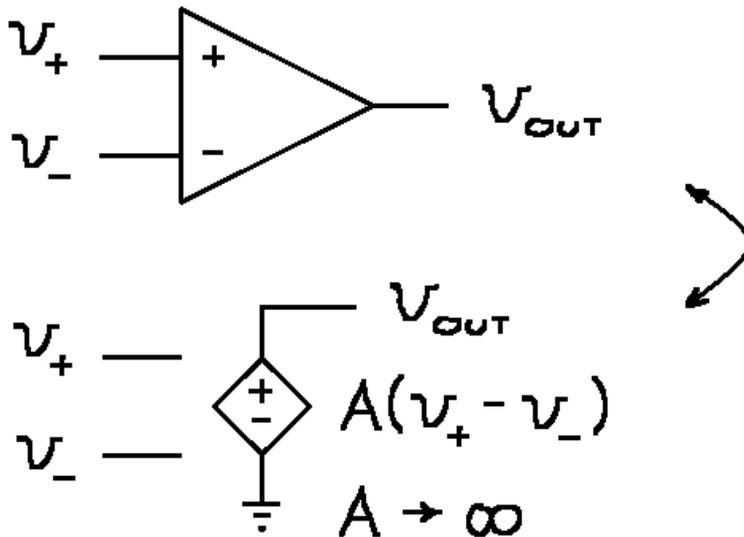


6.200 - Lecture 08

Op Amps & Amplifiers

- Ideal Op Amps
- Superposition
- Op-Amp-Based Amplifiers
- Analog Computing

Ideal Op Amp



v_+ , v_- and v_{out} are referenced to ground

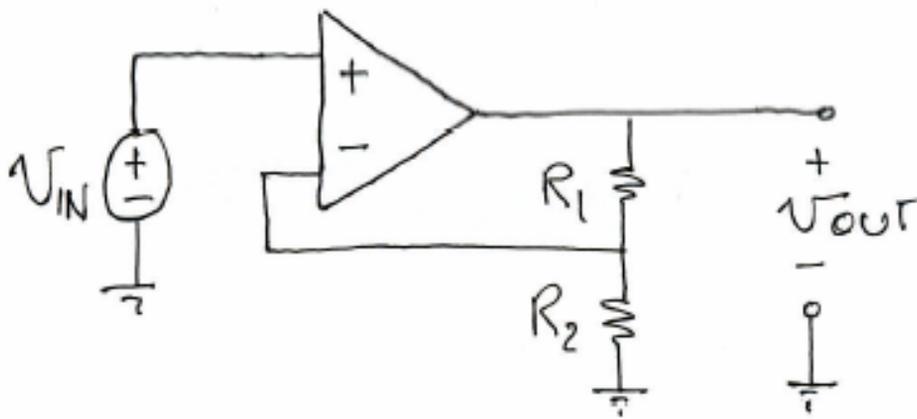
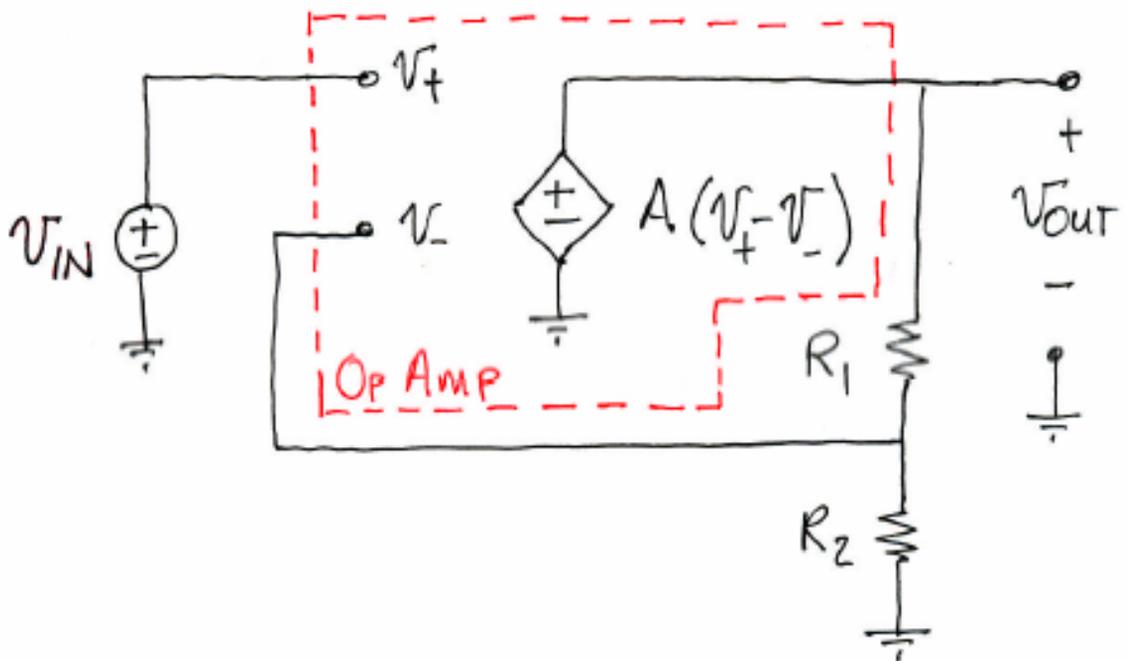
- Zero input currents
- Zero output resistance
- Algebraic differential output voltage
- Differential gain $A \rightarrow \infty$

$$\underline{V_+ = V_-}$$

- Ideal op amp $\Rightarrow v_+ - v_- = v_{\text{OUT}}/A$
- Stable system \Rightarrow finite v_{OUT}
- $\lim A \rightarrow \infty \Rightarrow v_+ - v_- = v_{\text{OUT}}/A \rightarrow 0$
 \Rightarrow $v_+ = v_-$

Non-Inverting Amplifier

Use negative feedback to trade gain for more ideal performance.



Power supply connections not shown.

