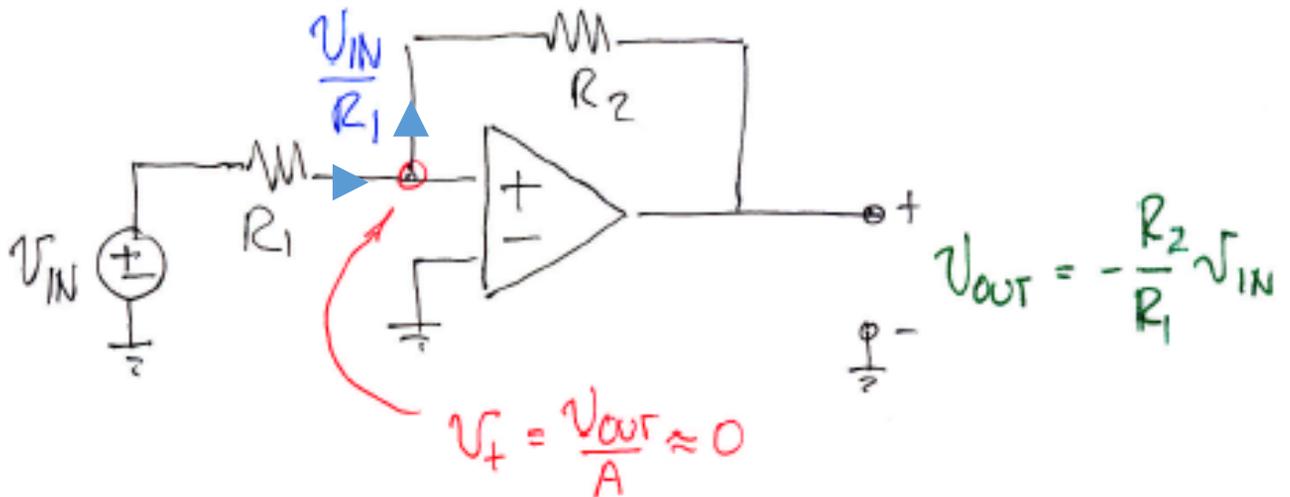
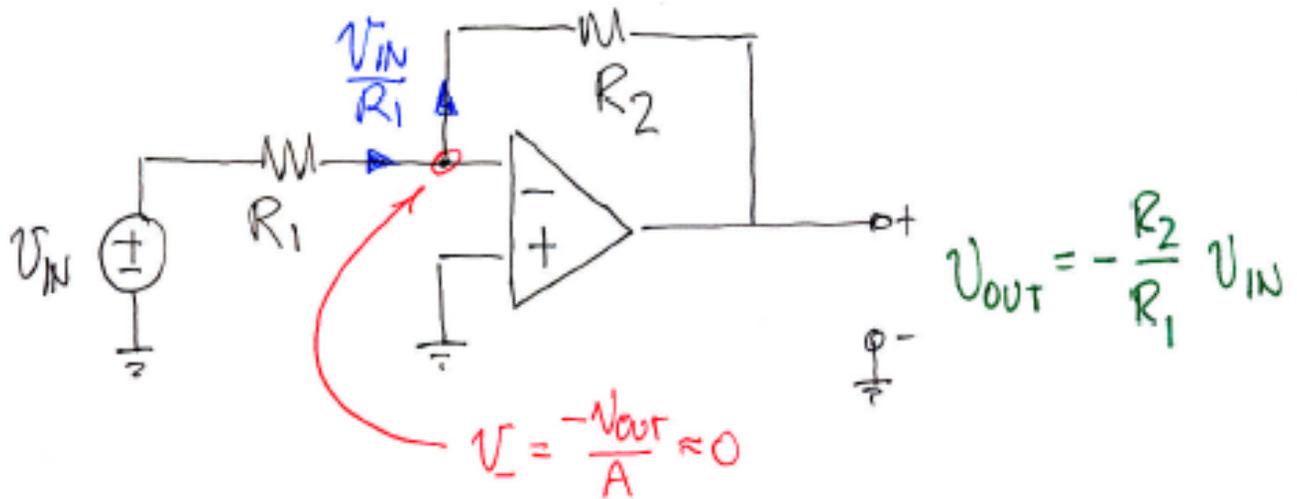


