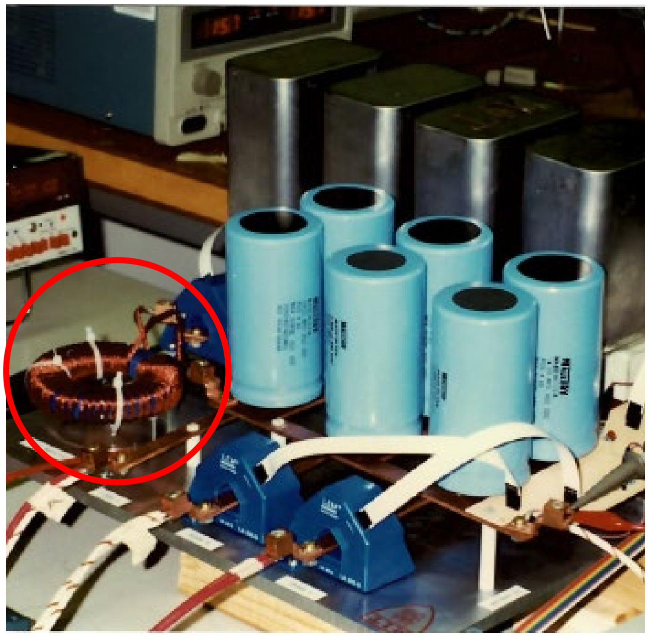
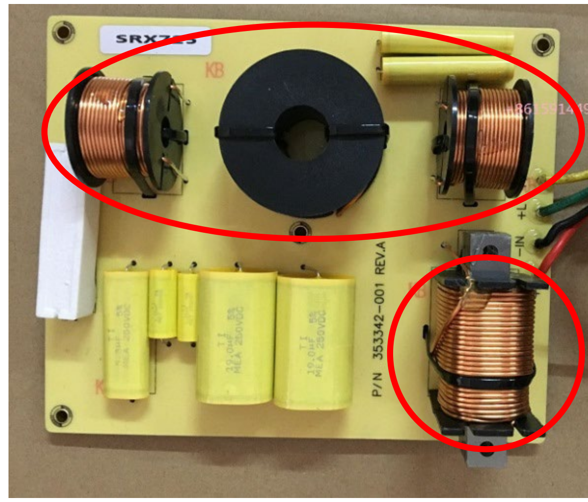
6.200 - Lecture 10

Inductors

- Inductors are Everywhere
- Uses
- Basic Physics
- First-Order ODEs

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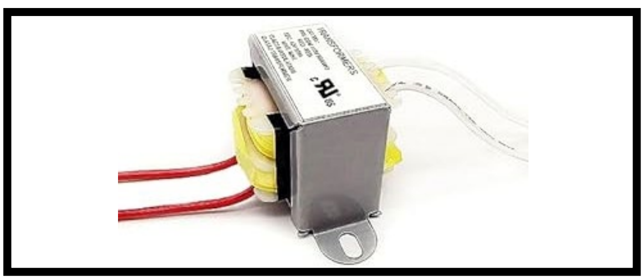