Old Analysis

Node Method: $\frac{v_-}{R_2} + \frac{v_- - A(v_+ - v_-)}{R_1} = 0$



$$v_- = \frac{A R_2}{R_1 + (A+1) R_2} v_{IN}$$

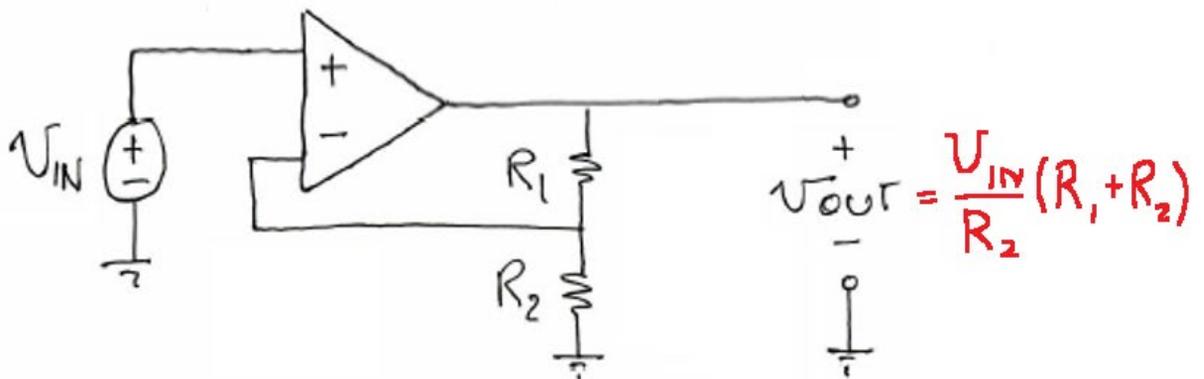
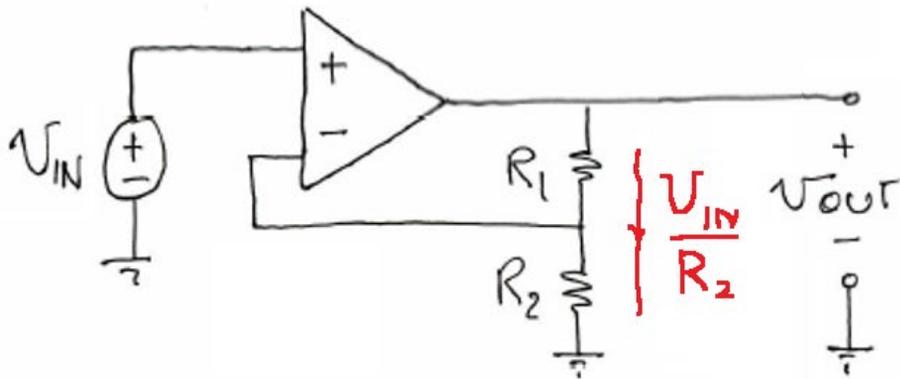
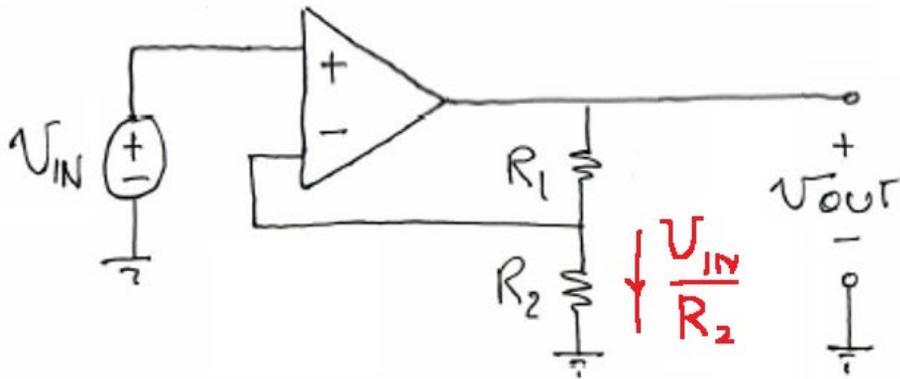
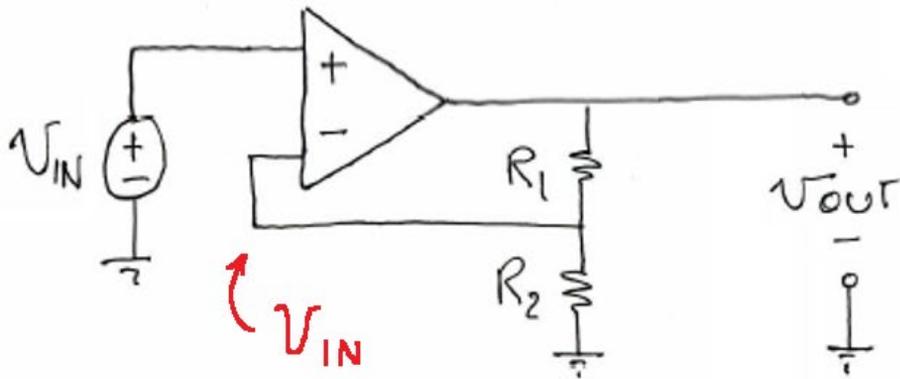
$$v_{out} = A (v_+ - v_-)$$


$$= \frac{A \cdot G}{A + G} v_{IN} \quad G \equiv \frac{R_1 + R_2}{R_2}$$

If $A \gg G$ then $v_{out} = G v_{in}$

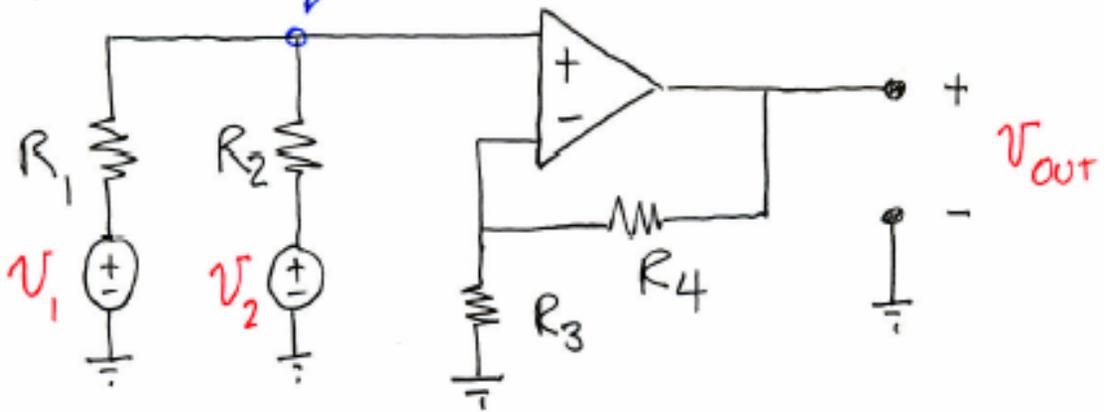

Dependent only
on resistors.

