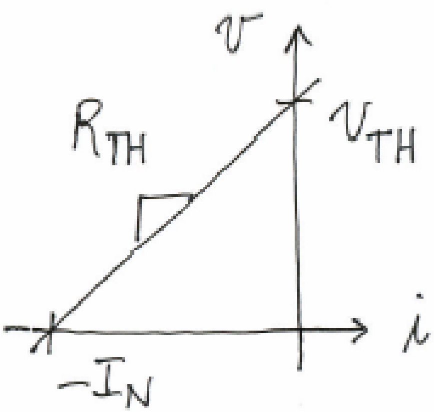
How to set a source to zero?



Thevenin & Norton Equivalence

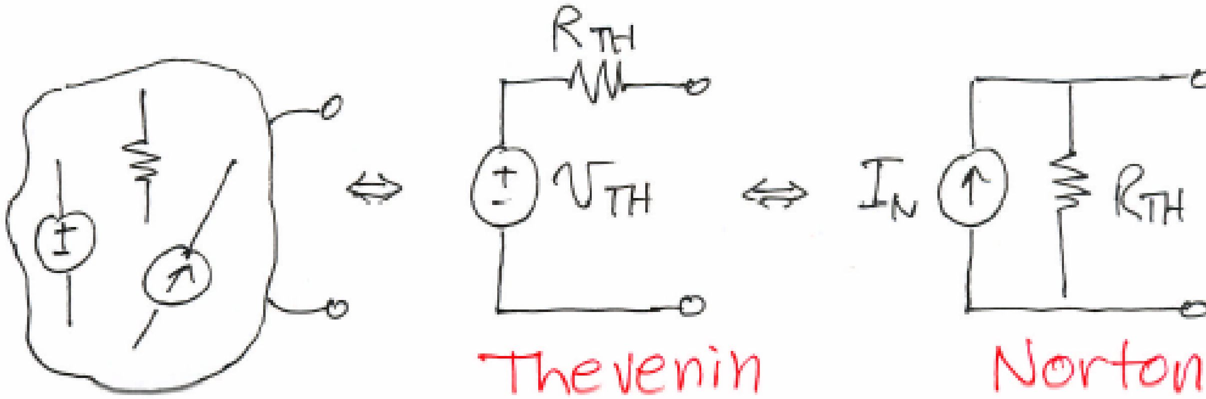


$$v = \underbrace{\sum_n \alpha_n V_n + \sum_m \beta_m I_m + R_{TH}\, i}_{\text{Superposition}}$$

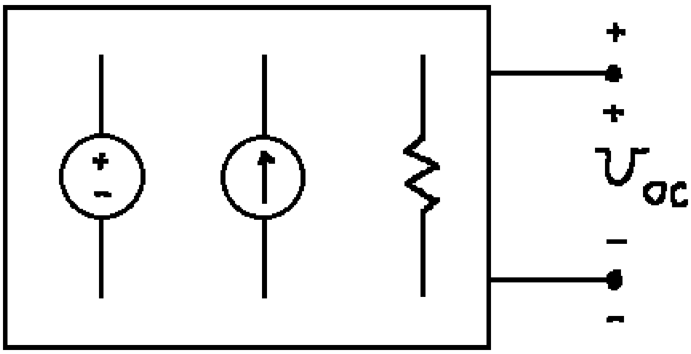


Open Circuit $(i = 0)$ $\Rightarrow$ $V_{TH} = \sum_n \alpha_n V_n + \sum_m \beta_m I_m$

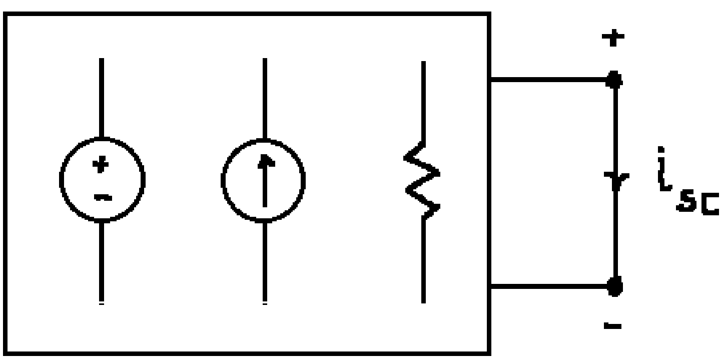
Short Circuit $(v = 0)$ $\Rightarrow$ $I_N = \dfrac{V_{TH}}{R_{TH}}$