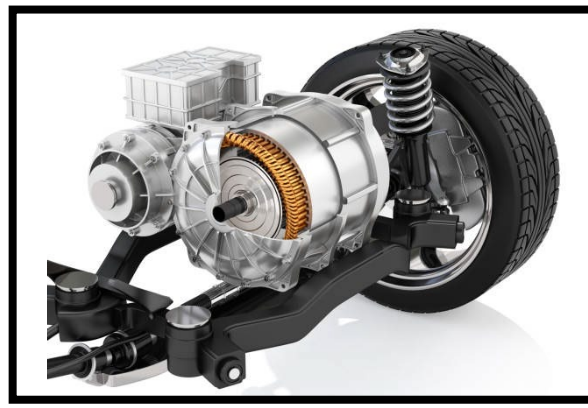


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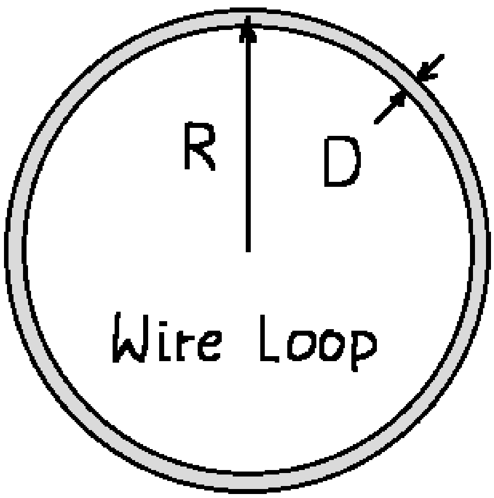
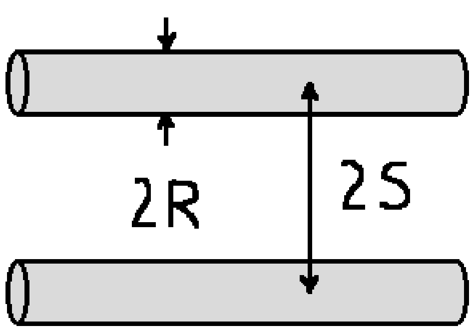
Wiring Inductance



$$L \approx \mu_0 R \left[\ln\left(\frac{16R}{D}\right) - 2 \right]$$



$$L = \frac{\mu_0}{\pi} \ln\left[\frac{S}{R} + \sqrt{\left(\frac{S}{R}\right)^2 - 1} \right]$$

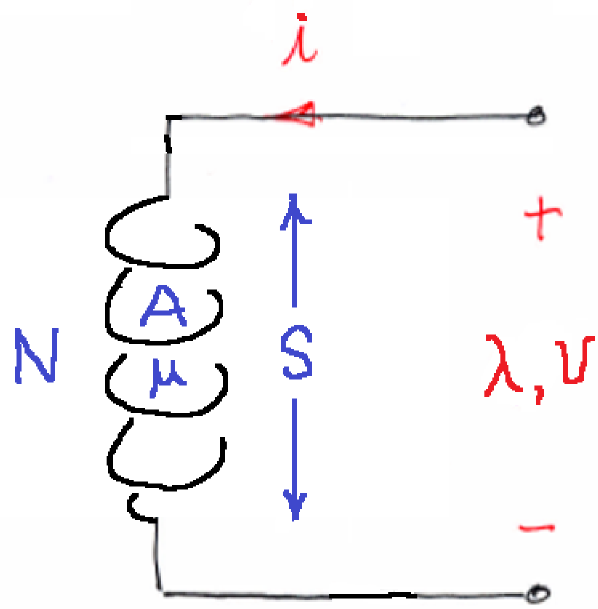
↑ Per Length

Uses

Inductor Uses:

- Energy Storage
- Energy Transduction (Transformers)
- Position and Motion Sensing
- Actuation (Motors)
- Generation (Generators)
- Memory (Original Core Memory!)
- Frequency-Domain Filtering
- Time-Domain Dynamics
- Resonators
- Timing

Ideal Inductor



Henries

$$\lambda = L\, i$$

Webers — Amps

$$L = \frac{\mu N^2 A}{S}$$

$$\lambda = \int_{-\infty}^{t} v\, dt \qquad v = \frac{d\lambda}{dt} = \frac{d}{dt}(L i) = L \frac{di}{dt}$$

Memory — 6.200

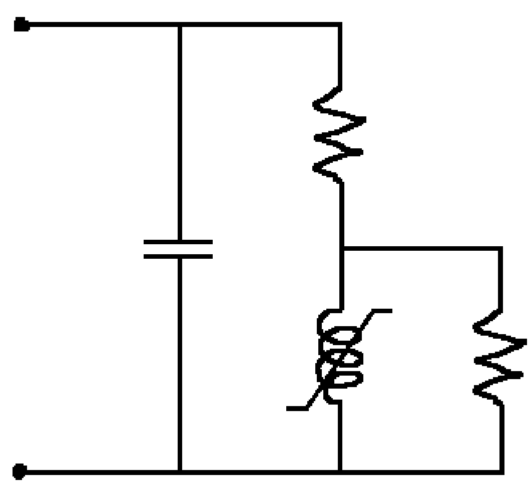
$$P = v i = L i \frac{di}{dt} = \frac{d}{dt}\left(\frac{1}{2} L i^2\right)$$