New Analysis Using $v_+ = v_-$

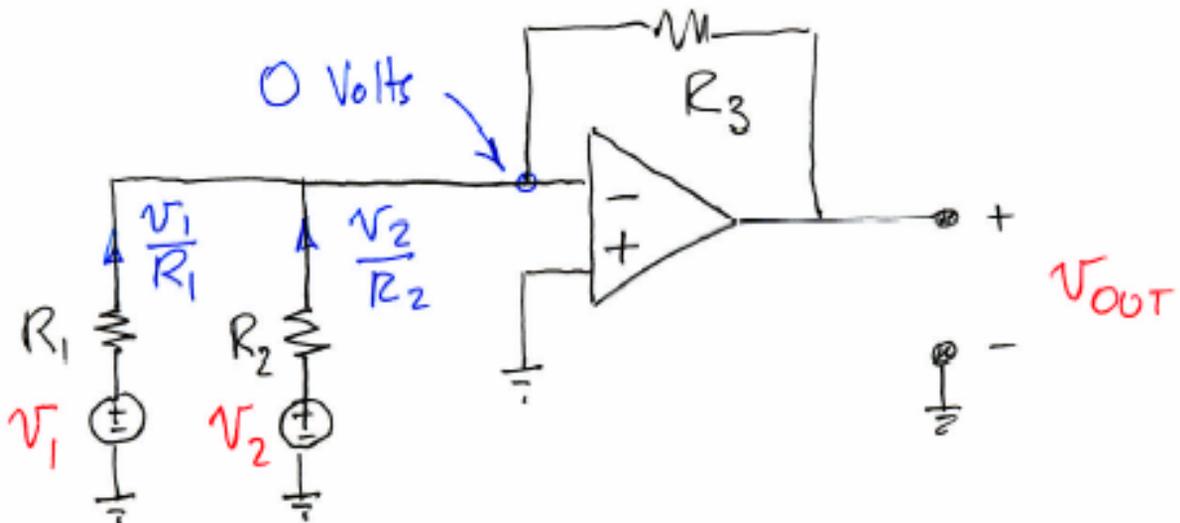


Addition

$$\frac{R_2 V_1 + R_1 V_2}{R_1 + R_2}$$

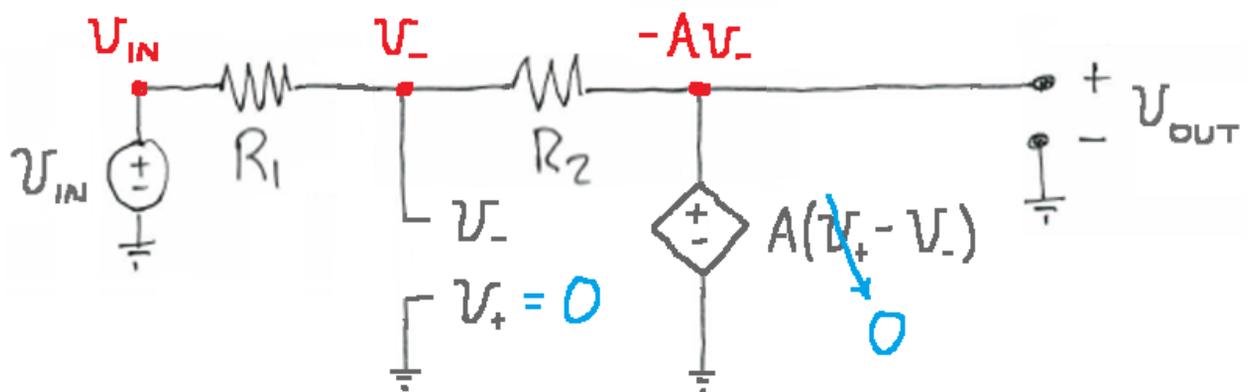
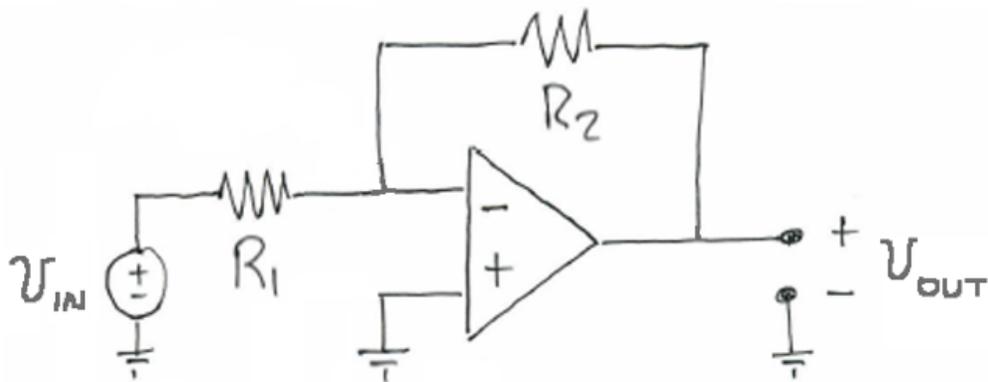


$$V_{OUT} = \frac{R_4 + R_3}{R_3} \left[\frac{R_2 V_1 + R_1 V_2}{R_1 + R_2} \right]$$



$$V_{OUT} = -R_3 \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} \right]$$