6.200 - Lecture 13

Stability

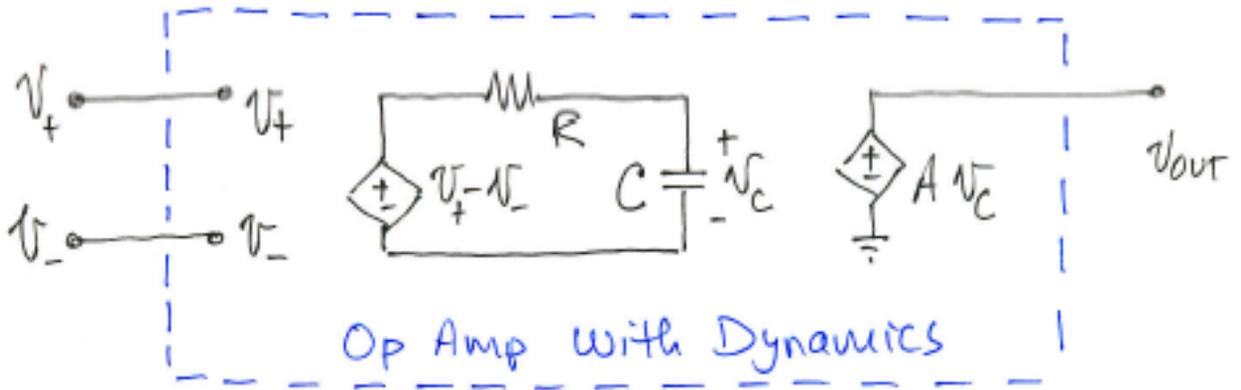
- Static Equilibria
- Pos/Neg Feedback
- Dynamics & Stability
- Comparators
- Schmitt Trigger
- Relaxation Oscillator

Equilibria

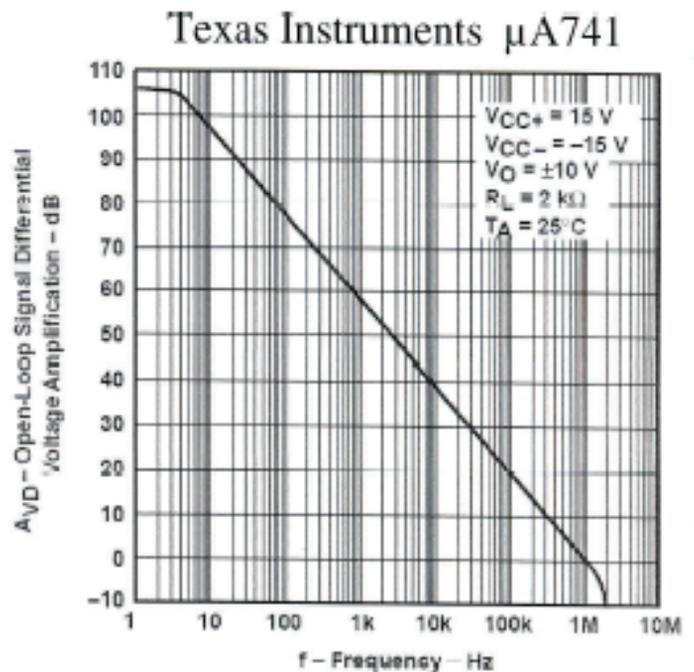
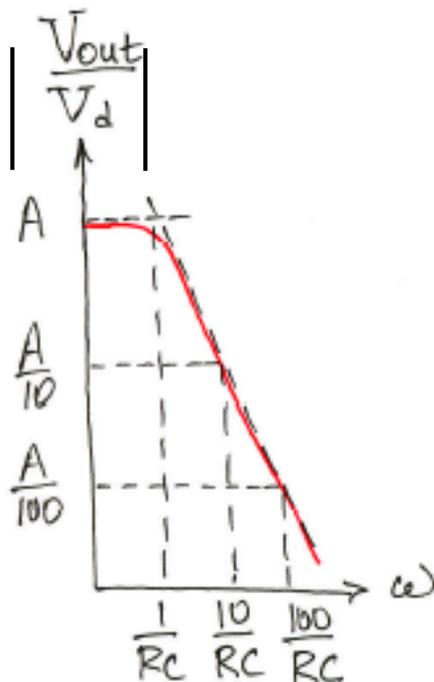


