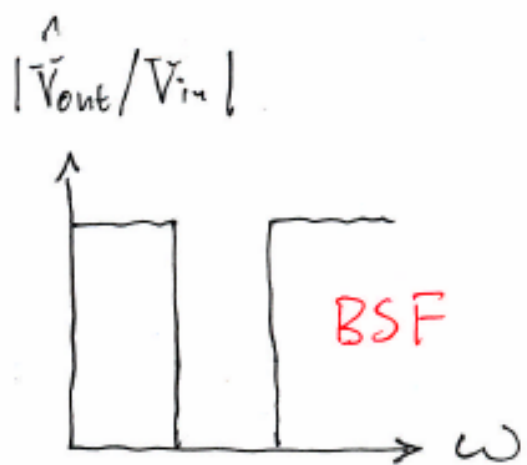
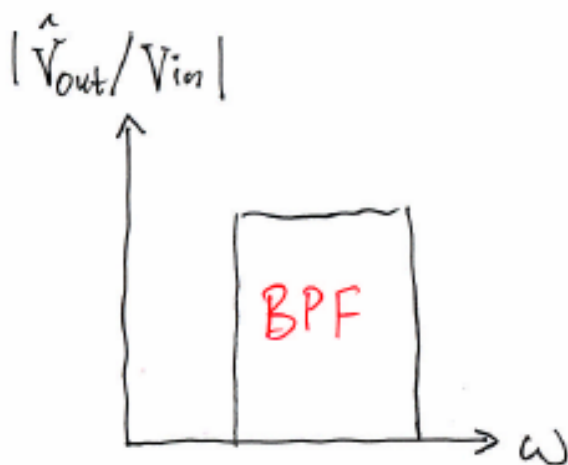
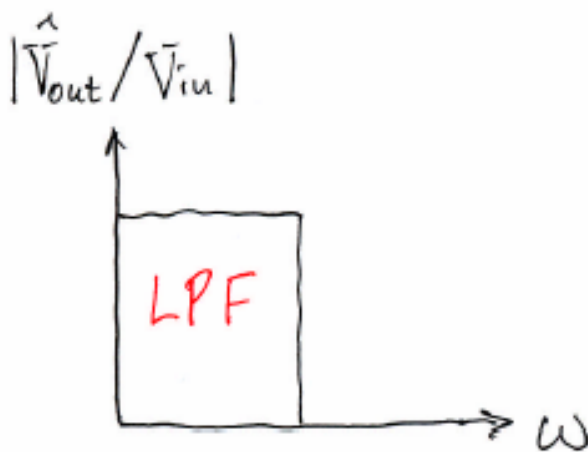
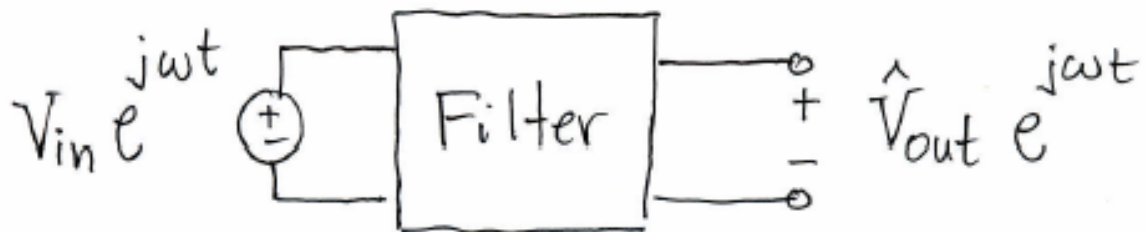


# 6.200 - Lecture 21

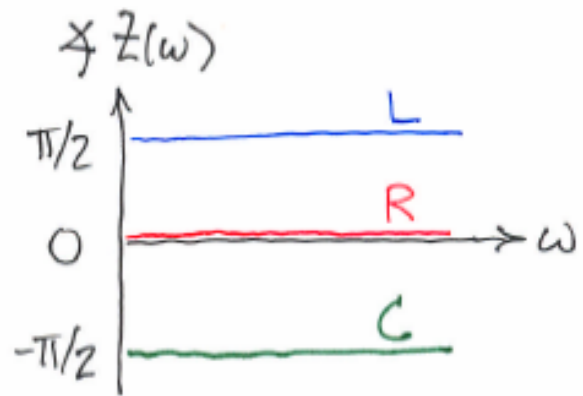
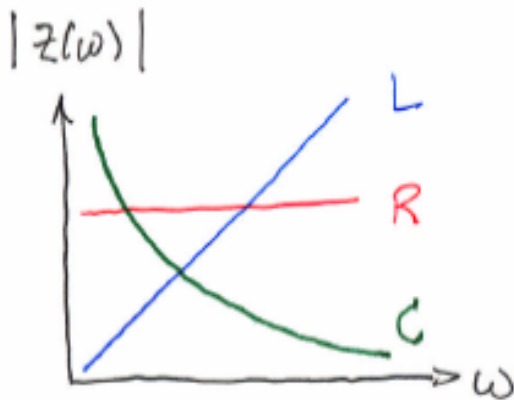
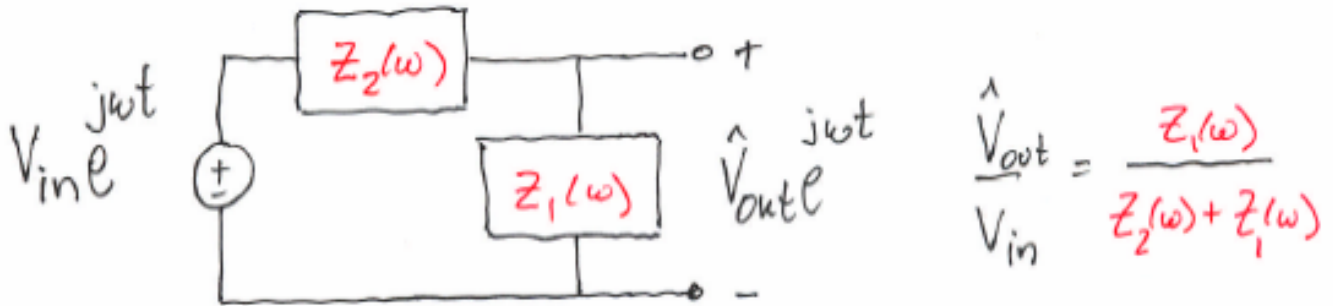
Filtering:

- Review Simple Objectives (LPF , HPF , BPF , BSF)
- Review Simple Synthesis
- LC Resonators
- Tesla Coil

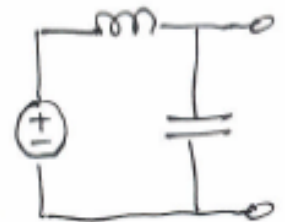
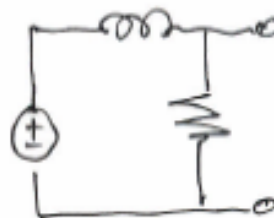
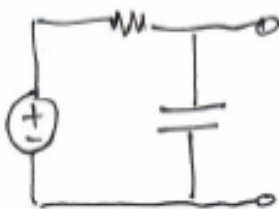
# Simple Filtering Objectives



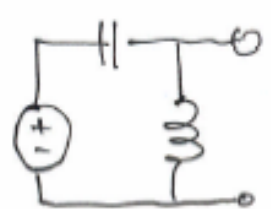
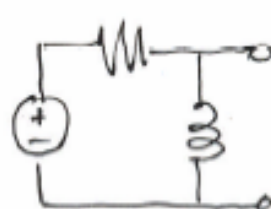
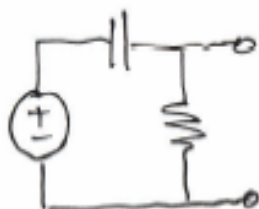
# Simple Filtering Synthesis



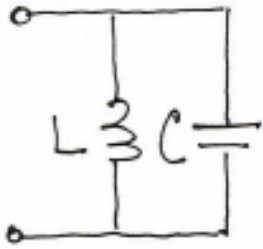
LPF:



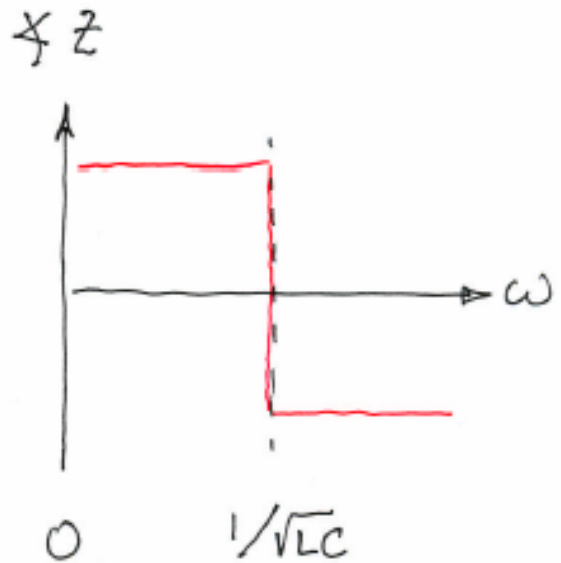
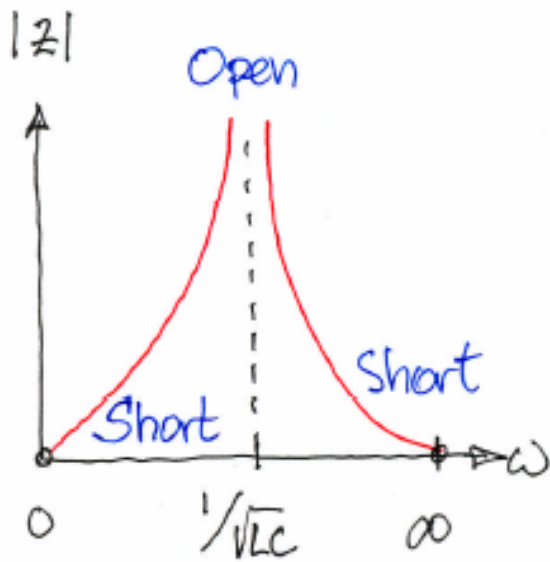
HPF:



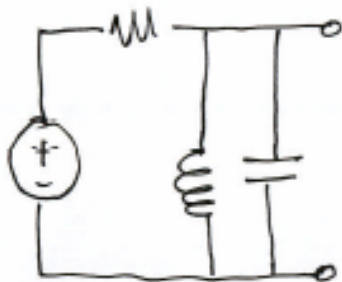
# Parallel LC Resonator



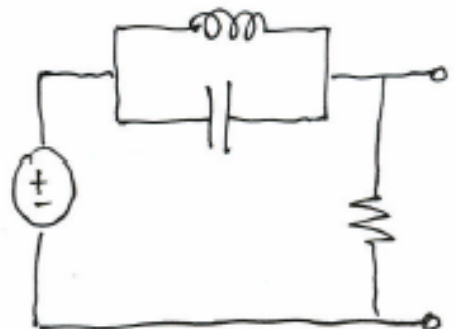
$$Z = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$



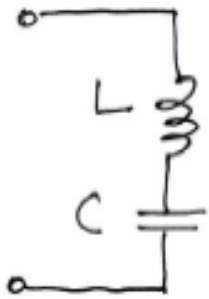
BPF:



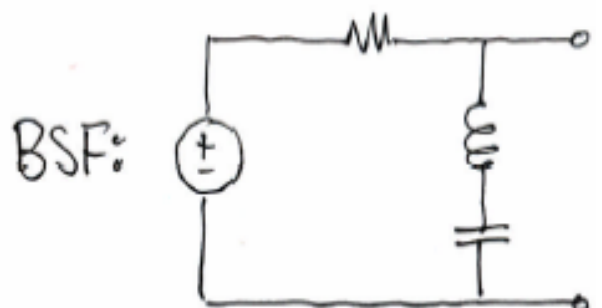
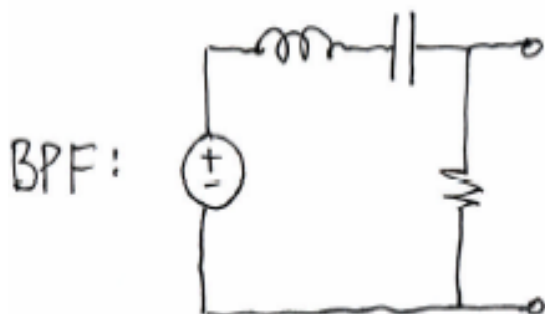
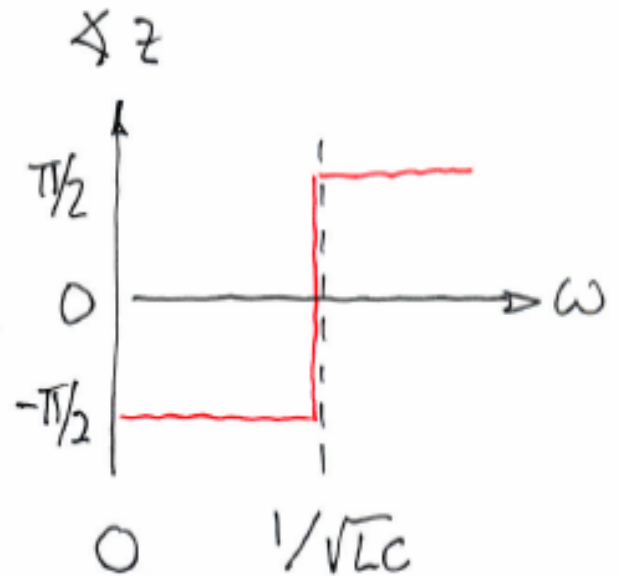
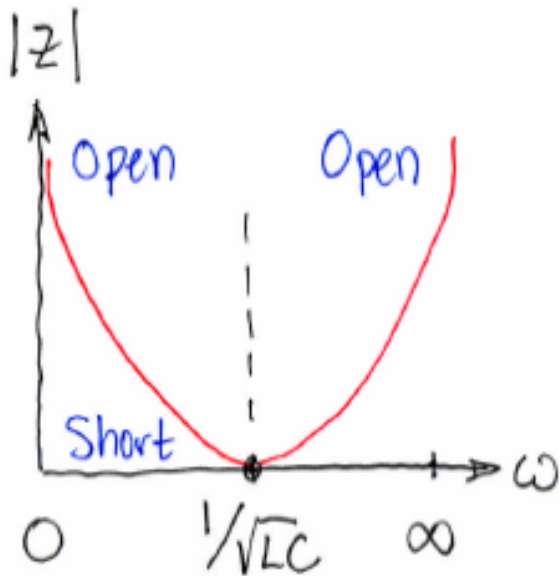
BSF:



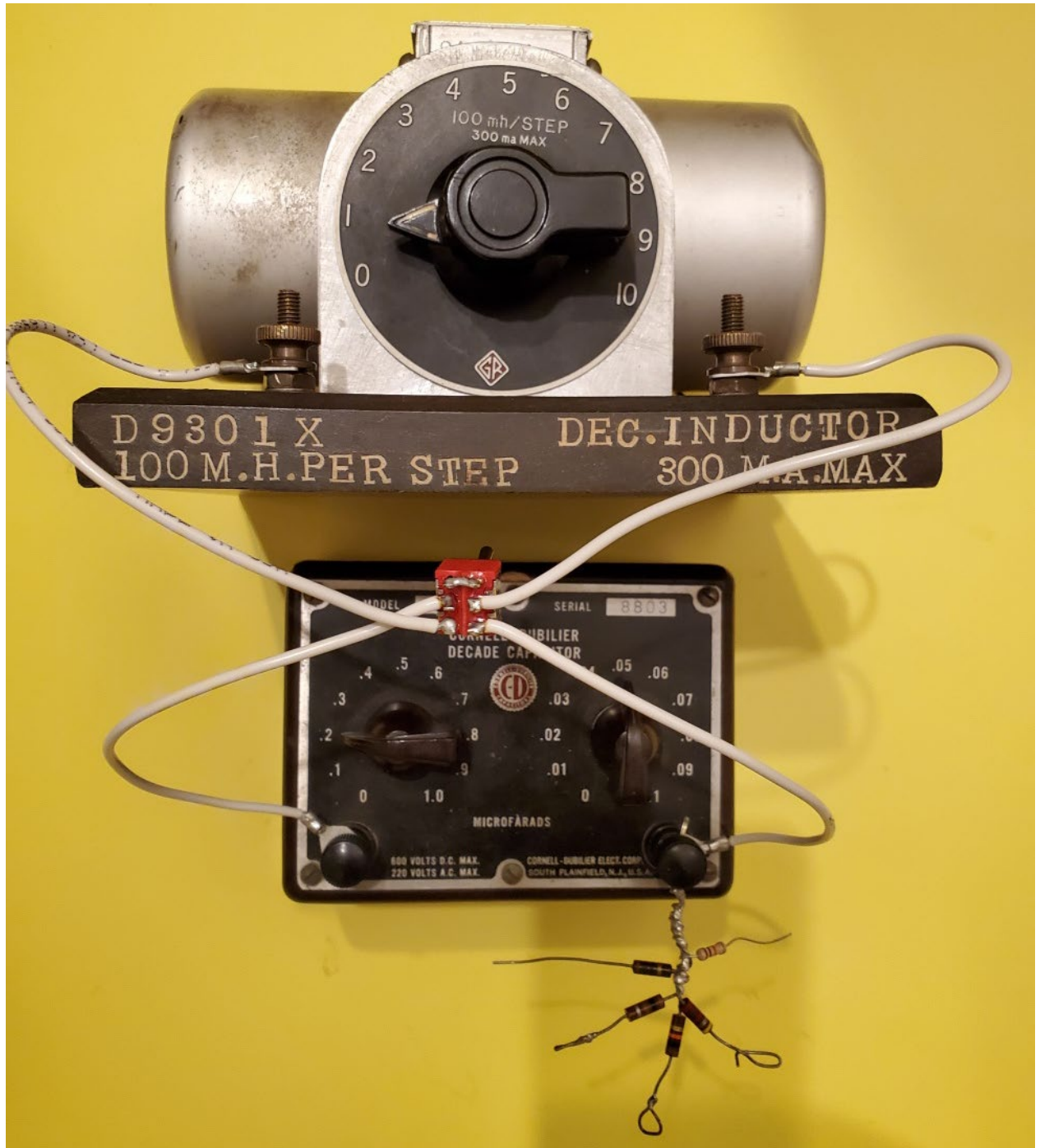
# Series LC Resonator



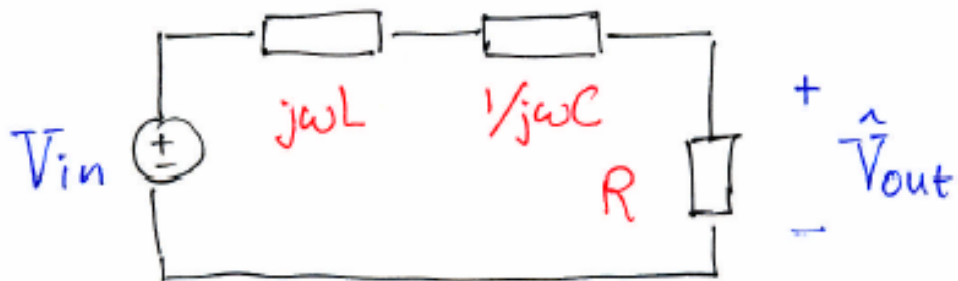
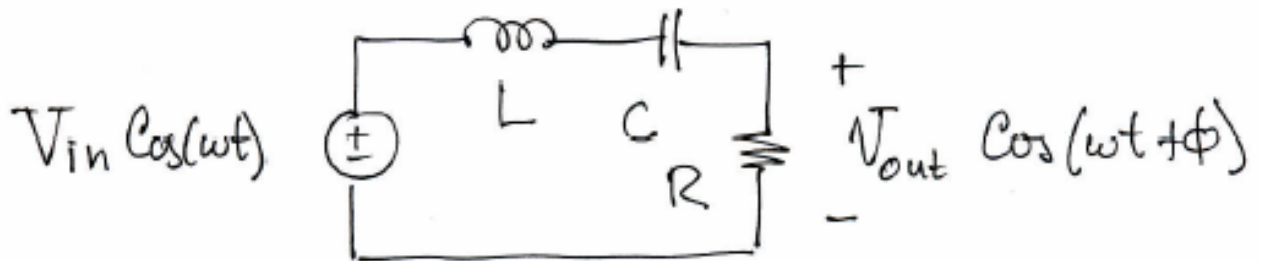
$$Z = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$



# RLC Demos



# Theory: Series-LC BPF

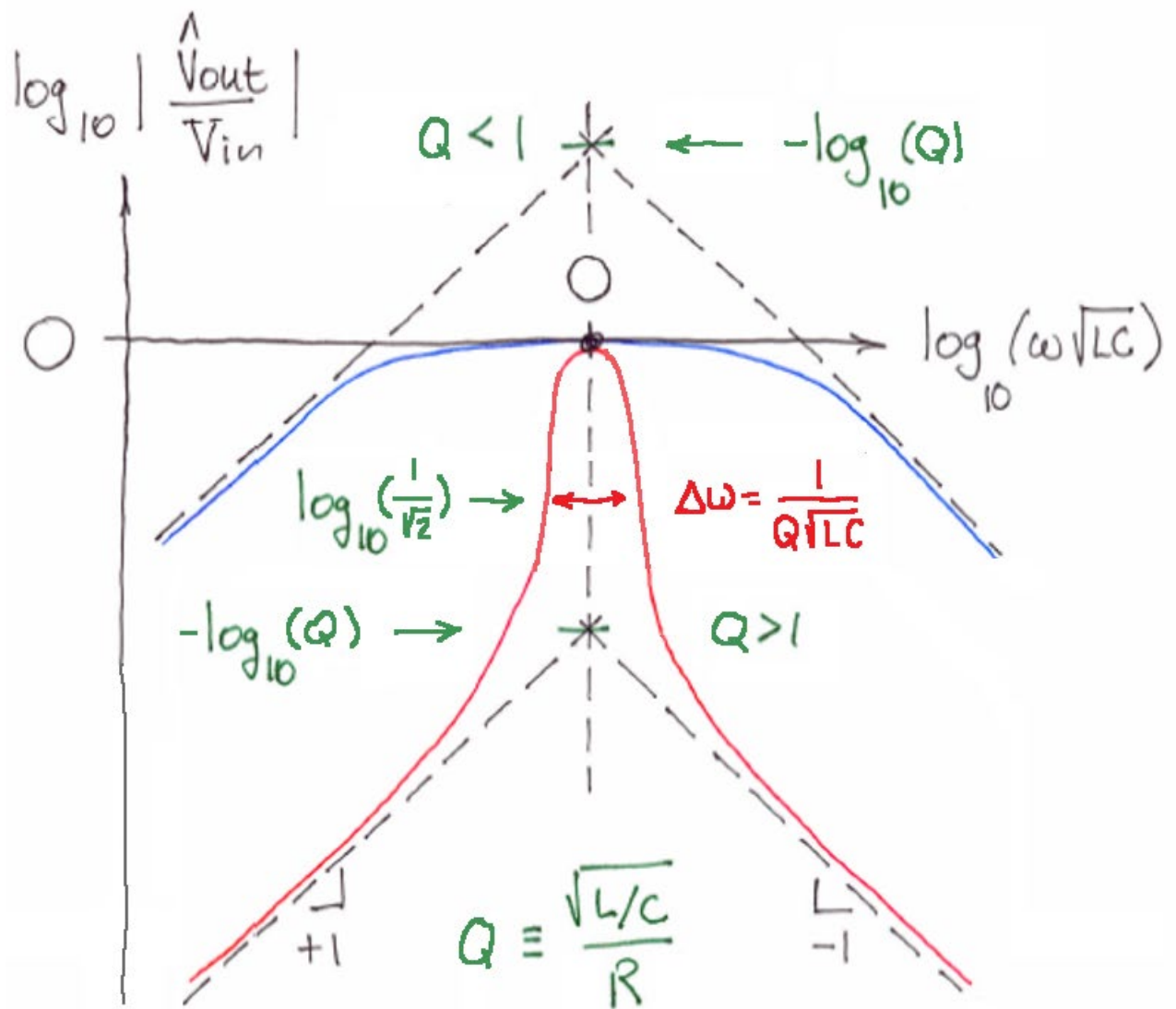


$$\hat{V}_{out} = \frac{R}{R + j\omega L + 1/j\omega C} \hat{V}_{in} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC} \hat{V}_{in}$$

$$V_{out} = |\hat{V}_{out}| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}} V_{in}$$

$$\phi = \angle \hat{V}_{out} = \tan^{-1} \left( \frac{1 - \omega^2 LC}{\omega RC} \right)$$