Inverting Amplifier

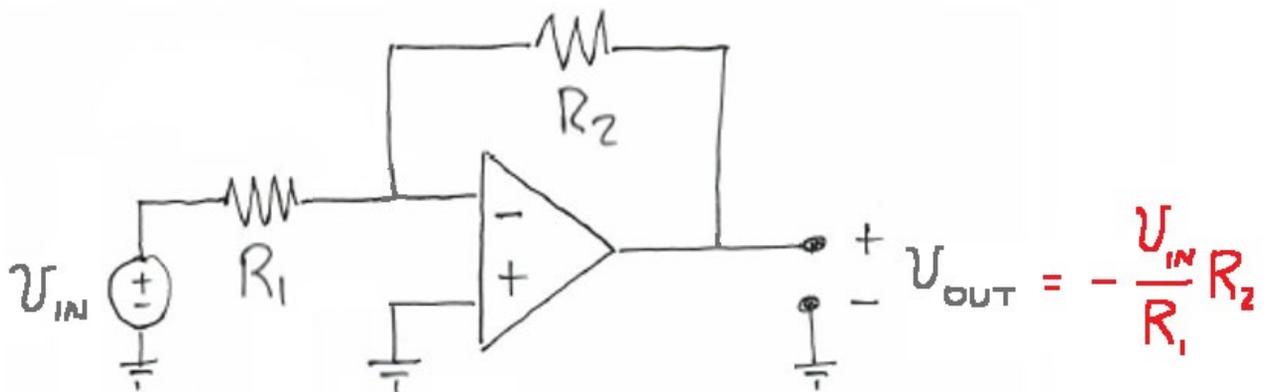
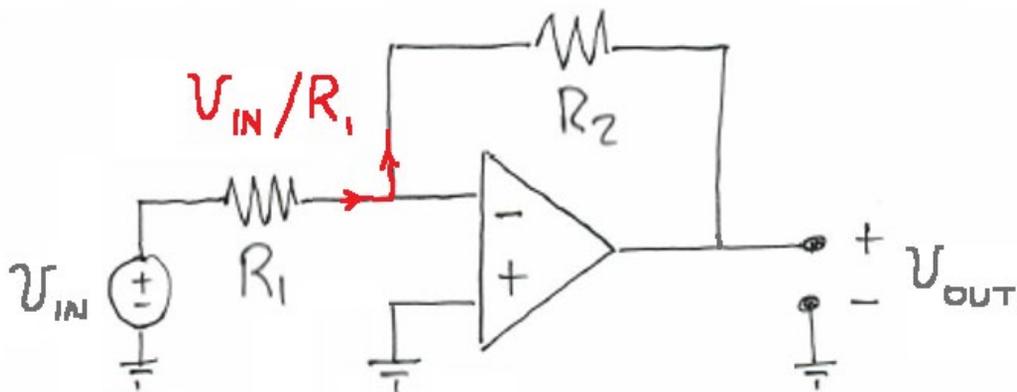
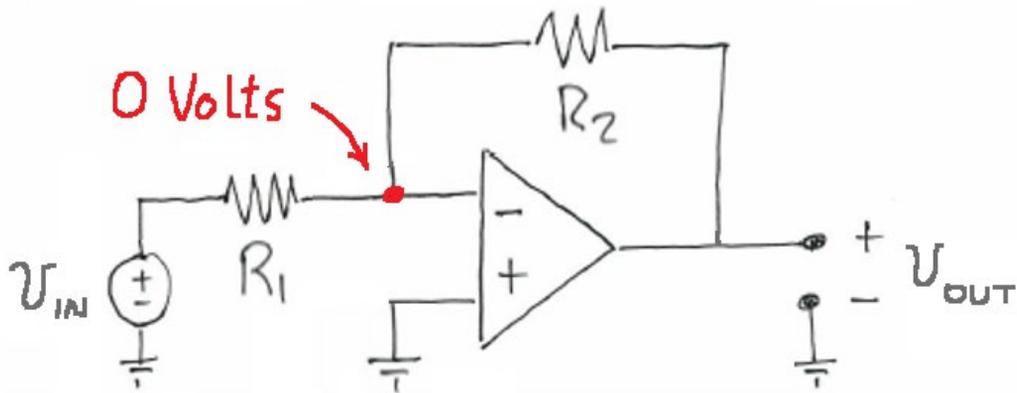


$$\frac{V_- - V_{IN}}{R_1} + \frac{V_- + AV_-}{R_2} = 0 \rightarrow V_- = \frac{R_2 V_{IN}}{(A+1)R_1 + R_2}$$

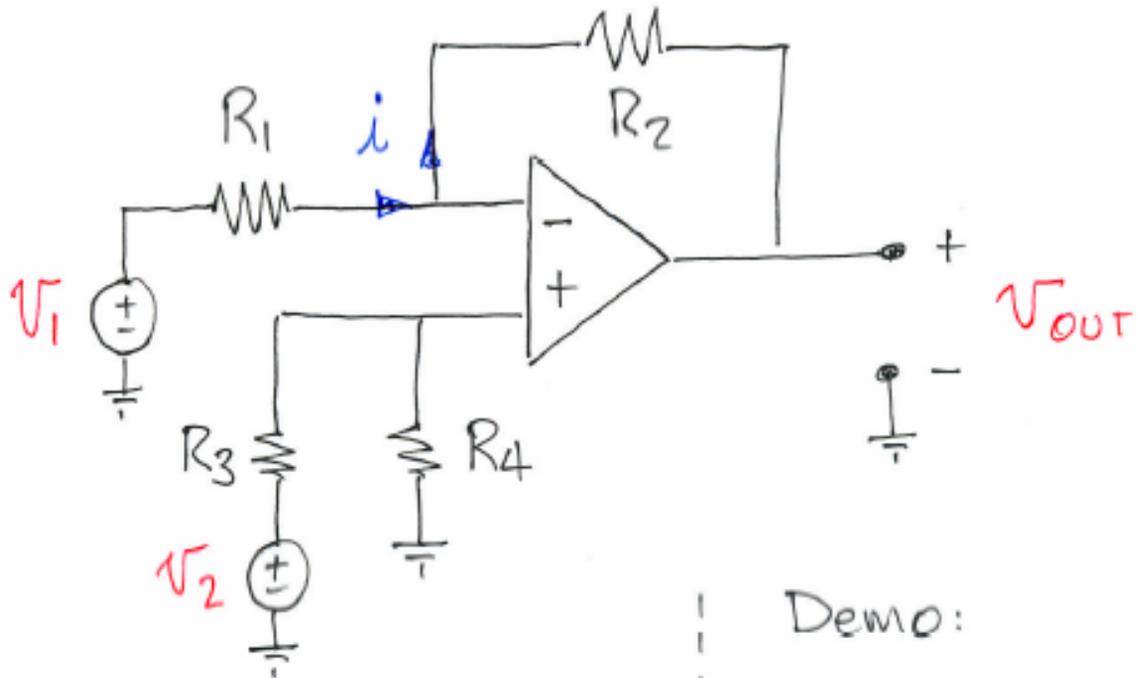
$$V_{OUT} = -AV_- = -\frac{AG}{A+1+G} V_{IN}, \quad G = R_2/R_1$$