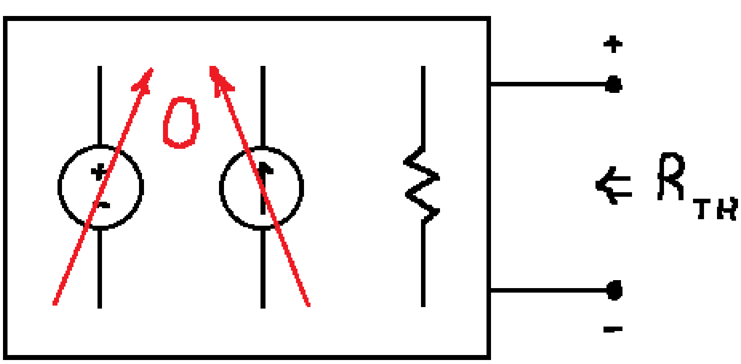
Thevenin / Norton Summary



The Thevenin voltage v_{TH} is the open-circuit voltage v_{OC}.

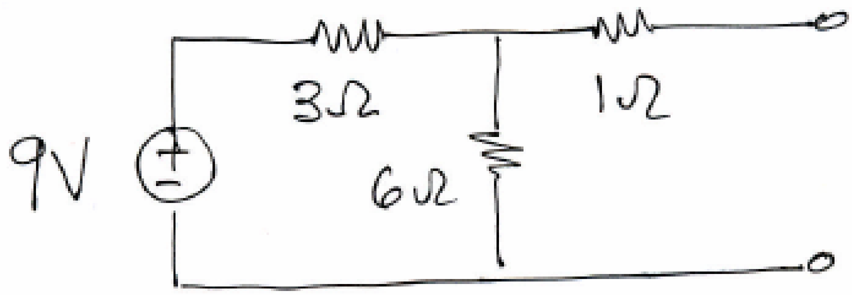


The Norton current i_N is the short-circuit current i_{SC}.



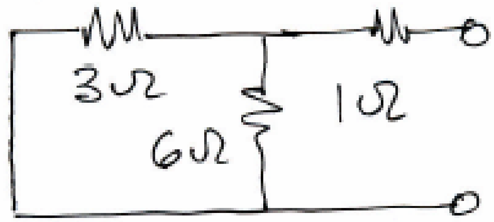
The Thevenin/ Norton resistance R_{TH} is the terminal resistance computed with zeroed sources.

Thevenin / Norton Example #1



Open-circuit voltage = 6 V via voltage divider $\Rightarrow$ $v_{TH} = 6\ V$

$R_{TH} \Rightarrow$



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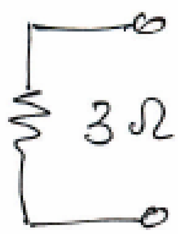
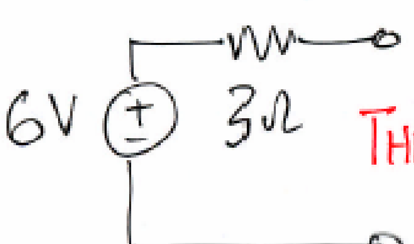
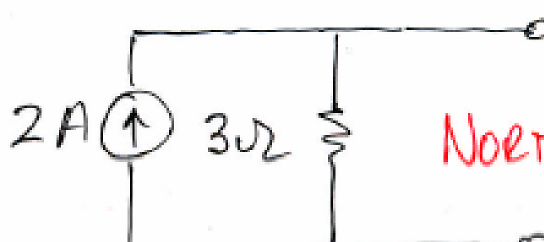