# Theory: Series-LC BPF



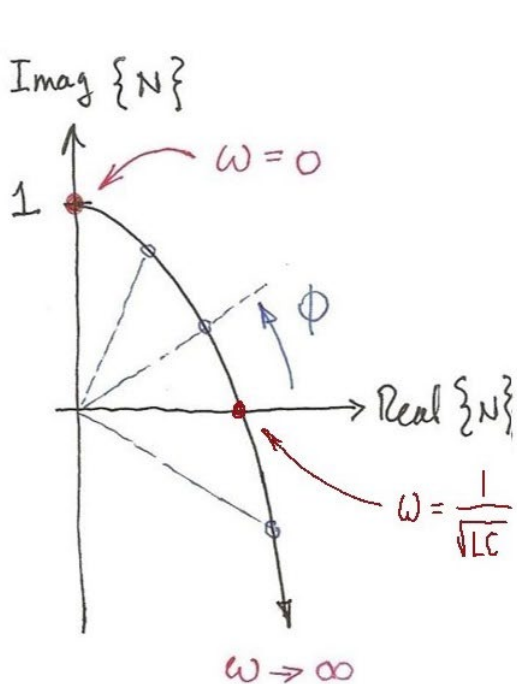
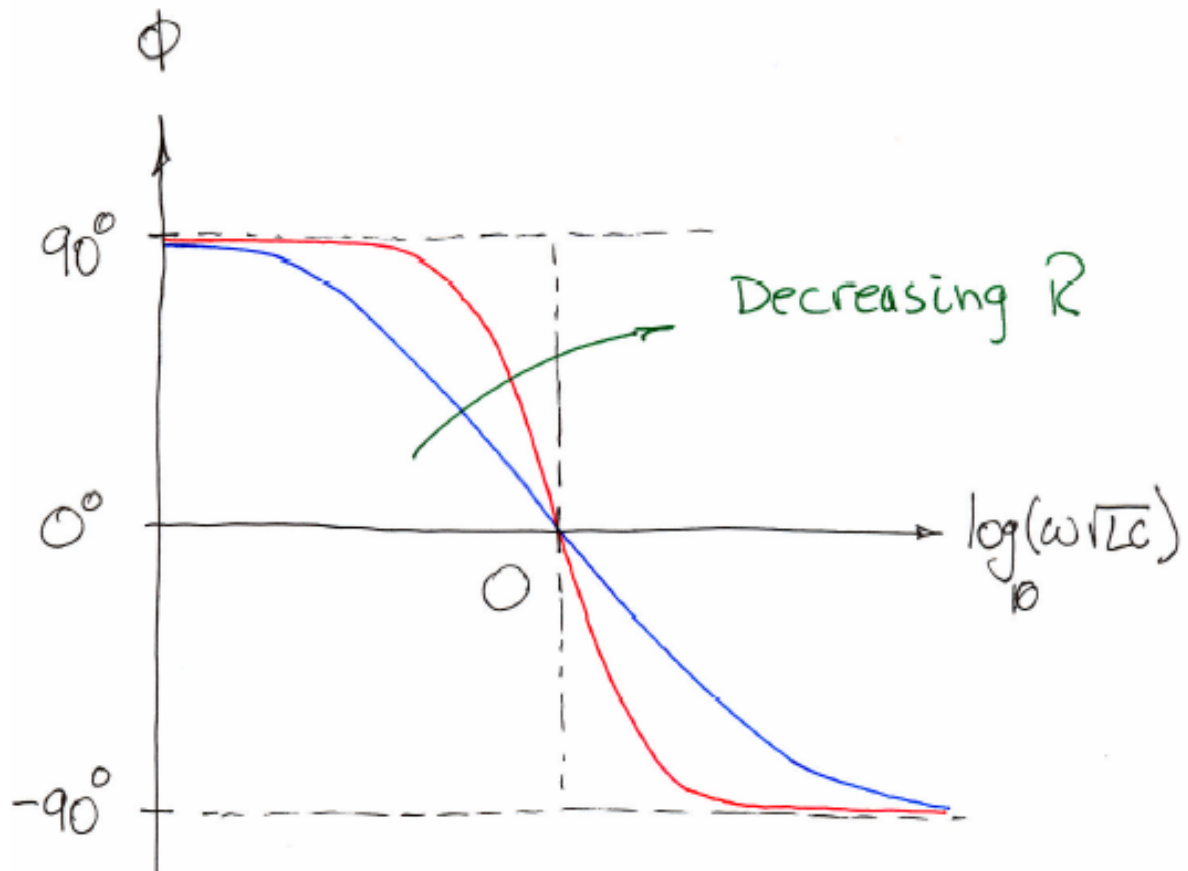
Low Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \omega RC = \frac{\omega \sqrt{LC}}{Q}$$

High Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \frac{R}{\omega L} = \frac{1}{\omega \sqrt{LC} Q}$$

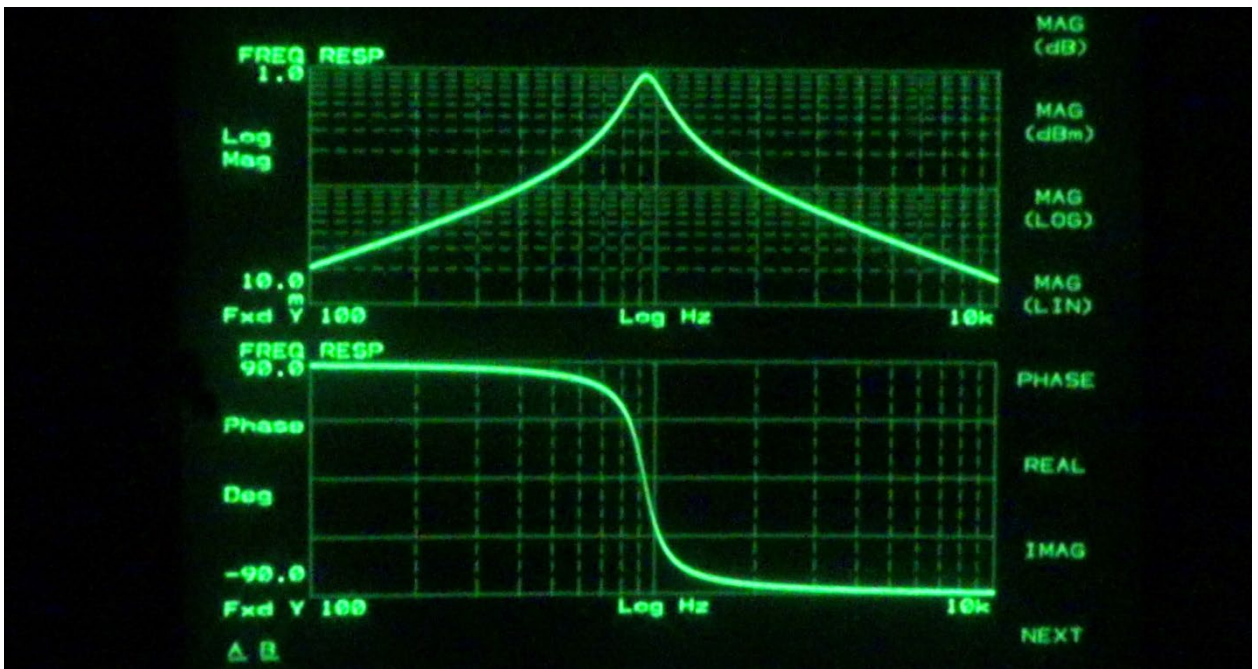
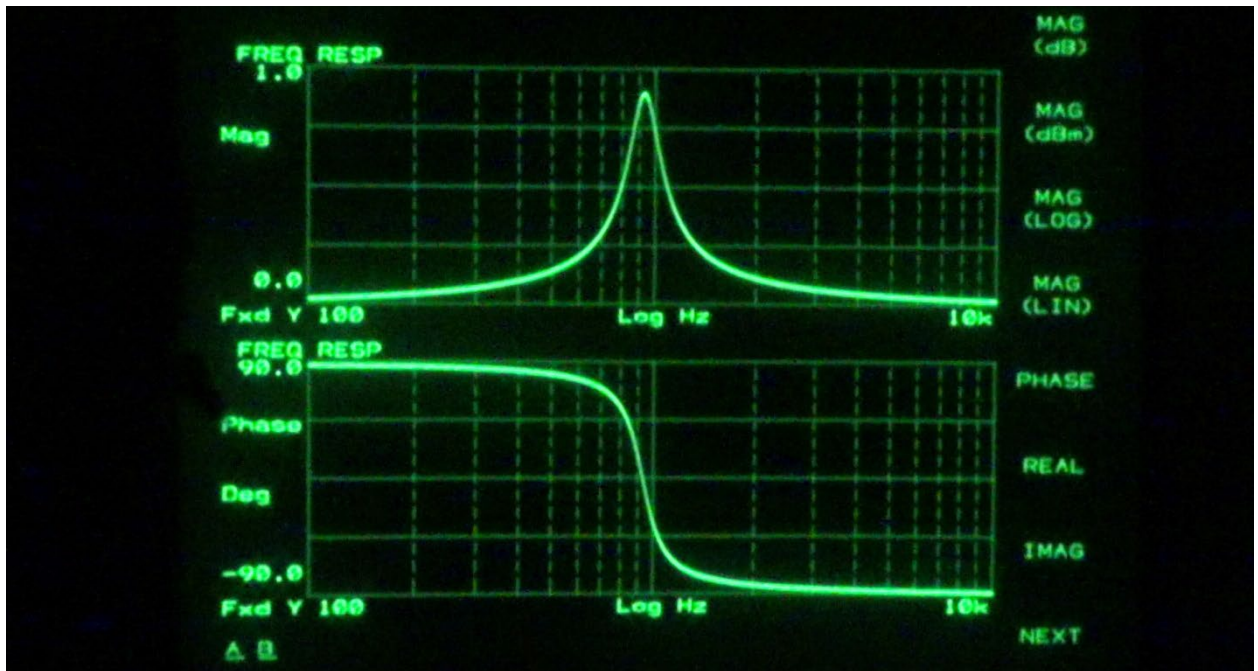
# Theory: Series-LC BPF