If $A \gg G$ then $V_{OUT} = -G V_{IN}$

New Analysis Using $v_+ = v_-$



Subtraction



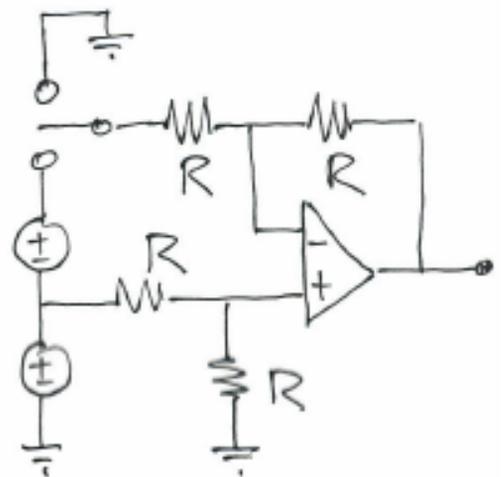
$$V_- \approx V_+ = \frac{R_4}{R_3 + R_4} V_2$$

$$i = \frac{V_1 - V_-}{R_1}$$

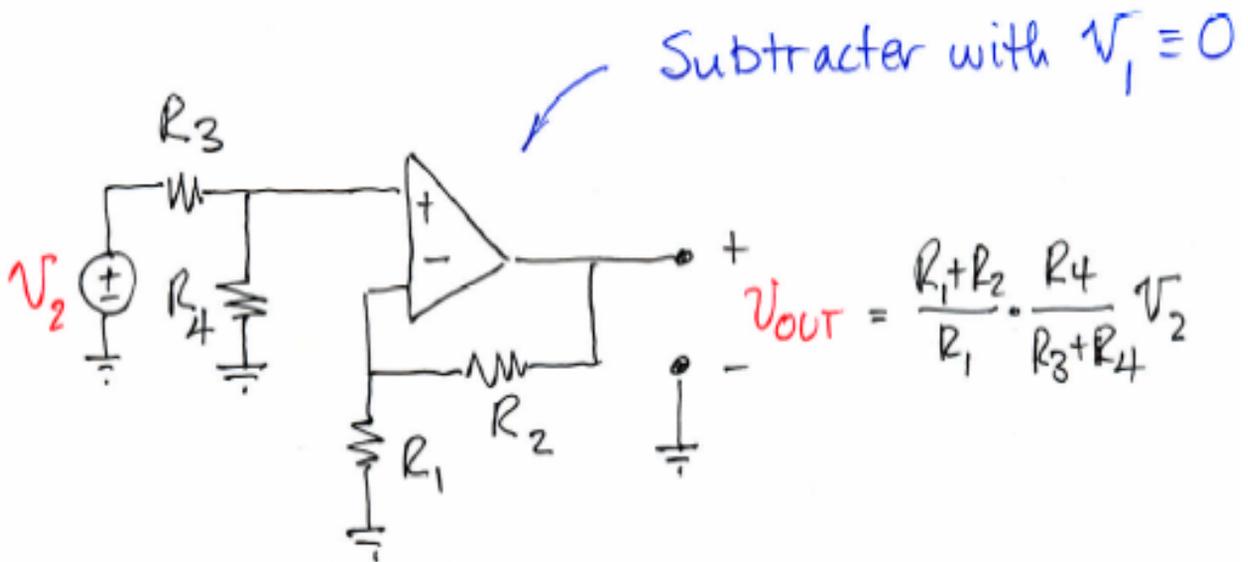
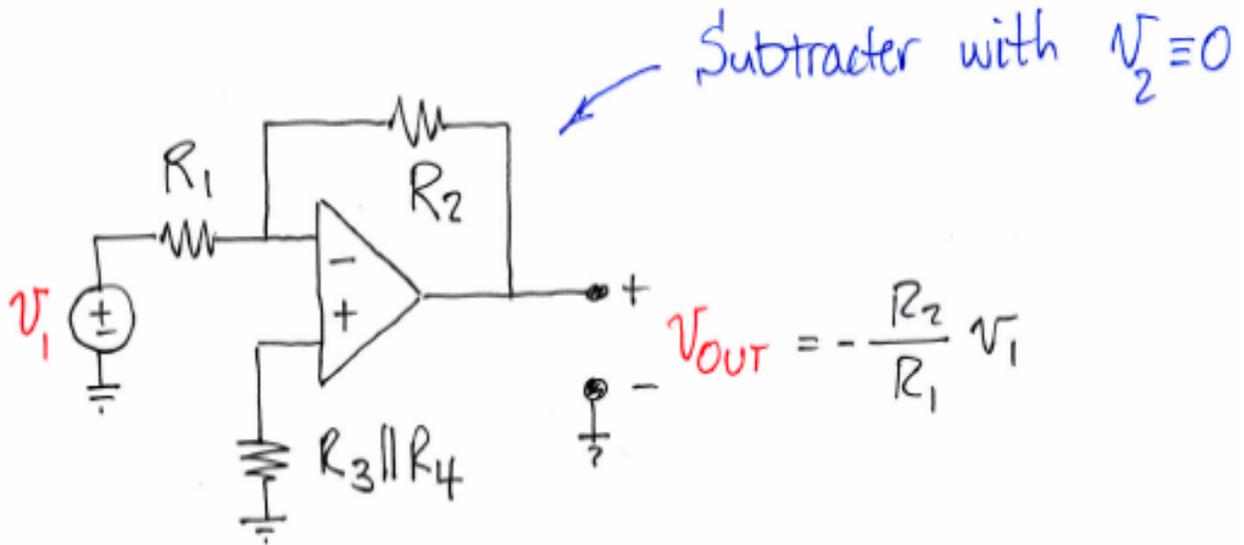
$$V_{OUT} = V_- - R_2 i$$

$$V_{OUT} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_1 + R_2}{R_1} V_2 - \frac{R_2}{R_1} V_1$$