Two signs Possible ⇒ Reversible — Stored Energy

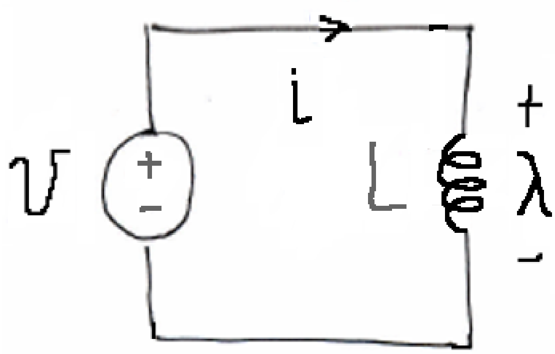
Common Nonidealities

- Winding series resistance
- Core loss and saturation
- Winding capacitance

A lumped-parameter model can often represent these non ideal behaviors.



Dynamics & Memory



$$\lambda = Li$$

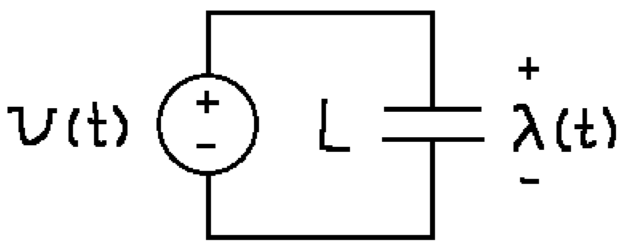
$$v = d\lambda/dt$$

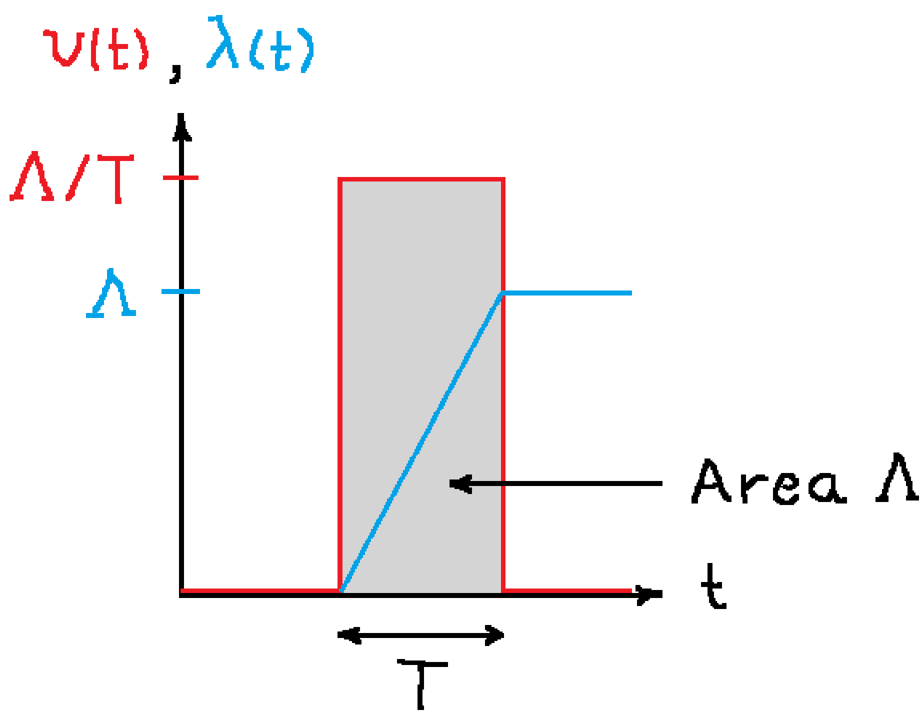
$$\lambda(t) = \int_{-\infty}^{t} v(t)\,dt = \int_{-\infty}^{t_0} v(t)\,dt + \int_{t_0}^{t} v(t)\,dt$$

$$= \underbrace{\lambda(t_0)}_{\text{Memory}} + \int_{t_0}^{t} v(t)\,dt$$

- Flux linkage (and current) depends on the entire voltage history.
- Flux linkage summarizes (memorizes) the voltage history relevant to the future.