$$\begin{aligned} \phi &\equiv \angle \left| \frac{\hat{V}_{out}}{V_{in}} \right| \\ &= \angle \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC} \\ &= \angle \frac{j\omega RC (1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + (\omega RC)^2} \\ &= \angle j(1 - \omega^2 LC) + \omega RC \\ &\equiv \angle N \end{aligned}$$

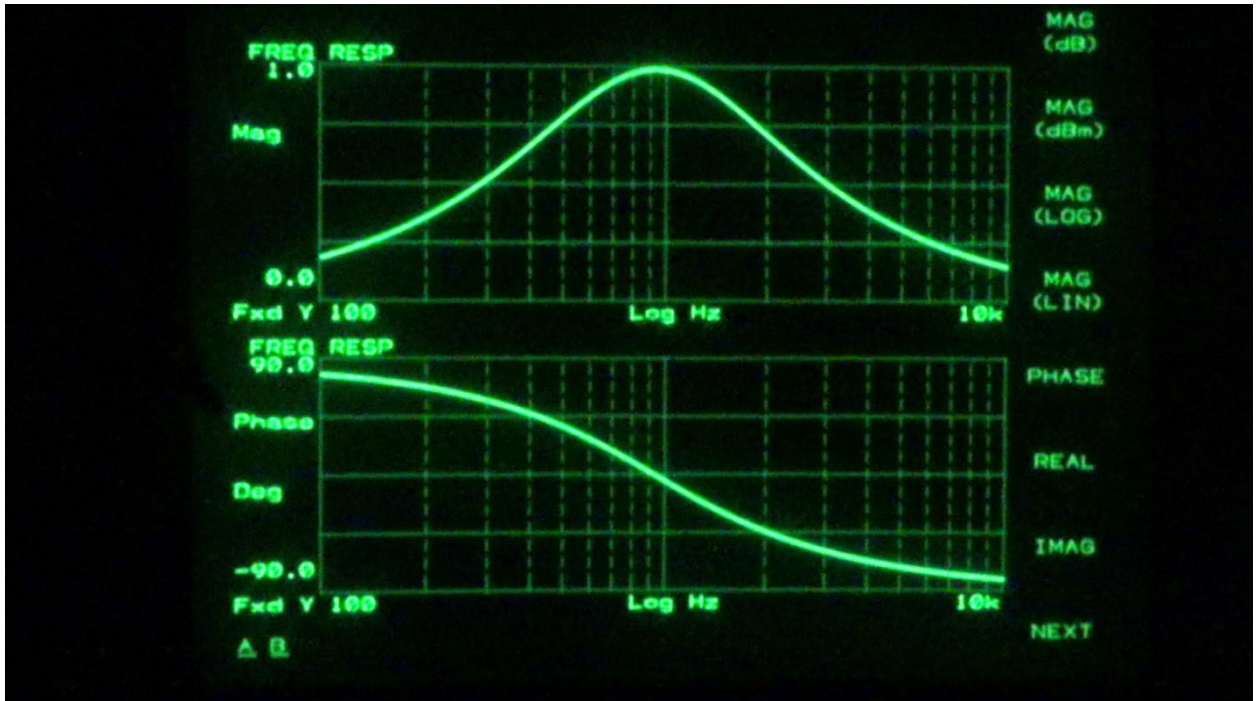
# Demo: Series-LC BPF

$L = 0.1 \text{ H}$  ;  $C = 0.25 \mu\text{F}$  ;  $R = 100 \Omega$   
 $(2\pi\sqrt{LC})^{-1} = 1 \text{ kHz}$  ;  $\sqrt{L/C} = 632 \Omega$

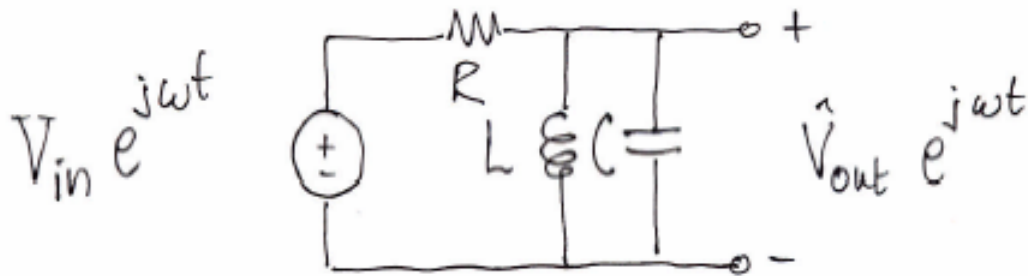


# Demo: Series-LC BPF

$L = 0.1 \text{ H}$  ;  $C = 0.25 \mu\text{F}$  ;  $R = 1 \text{ k}\Omega$   
 $(2\pi\sqrt{LC})^{-1} = 1 \text{ kHz}$  ;  $\sqrt{L/C} = 632 \Omega$

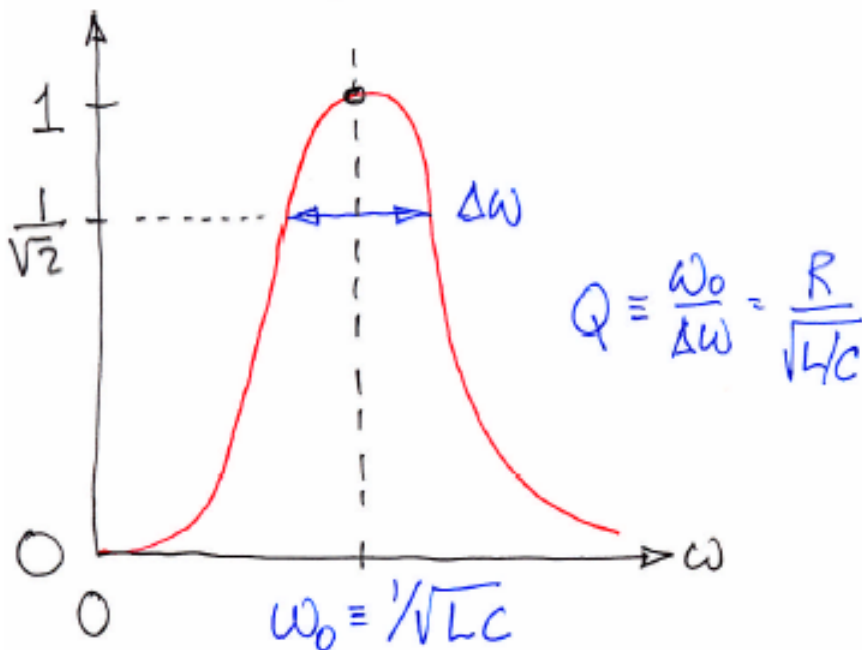


# Theory: Parallel-LC BPF



$$\frac{\hat{V}_{out}}{V_{in}} = \frac{1/R}{1/R + 1/j\omega L + j\omega C} = \frac{j\omega L/R}{1 - \omega^2 LC + j\omega L/R}$$

$$\left| \frac{\hat{V}_{out}}{V_{in}} \right| = \frac{(\omega L/R)}{\sqrt{(1 - \omega^2 LC)^2 + (\omega L/R)^2}}$$