Demo:

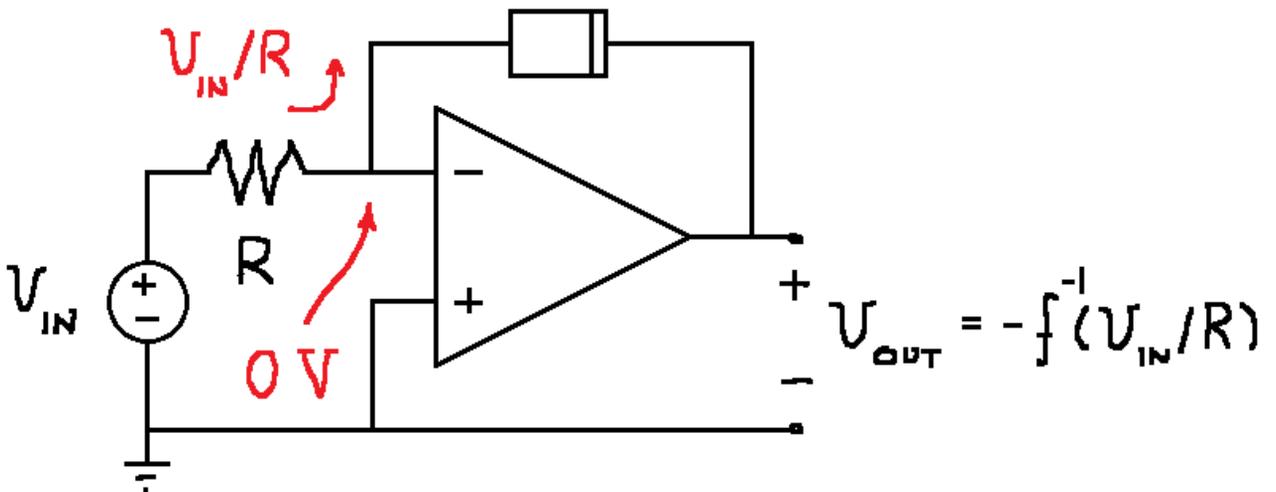
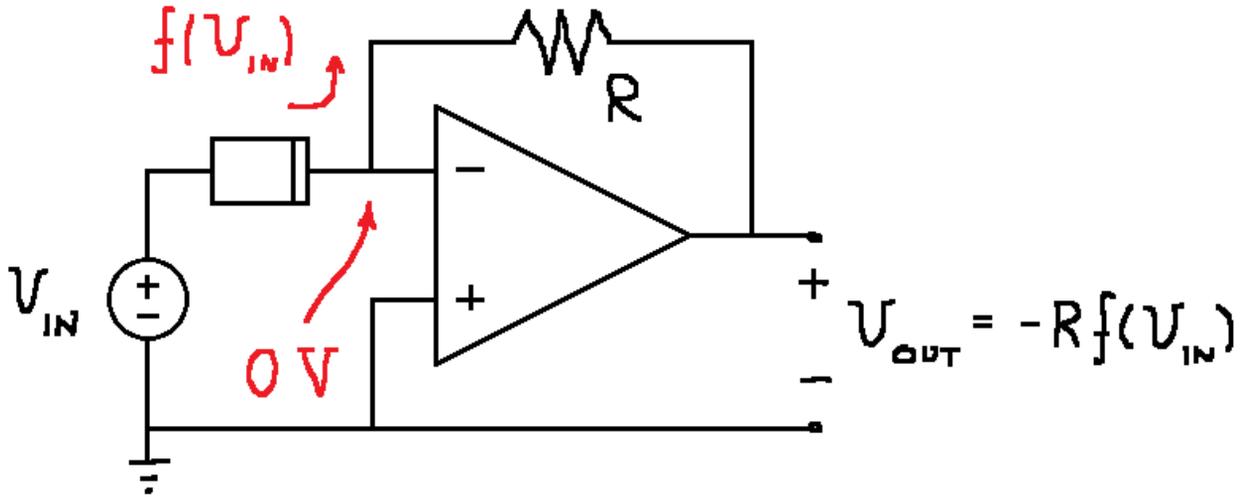
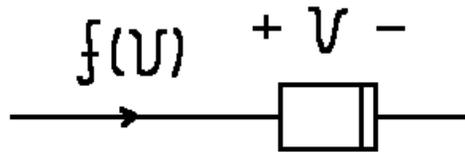


Subtraction By Superposition

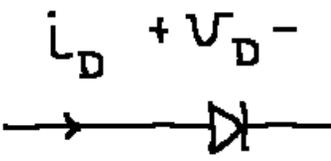


Subtractor:
$$V_{OUT} = \frac{R_1 + R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1$$

Nonlinear Functions



Exponential Diode

Exponential Diode: 