Do these two circuits really behave in the same way?

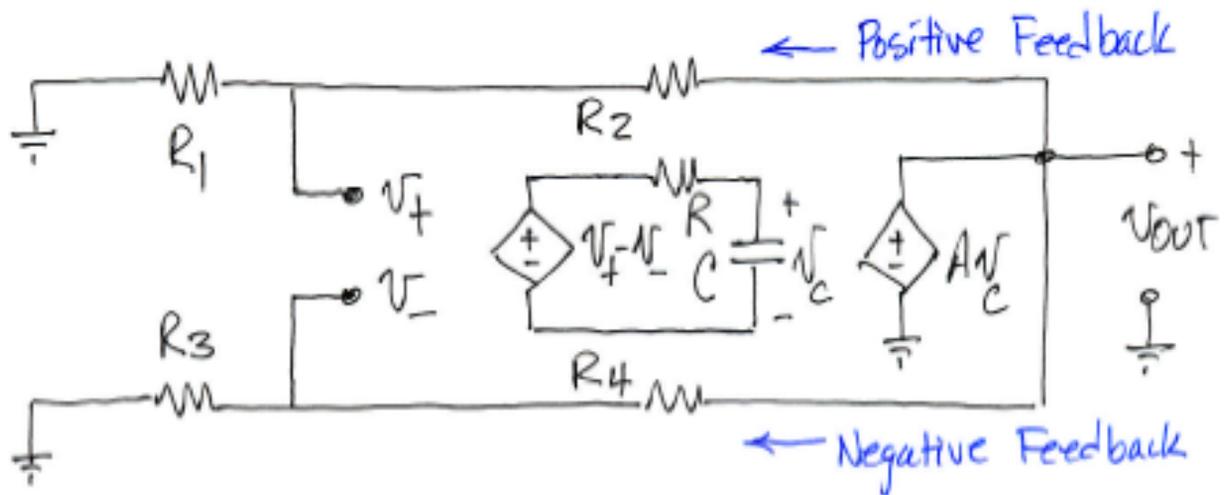
Op Amp Dynamics



$$v_+ - v_- \equiv v_D \rightarrow v_D e^{j\omega t} \Rightarrow v_{out} \rightarrow \frac{A v_D}{1 + j\omega RC} e^{j\omega t}$$



Amplifier Dynamics I



$$V_{out} = A v_c$$

$$RC \frac{dv_c}{dt} + v_c = v_+ - v_-$$

$$v_+ = \frac{R_1}{R_1 + R_2} v_{out} \equiv \delta^+ v_{out} \quad (\text{Positive Feedback})$$

$$v_- = \frac{R_3}{R_3 + R_4} v_{out} \equiv \delta^- v_{out} \quad (\text{Negative Feedback})$$

$$\frac{RC}{A} \frac{dv_{out}}{dt} + \left[\frac{1}{A} + \delta^- - \delta^+ \right] v_{out} = 0$$