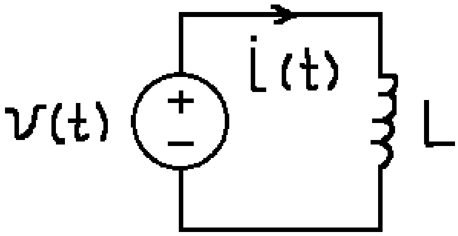
State Continuity





- Lim $T \rightarrow 0$: $\lambda(t) \rightarrow$ step and $v(t) \rightarrow \infty$
- Without infinite voltage, $\lambda(t)$ cannot step and so is continuous

Power & Energy

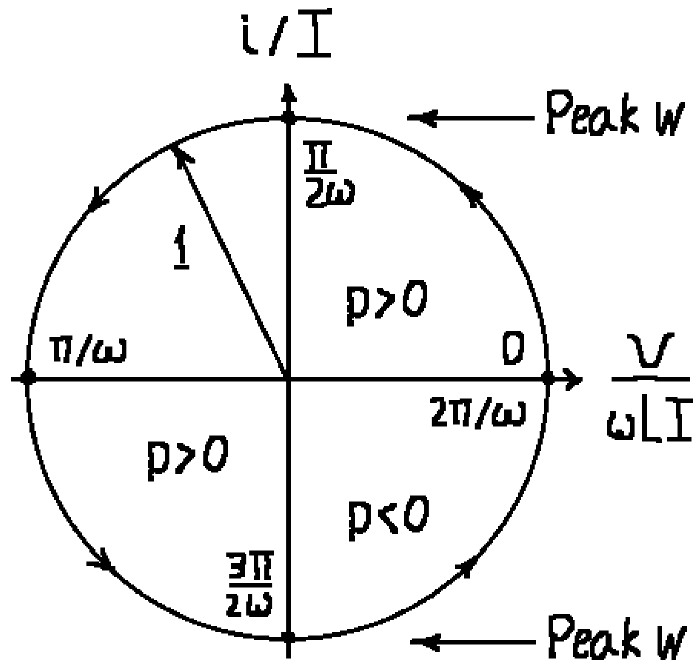


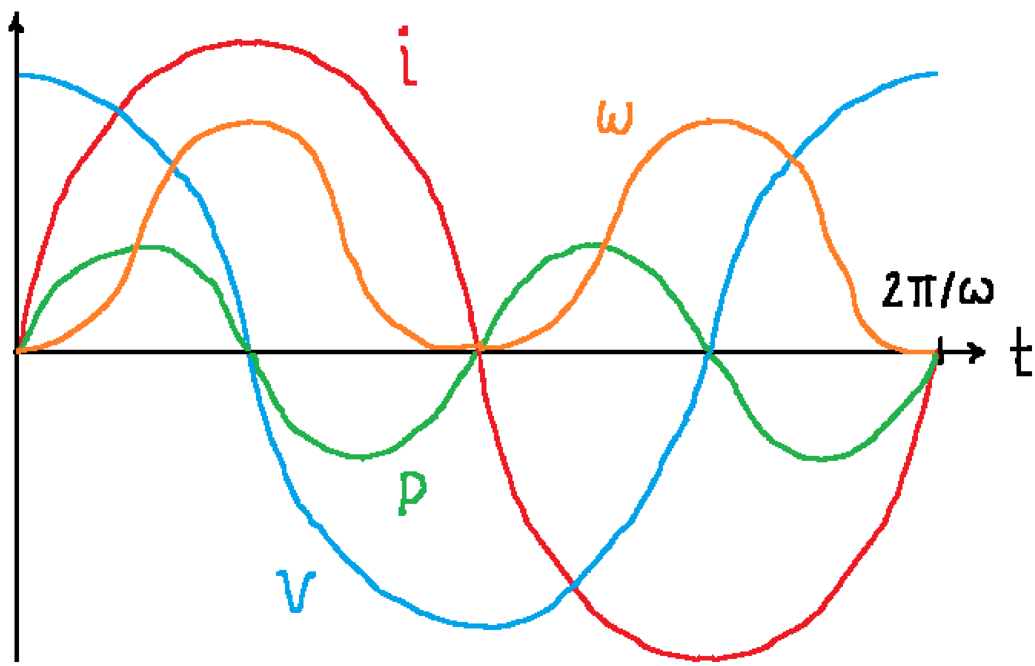
$i = I\sin(\omega t)$

$v = \omega L I\cos(\omega t)$

$p = \omega L I^2\sin(2\omega t)/2$

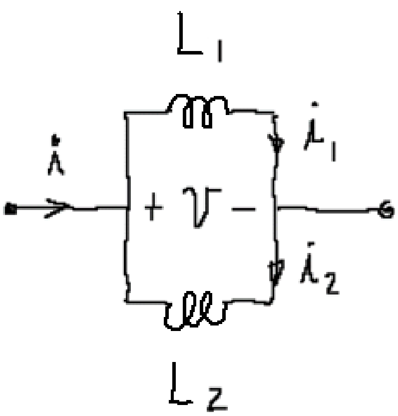
$w = L I^2\sin^2(\omega t)/2$