$$i_D = I_s \left[e^{v_D / n V_T} - 1 \right]$$

I_s = Saturation Current $\approx 1 \text{ pA} \dots 10 \text{ nA}$

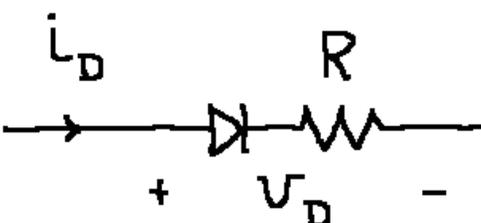
$$V_T = k_B T / q$$

$$k_B = 1.38 \cdot 10^{-23} \text{ J/K}$$

T = Temperature

$$q = 1.6 \cdot 10^{-19} \text{ C}$$

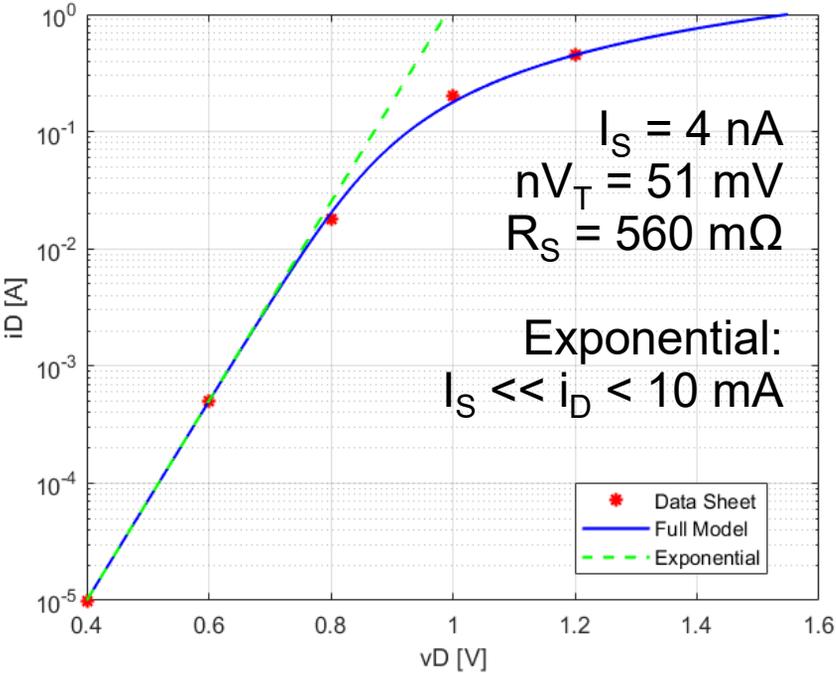
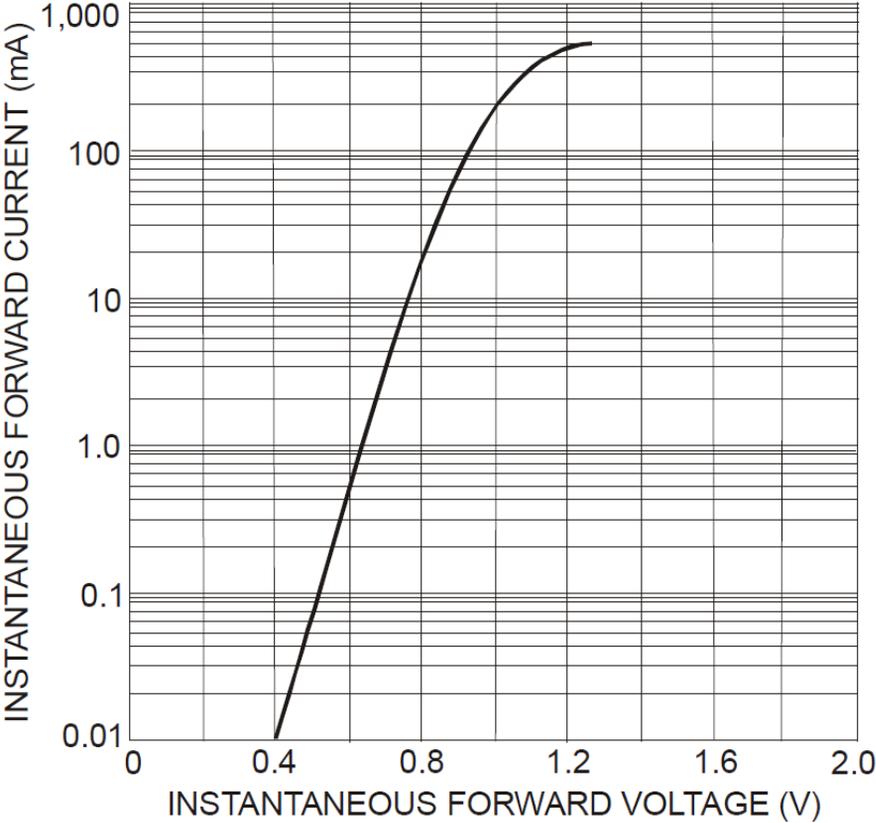
n = Ideality Factor $\approx 1 \dots 2$

Practical Diode: 