↖ Small as $A \rightarrow \infty$

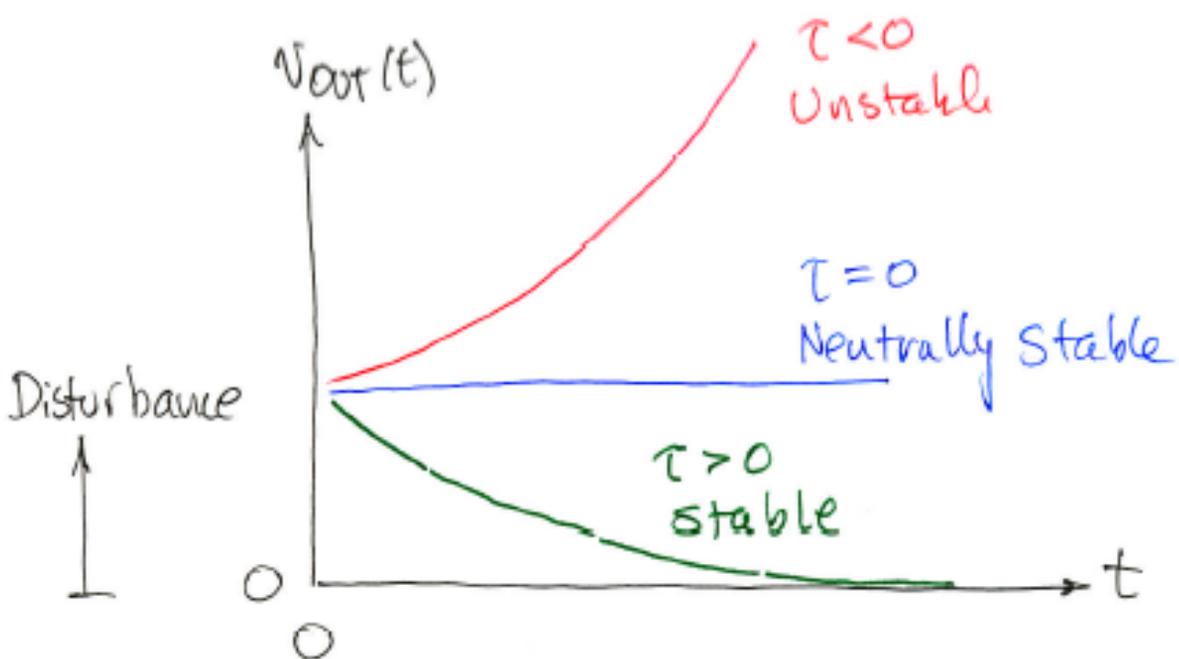
$$\frac{dv_{out}}{dt} + \frac{A(\delta^- - \delta^+)}{RC} v_{out} = 0$$