$I_N = v_{TH} / R_{TH} = 2\ A$

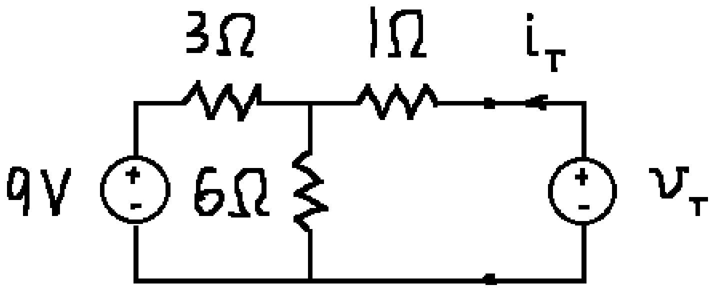


THEVENIN



NORTON

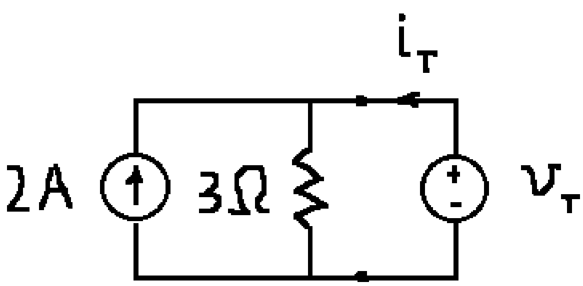
Confirm Equivalence



$$i_T = \frac{v_T}{3\Omega} - \frac{6}{7}\frac{9V}{(3+6/7)\Omega}$$

$$= v_T/3\Omega - 2A$$

$$= (v_T - 6V)/3\Omega$$



$$i_T = v_T/3\Omega - 2A$$

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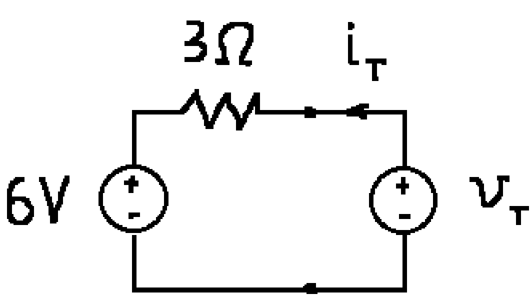
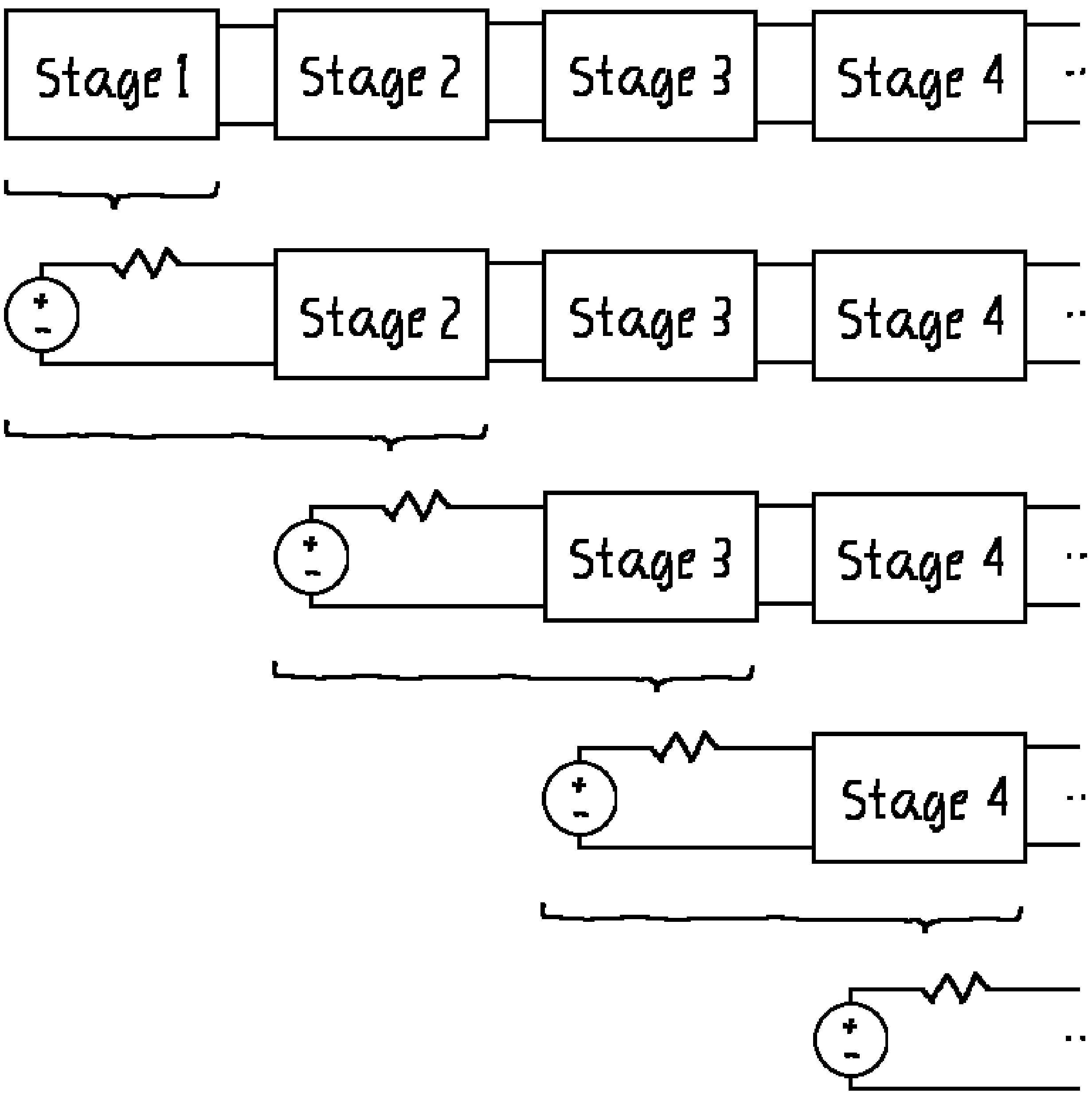