Numbers:

$$L = 0.1 \text{ H}$$

$$C = 0.25 \mu\text{F}$$

$$R = 10 \text{ k}\Omega$$

$$\frac{1}{2\pi\sqrt{LC}} = 1 \text{ kHz}$$

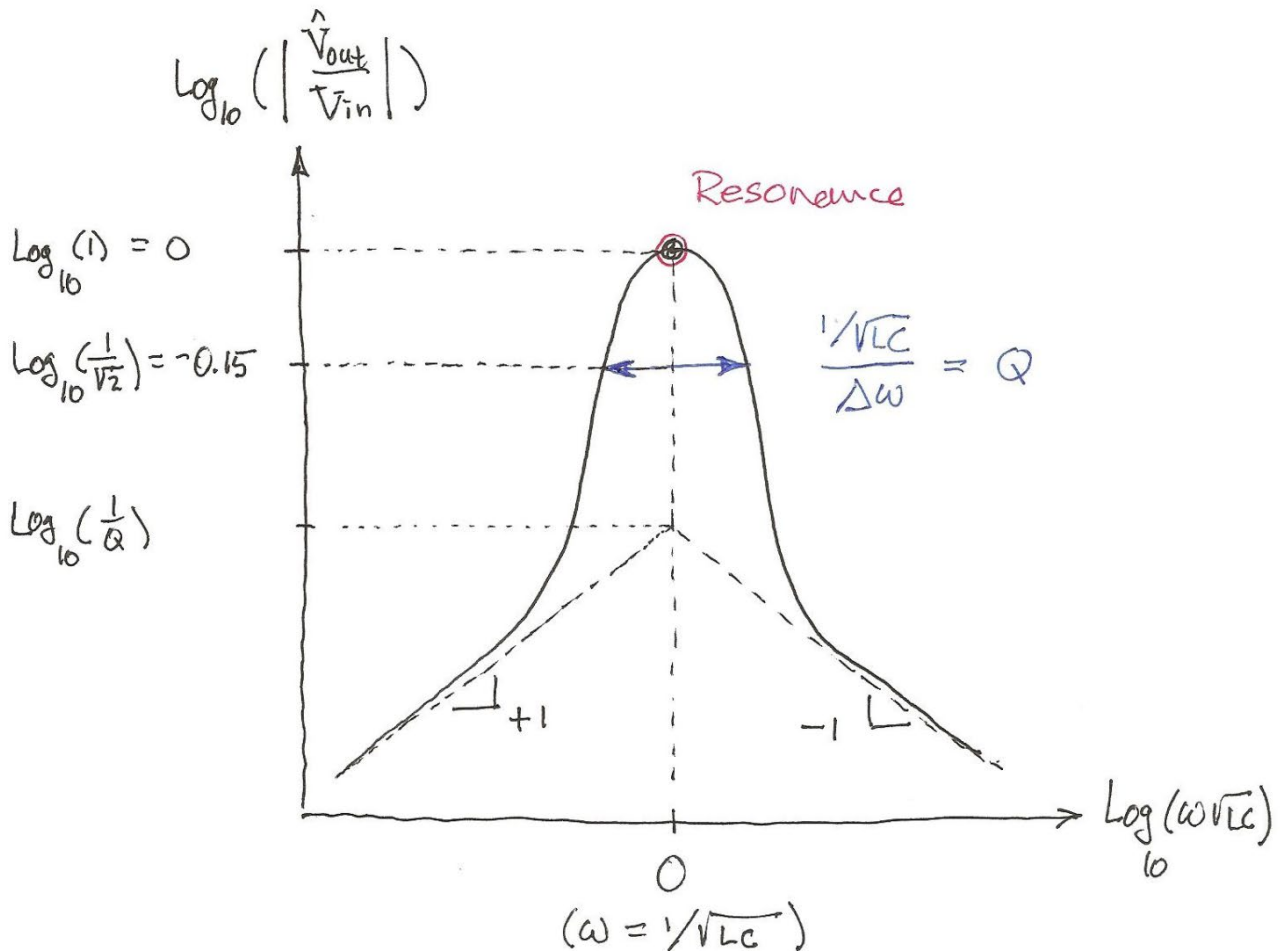
$$\sqrt{L/C} = 632 \Omega$$

$$Q = 16$$

# Theory: Parallel-LC BPF

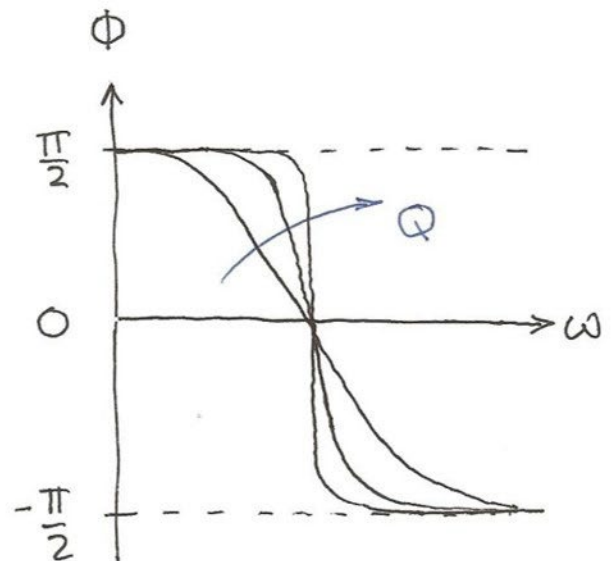
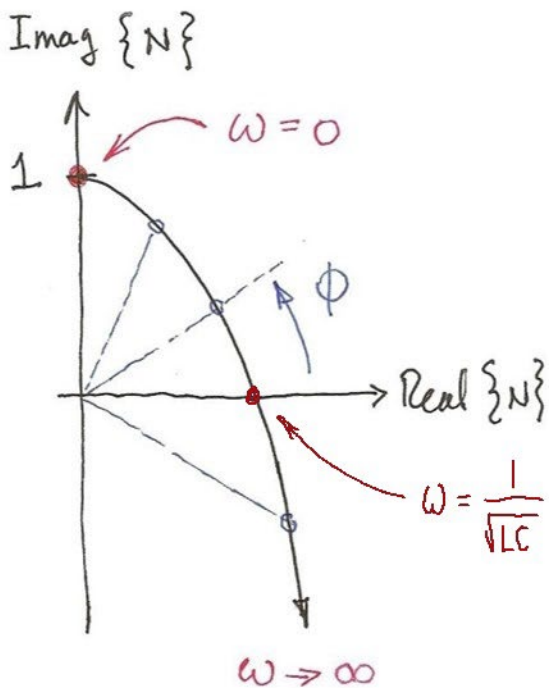
$$\left. \begin{array}{l}
 \text{Low } \omega : \left| \frac{\hat{V}_{out}}{V_{in}} \right| \approx \frac{\omega L}{R} = \frac{\omega \sqrt{LC}}{Q} \\
 \text{High } \omega : \left| \frac{\hat{V}_{out}}{V_{in}} \right| \approx \frac{1}{\omega RC} = \frac{1}{\omega \sqrt{LC} Q}
 \end{array} \right\} \text{Asymptotes}$$

Asymptotes cross at  $\omega \sqrt{LC} = 1$  with value  $1/Q$ .