Amplifier Dynamics II

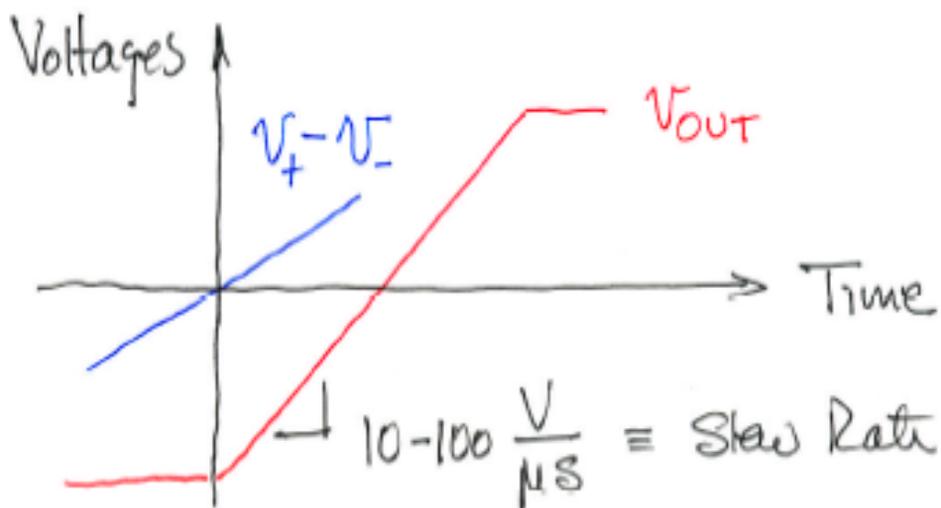
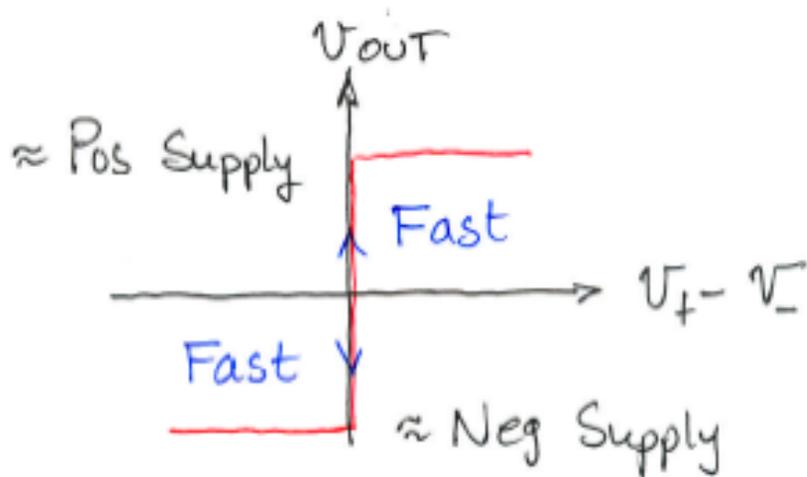
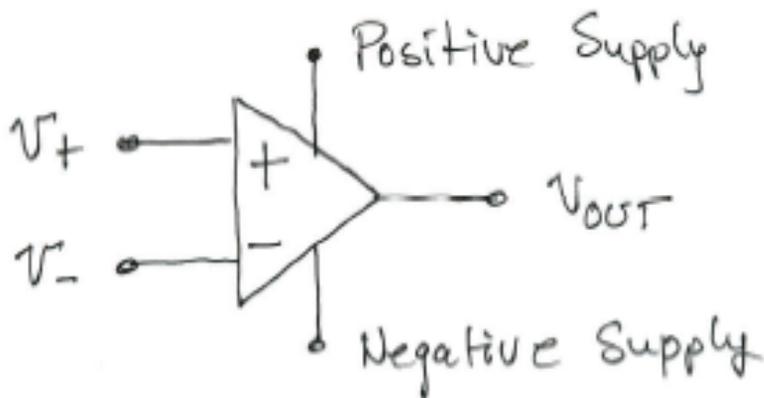
$$\frac{dV_{out}}{dt} + \frac{1}{\tau} V_{out} = 0 \quad \tau \equiv \frac{RC}{A(\bar{\delta} - \delta^+)}$$

$$V_{out}(t) = V_{out}(0) e^{-t/\tau}$$

$$\tau \begin{cases} > 0 & \bar{\delta} > \delta^+ & \text{Negative Feedback} \\ = 0 & \bar{\delta} = \delta^+ & \text{Neutral Feedback} \\ < 0 & \bar{\delta} < \delta^+ & \text{Positive Feedback} \end{cases}$$