Inductor Combinations

Parallel

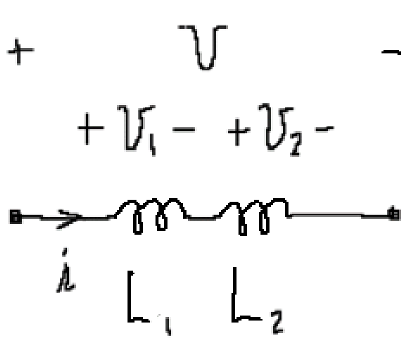


$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$
$$= \frac{1}{L_1} v + \frac{1}{L_2} v$$
$$= \underbrace{\left(\frac{1}{L_1} + \frac{1}{L_2} \right)} v$$

Effective reciprocal inductance

⇒ reciprocal parallel inductances add

Series

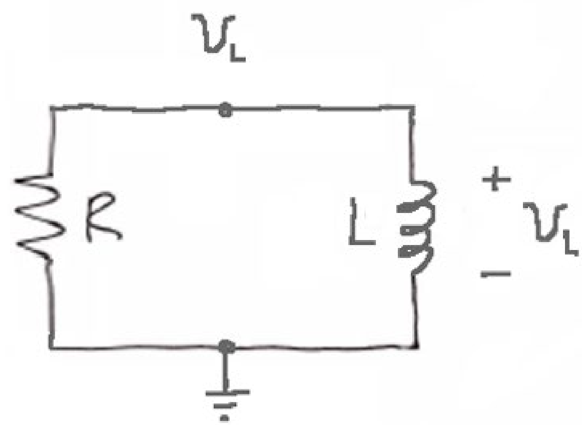


$$v = v_1 + v_2$$
$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$
$$= \underbrace{(L_1 + L_2)} \frac{di}{dt}$$