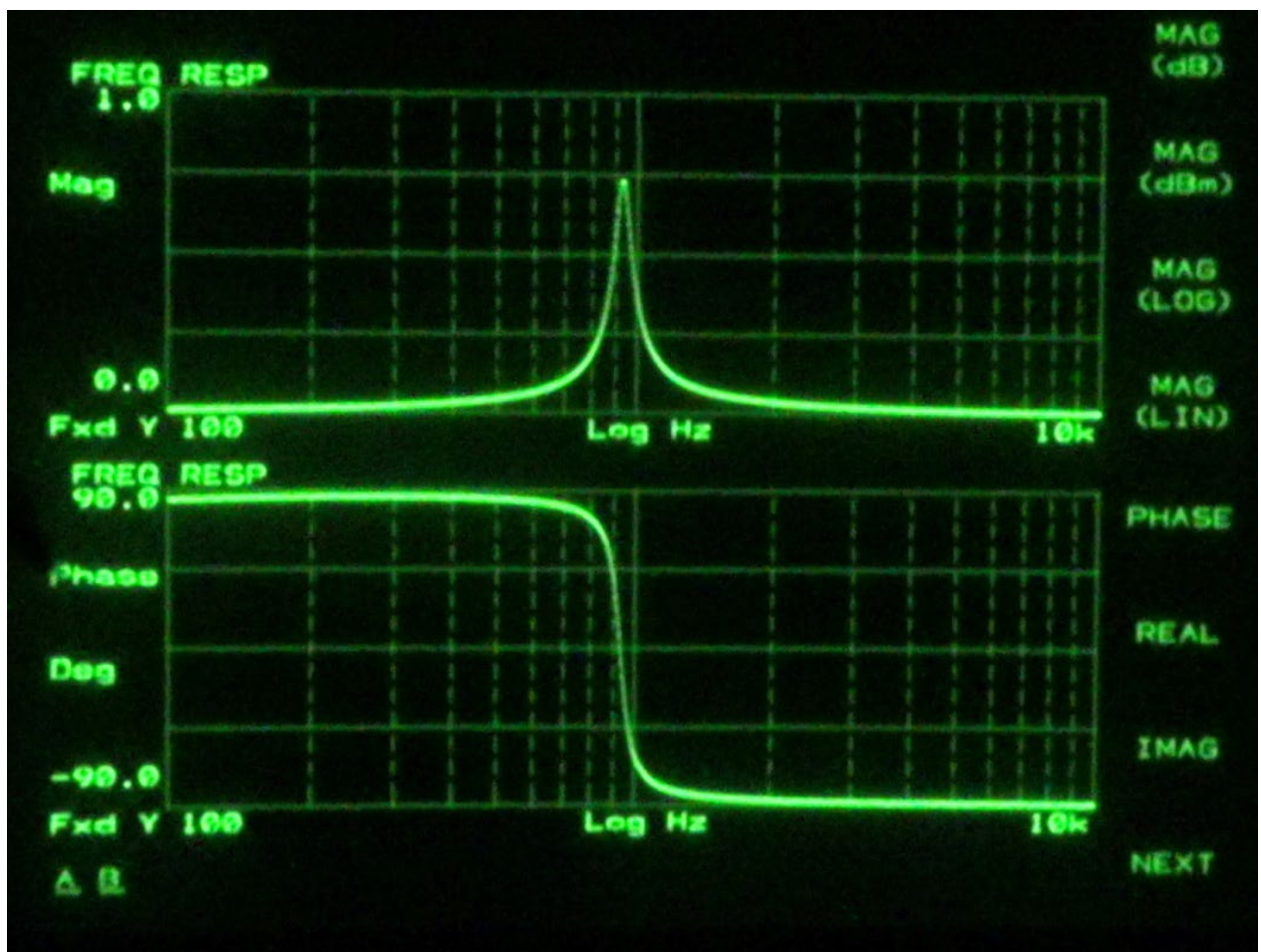
# Theory: Parallel-LC BPF

$$\begin{aligned}
 \angle \left| \frac{\hat{V}_{out}}{V_{in}} \right| \equiv \phi &= \angle \frac{j\omega L/R}{1 - \omega^2 LC + j\omega L/R} \\
 &= \angle \frac{j\omega L/R (1 - \omega^2 LC - j\omega L/R)}{(1 - \omega^2 LC)^2 + (\omega L/R)^2} \\
 &= \angle j(1 - \omega^2 LC) + \omega L/R \\
 &\equiv \angle N
 \end{aligned}$$

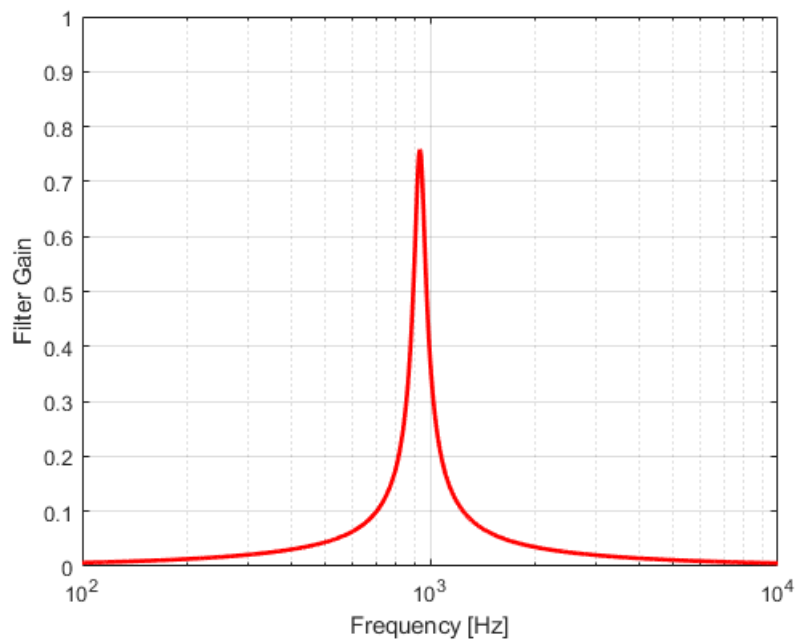
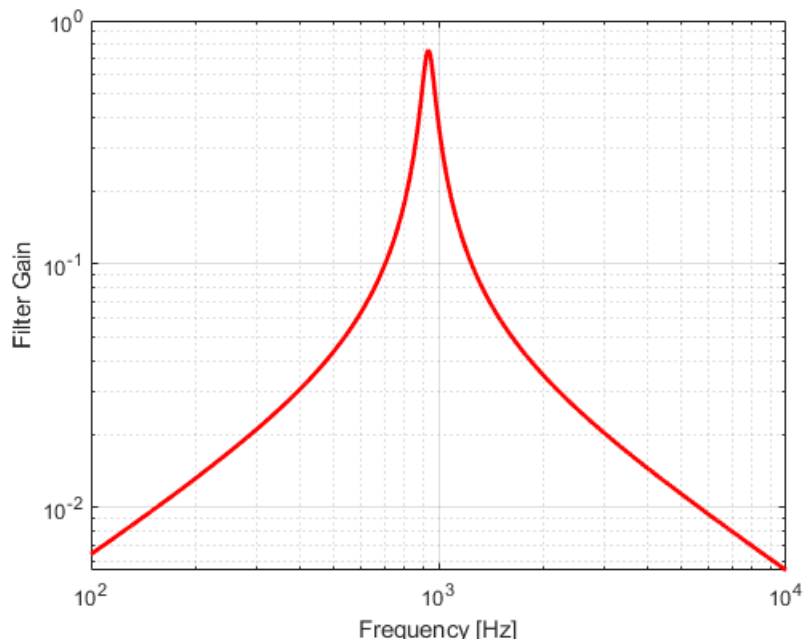
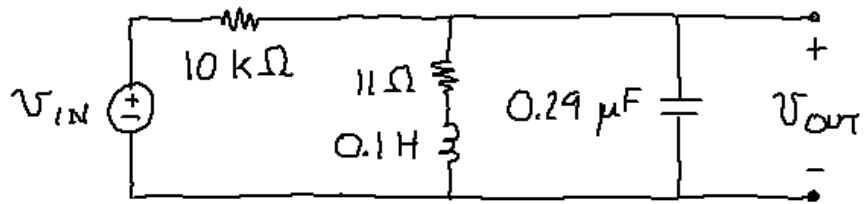