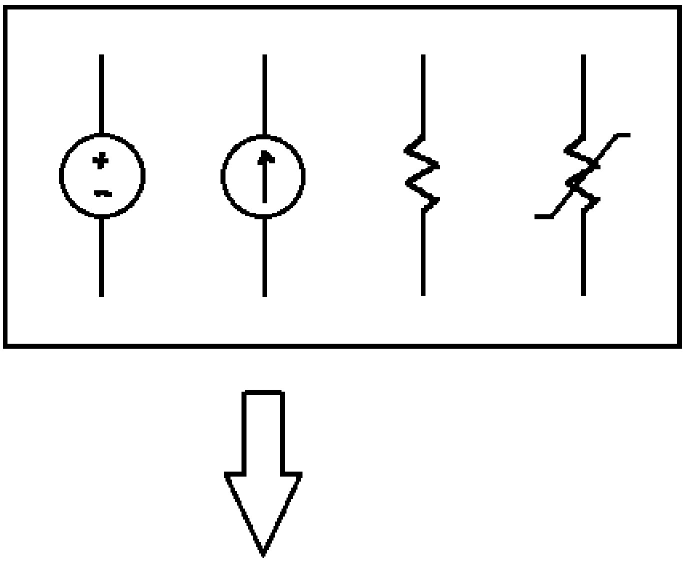
$$i_T = (v_T - 6V)/3\Omega$$

Use Case I

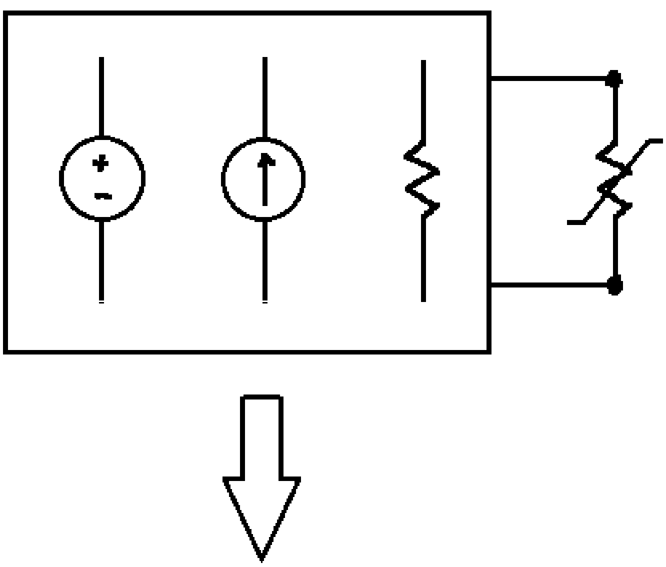
The successive use of Thevenin equivalence allows a complex circuit to be analyzed in steps as a cascade of simpler circuits.



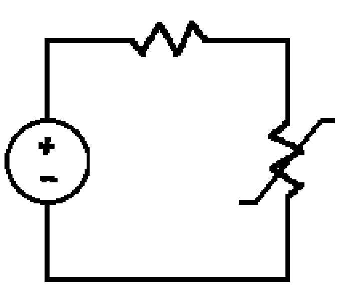
Use Case II



Arbitrary linear circuit plus one nonlinear resistor.