Effective inductance ⇒

Series inductances add

RL Node Analysis



Node Method $\Rightarrow$ $\dfrac{v_L}{R} + \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v_L(t)\,dt = 0$

$$\frac{L}{R}\frac{dv_L}{dt} + v_L = 0$$

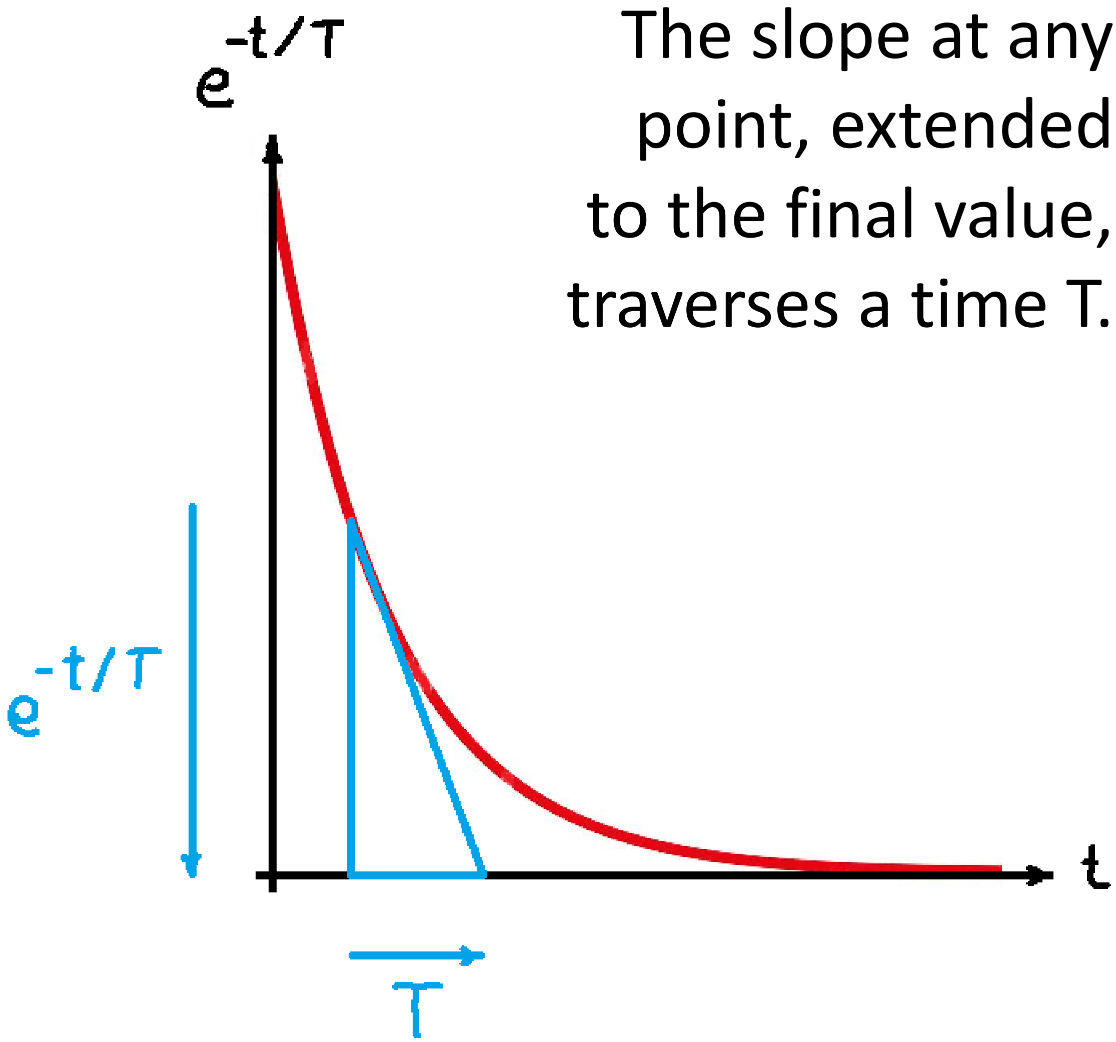
Time

Well-Posed Problem $\Rightarrow$ $v_L(0) = -R\,i_L(0)$ Given

Solution $\Rightarrow$ $v_L(t) = v_L(0)\, e^{-t/(L/R)}$

Exponential Decay

$$\frac{d}{dt} e^{-t/T} = \frac{-e^{-t/T}}{T} = \text{Slope} = \frac{\text{Rise}}{\text{Run}}$$



The slope at any point, extended to the final value, traverses a time T.