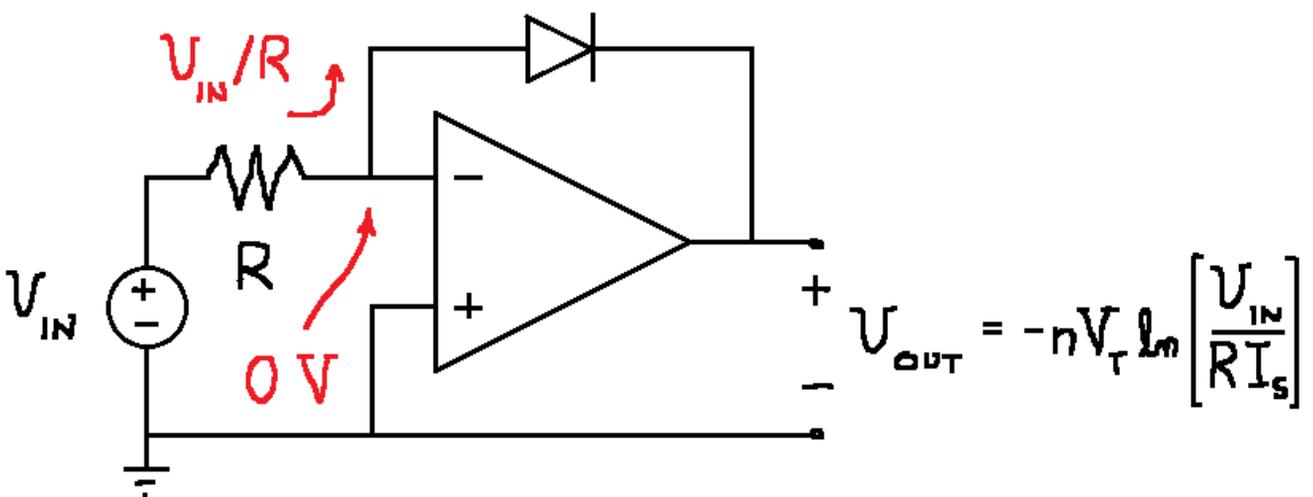
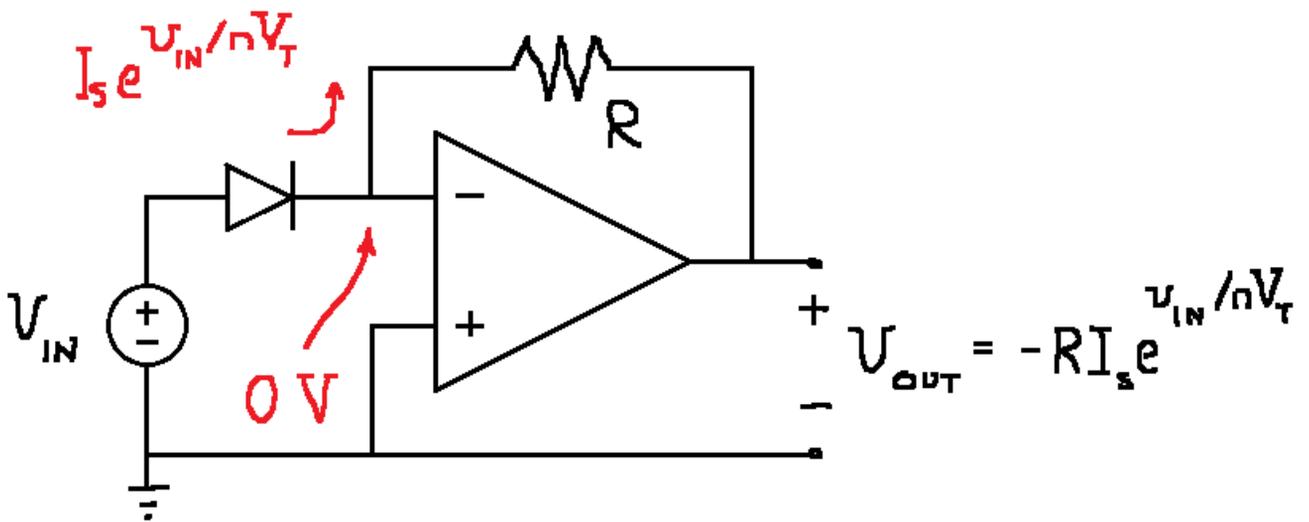
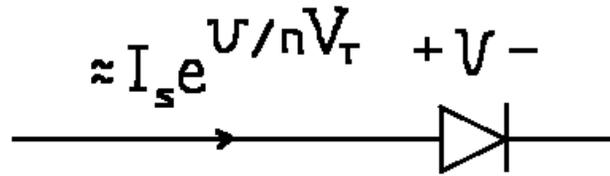
$$v_D = n V_T \ln (i_D / I_s + 1) + R i_D$$

R = Body Resistance $\approx \dots 1 \Omega \dots$

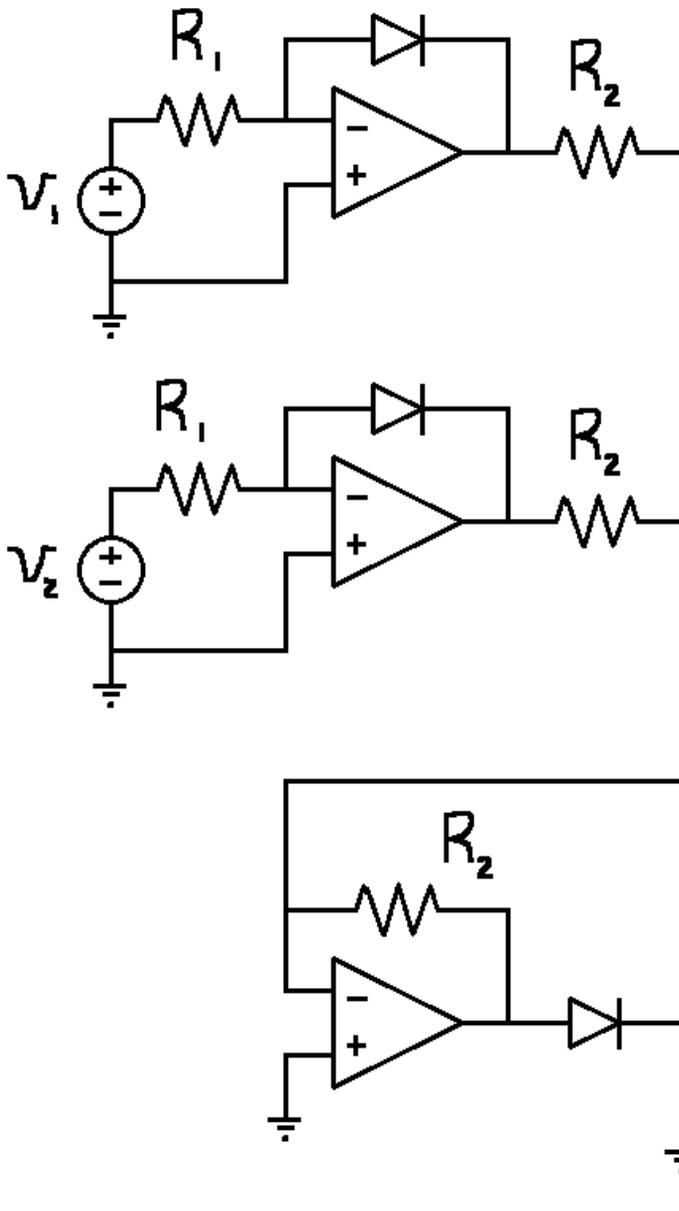
Exponential Diode – 1N4148



Exponential & Log Amps



Multiplier



$$R_1 = 500 \text{ k}\Omega$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_3 = 1 \text{ k}\Omega$$

$$I_s \approx 4 \text{ nA}$$

$$\begin{aligned}
 & - \frac{R_3 V_1 V_2}{R_1^2 I_s} \\
 & \approx - \frac{V_1 V_2}{1 \text{ V}}
 \end{aligned}$$