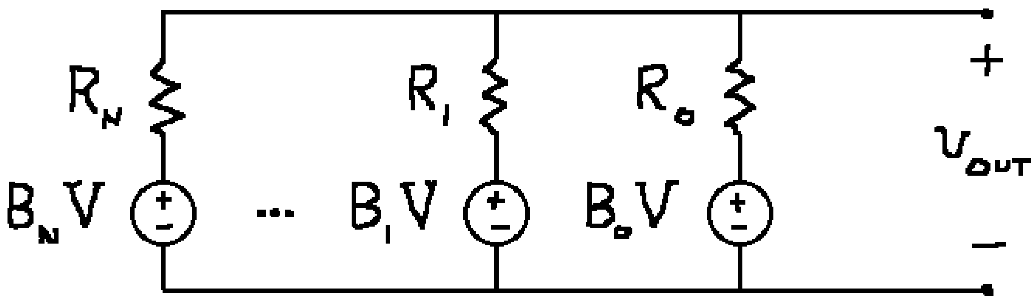


Pull the nonlinear resistor out from the arbitrary linear circuit.



Replace the arbitrary linear circuit with a Thevenin (or Norton) equivalent circuit.

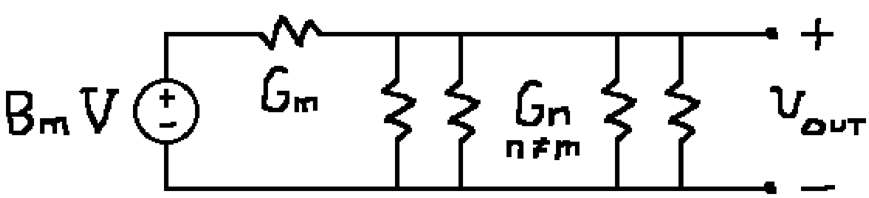
DAC Example



DAC ⇒ Digital Data = $\boxed{B_N \,|\, \cdots \,|\, B_1 \,|\, B_0}$

$$v_{OUT} = KV \sum_n B_n 2^n \; ; \; B_n = \{0, 1\}$$

Use superposition and voltage division:

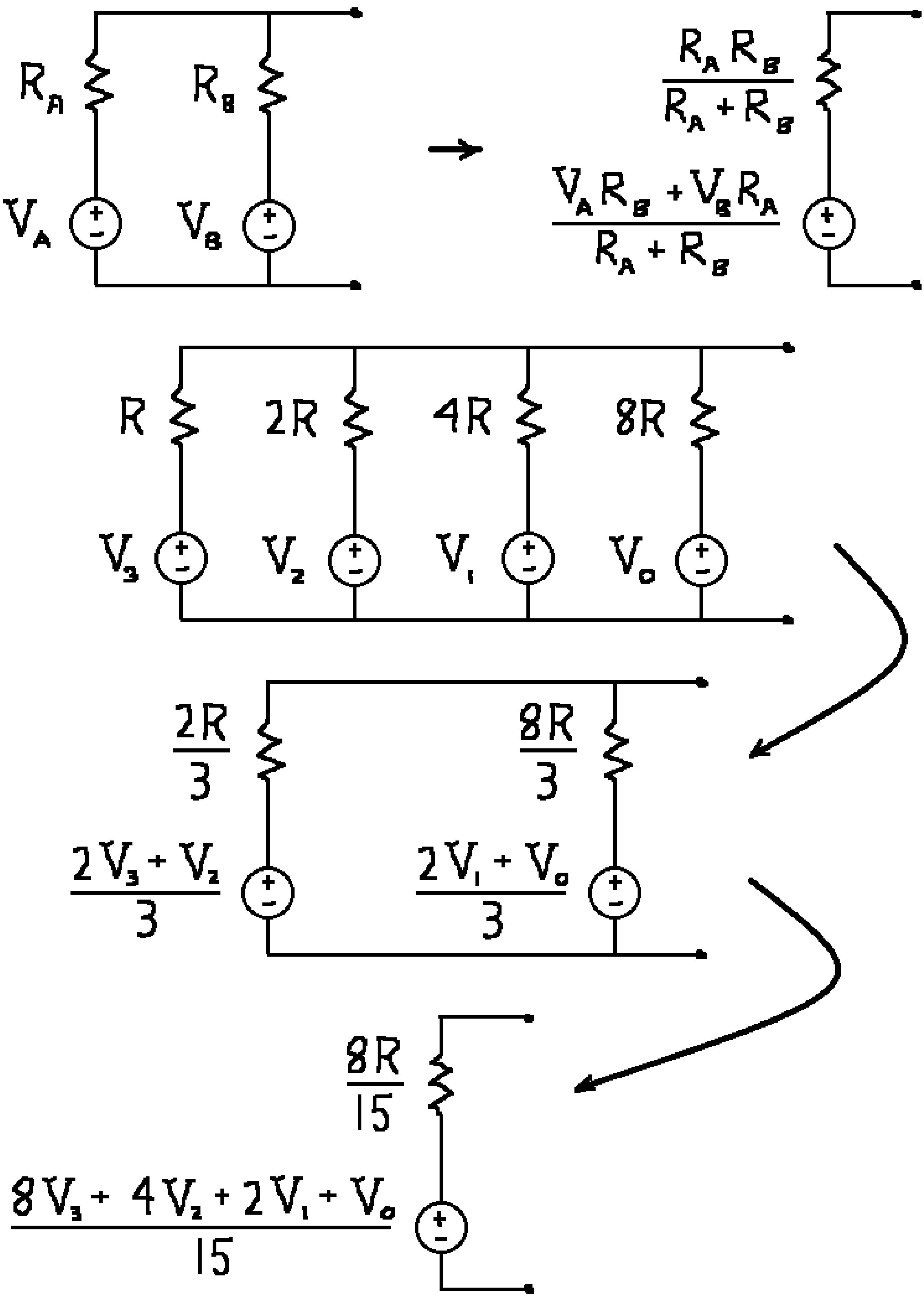


$$v_{OUT,m} = \frac{G_m B_m V}{\sum_{n \neq m} G_n + G_m} = \frac{G_m B_m V}{\sum_n G_n}$$

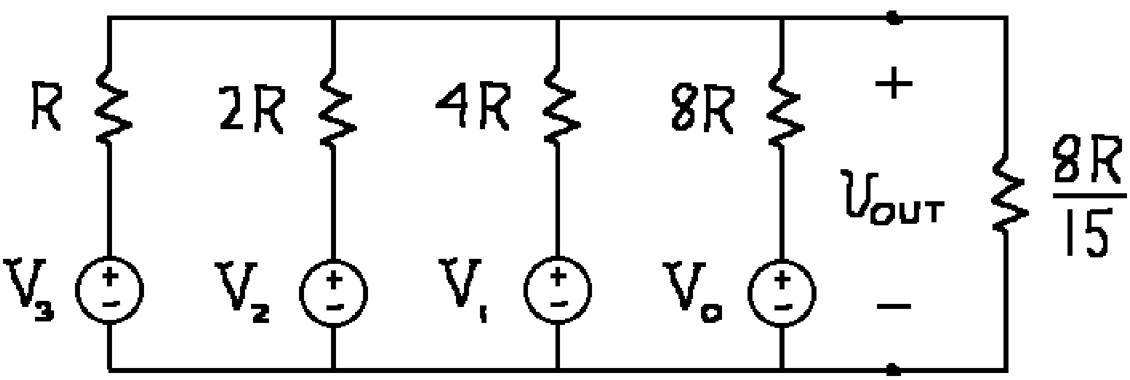
$$v_{OUT} = V\left(\sum_n G_n B_n\right) / \left(\sum_m G_m\right) \text{ ... by superposition}$$