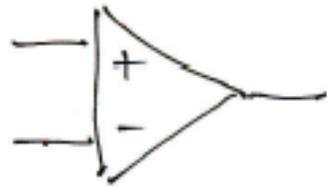


Comparators



Op Amp \leftrightarrow Comparator I

In 6.200, the device



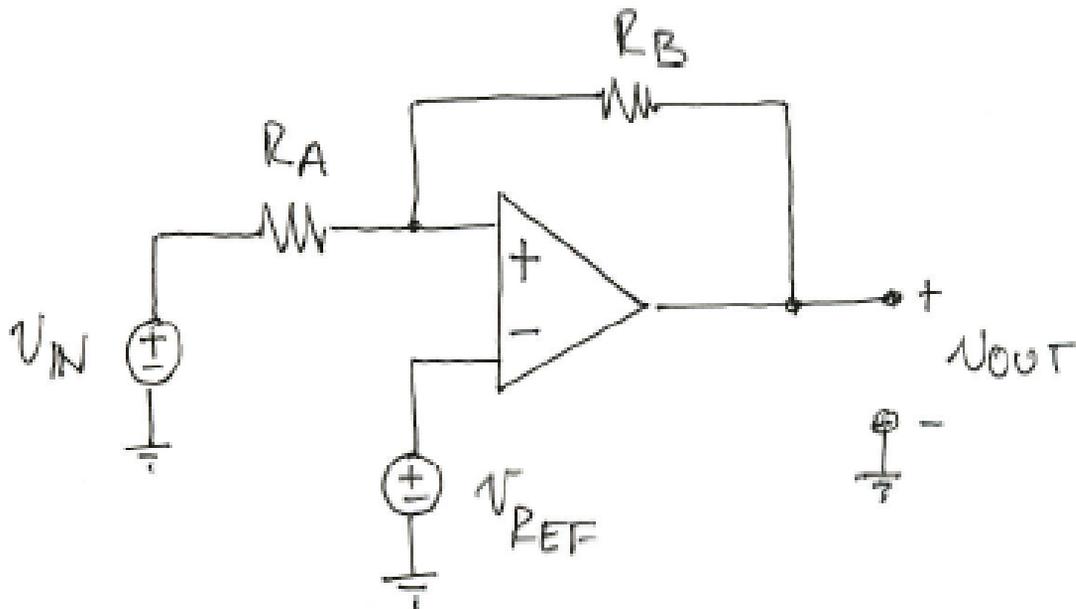
represents an amplifier, usually having ideal properties. It can be used in both negative-feedback amplification (Op-Amp) applications, and positive-feedback comparison (Comparator) applications.

Op Amp ↔ Comparator II

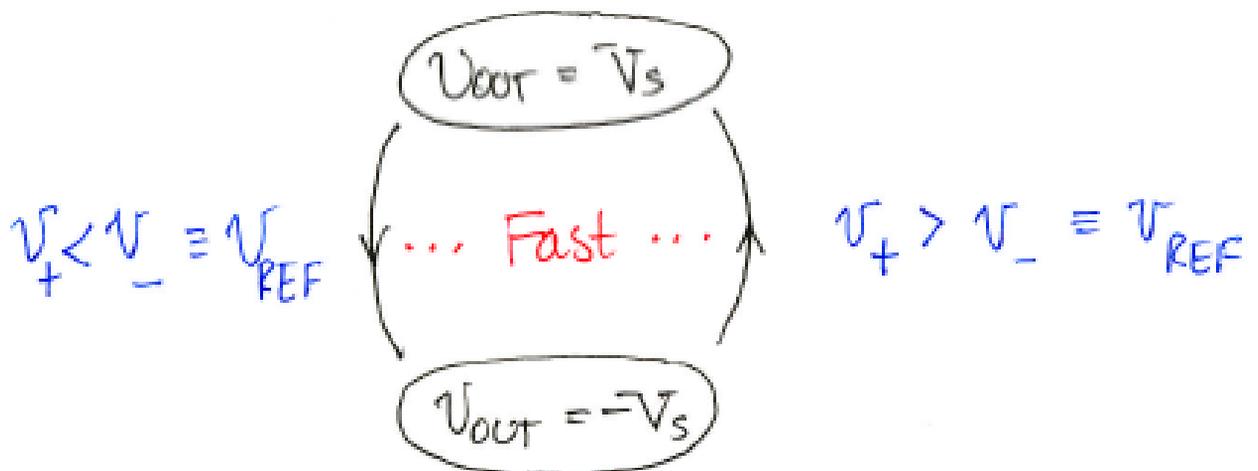
Op-Amp ⇒ Designed to be largely linear, and for use in negative feedback applications. Implemented with a few high-gain stages having modest bandwidth.

Comparators ⇒ Designed to be very fast, and for use in open-loop applications or with positive feedback. Often implemented with many very fast stages having modest gain. May employ positive feedback for high-speed.