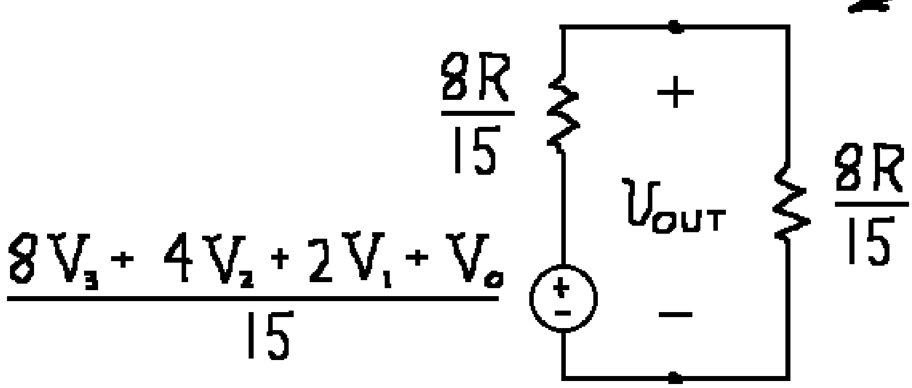
$$\text{Design} \Rightarrow G_n = G^* 2^n \Rightarrow K = 1/\left(\sum_m 2^m\right)$$

Thevenin / Norton Example #2



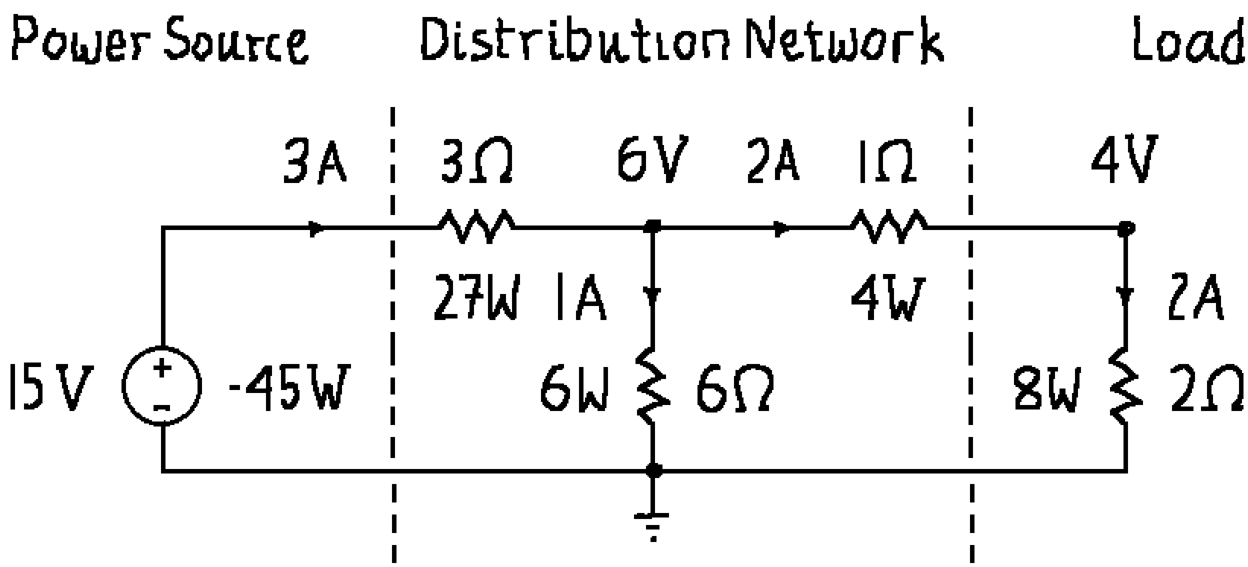
Thevenin / Norton Example #2

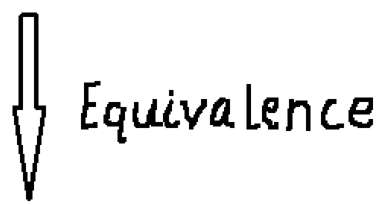


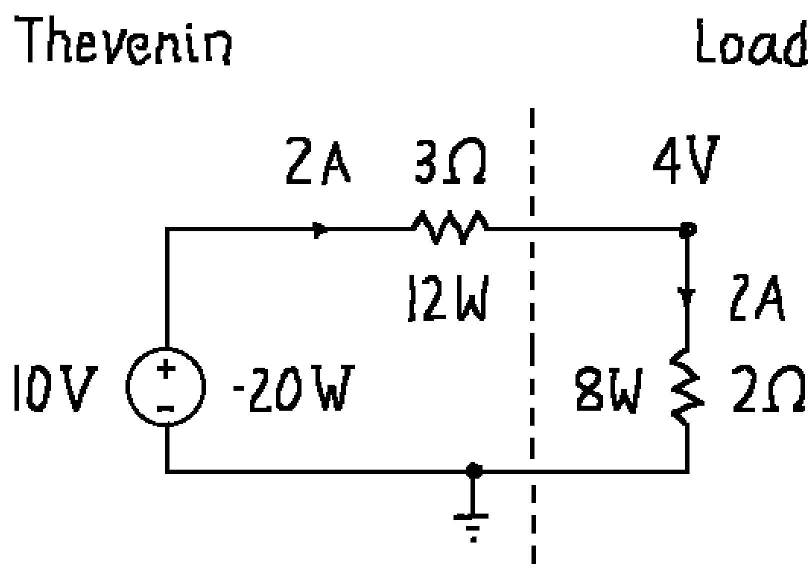


Loading the DAC with 8R/15 will reduce its output by 50%.

Thevenin / Norton Example #3

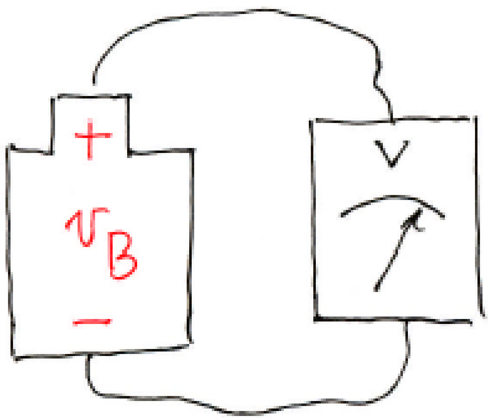


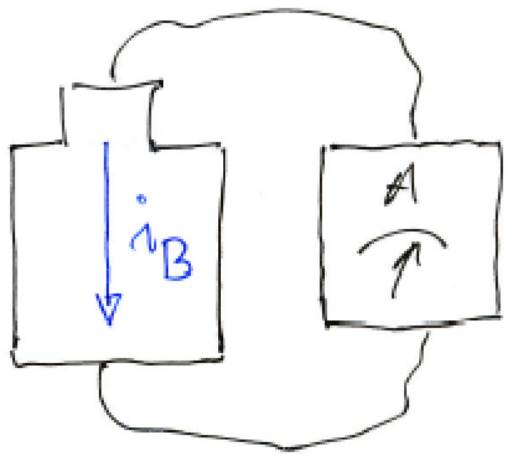




Thevenin and Norton equivalents do not preserve power considerations!

Experimental Battery Example



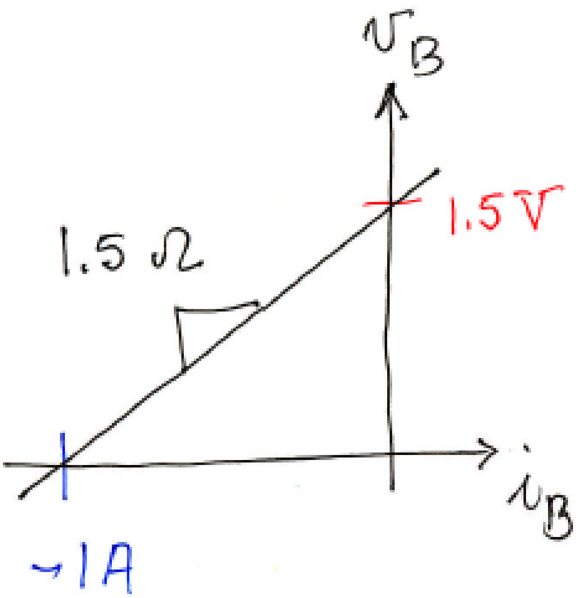


Open-Circuit Voltage

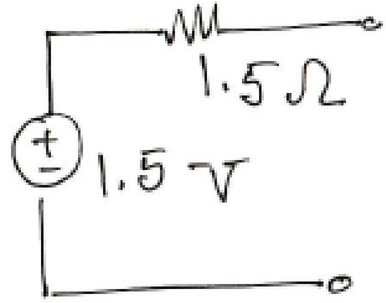
$v_B = 1.5\ \text{V}$

Short-Circuit Current

$i_B = -1\ \text{A}$



Thevenin Equivalent



5-Lecture Summary

- Two-terminal devices: V, I, R
- KCL and KVL
- Brute-force analysis
- Node analysis
- Parallel and series reductions
- Voltage and current dividers
- Superposition
- Thevenin and Norton equivalence