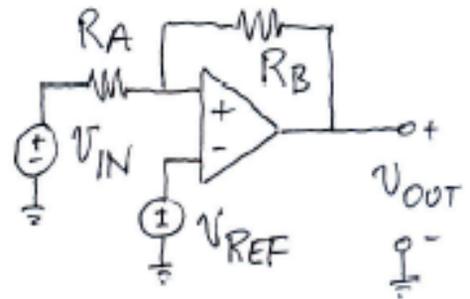
Schmitt Trigger I



Could interchange roles of V_{IN} and V_{REF} .



Schmitt Trigger II

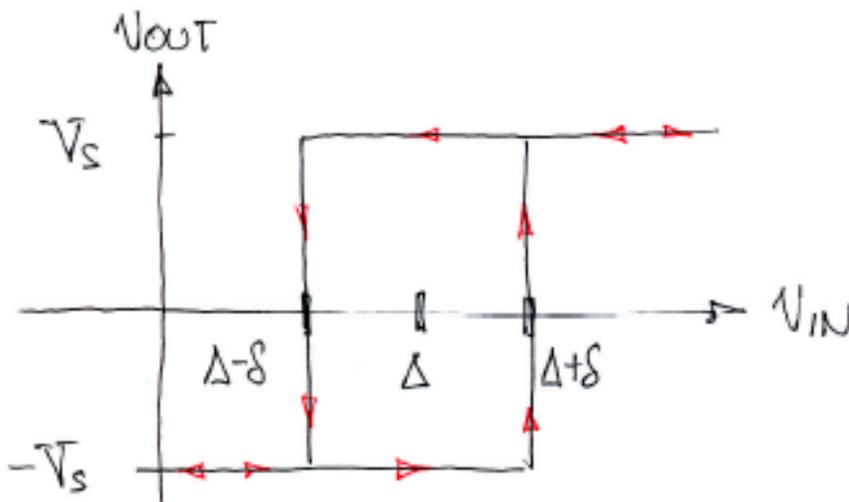


$-V_S \rightarrow +V_S$ Transition:

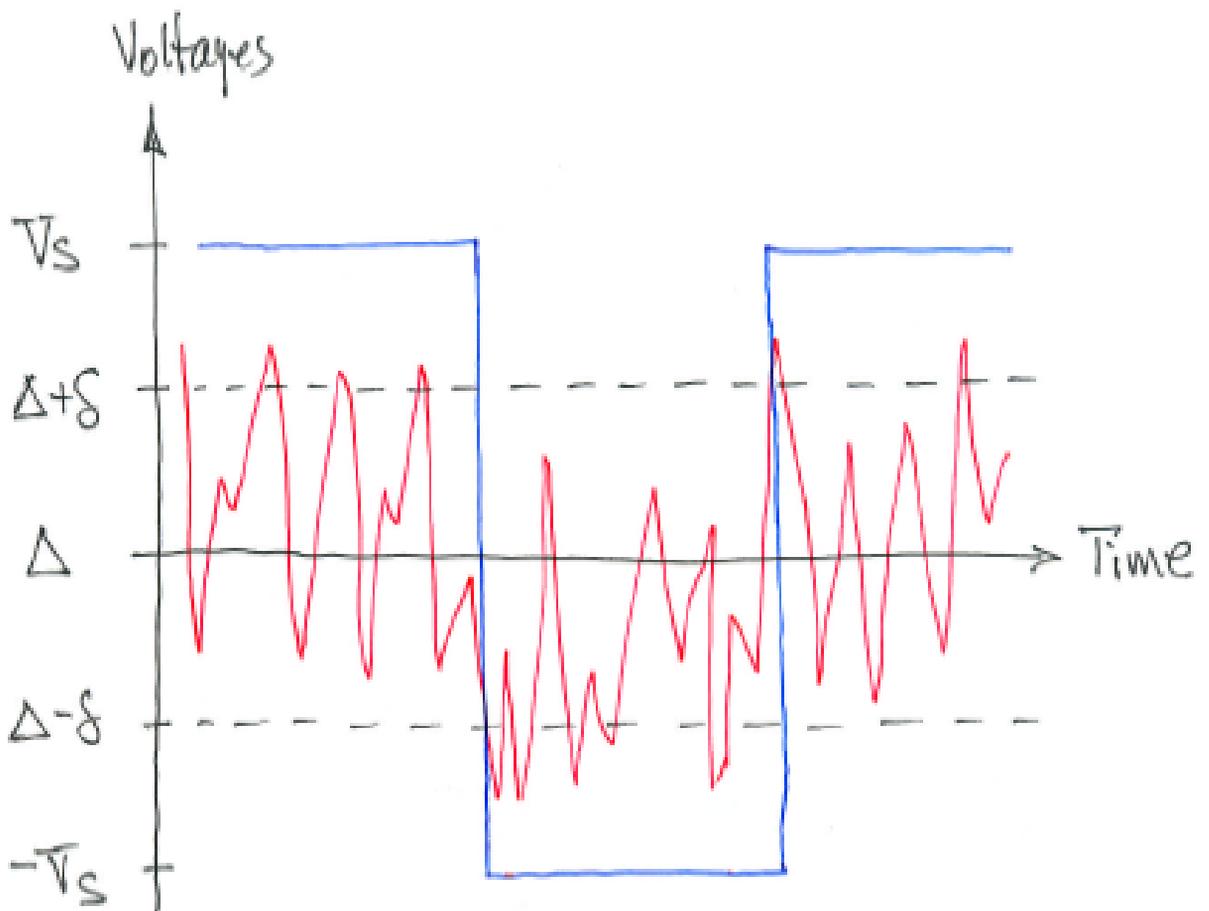
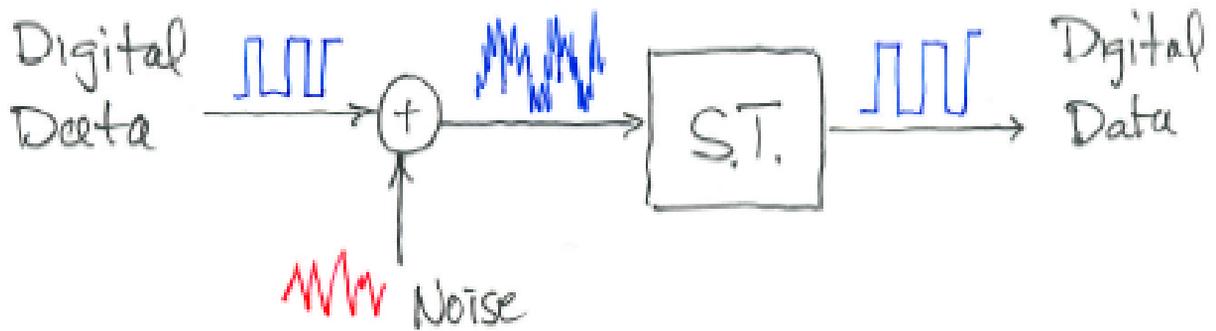
$$V_{OUT} \frac{R_A}{R_A + R_B} + V_{IN} \frac{R_B}{R_A + R_B} > V_{REF} \Rightarrow V_{IN} > \underbrace{\frac{R_A + R_B}{R_B} V_{REF}}_{\Delta} + \underbrace{\frac{R_A}{R_B} V_S}_{\delta}$$

$+V_S \rightarrow -V_S$ Transition:

$$V_{OUT} \frac{R_A}{R_A + R_B} + V_{IN} \frac{R_B}{R_A + R_B} < V_{REF} \Rightarrow V_{IN} < \underbrace{\frac{R_A + R_B}{R_B} V_{REF}}_{\Delta} - \underbrace{\frac{R_A}{R_B} V_S}_{\delta}$$



Schmitt Trigger Application



Oscillator