# Demo: Parallel-LC BPF

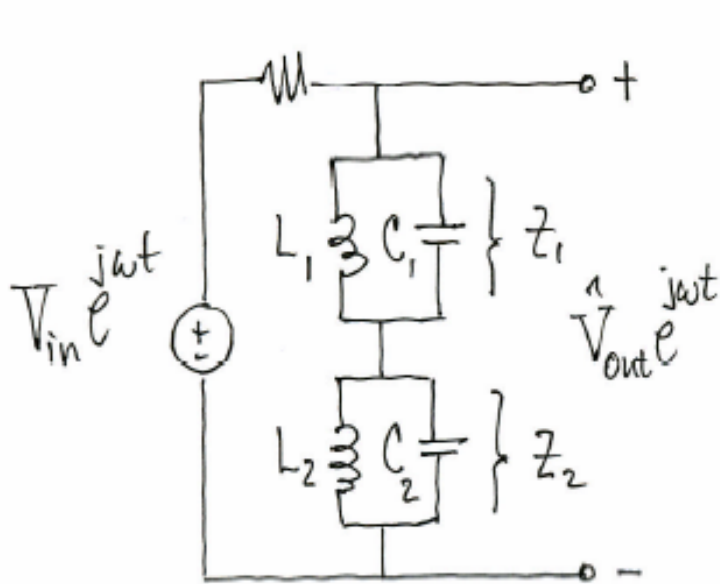
$$L = 0.1 \text{ H} ; C = 0.25 \mu\text{F} ; R = 10 \text{ k}\Omega$$
$$(2\pi\sqrt{LC})^{-1} = 1 \text{ kHz} ; R/\sqrt{L/C} = 16$$



# Numerical: Parallel-LC BPF



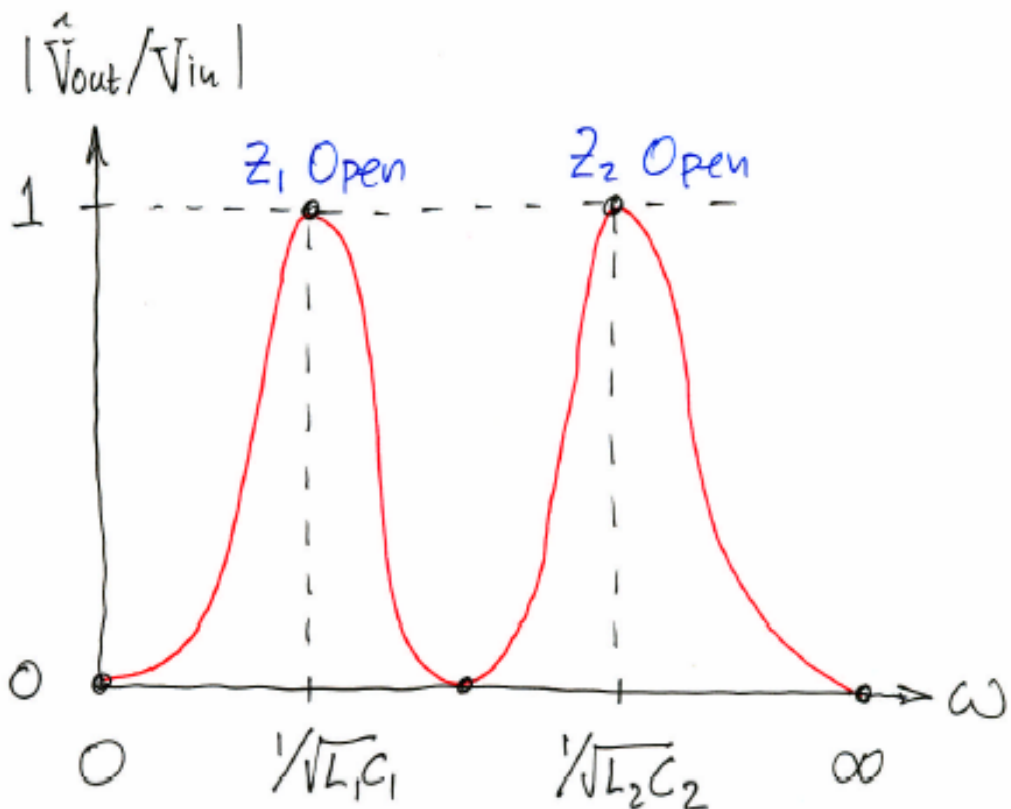
# LC Comb Filter



$$z_1 = \text{Open at } \omega = \frac{1}{\sqrt{L_1 C_1}}$$

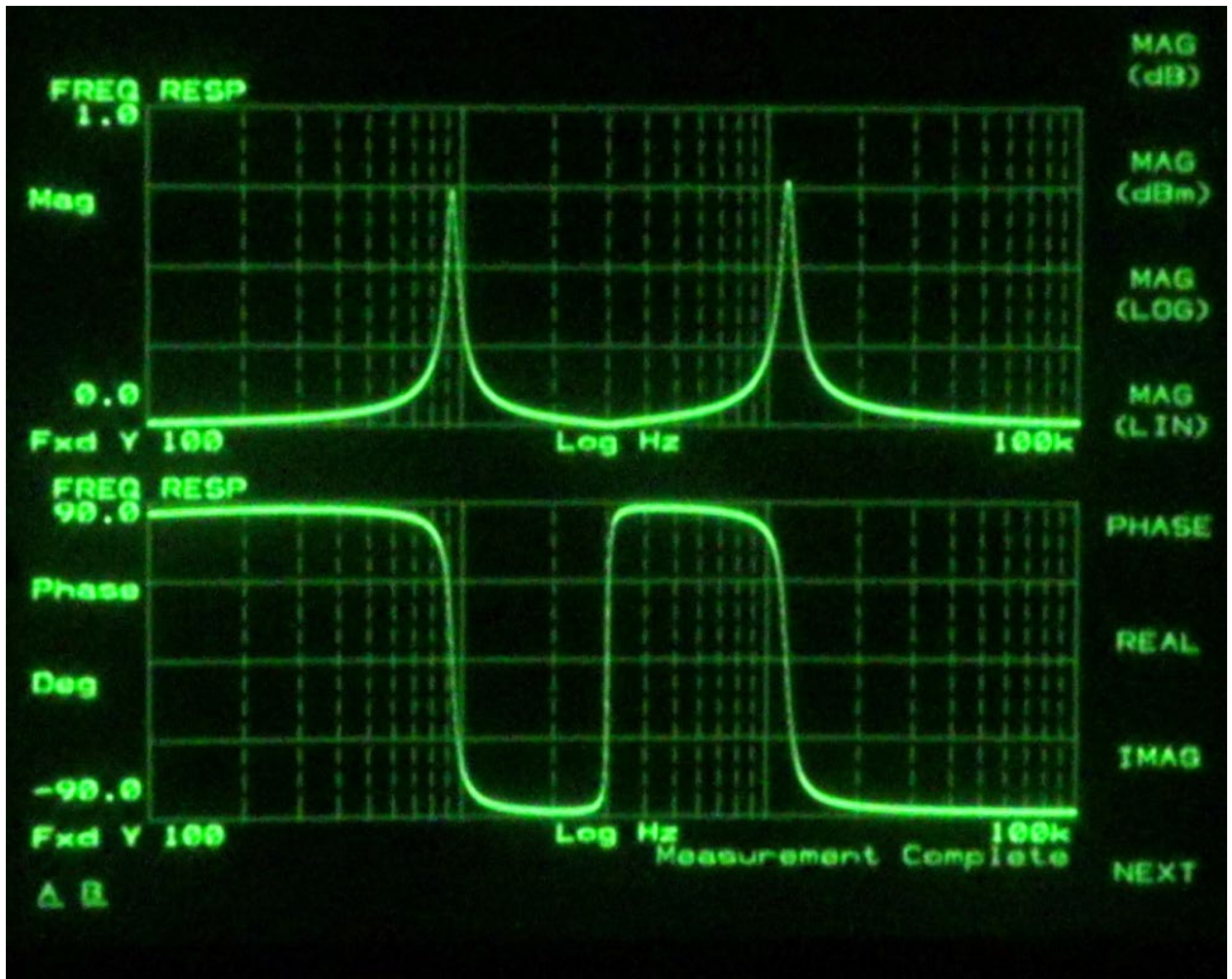
$$z_2 = \text{Open at } \omega = \frac{1}{\sqrt{L_2 C_2}}$$

$z_1 + z_2 = \text{Short at } \omega = 0, \infty$   
and between the two opens

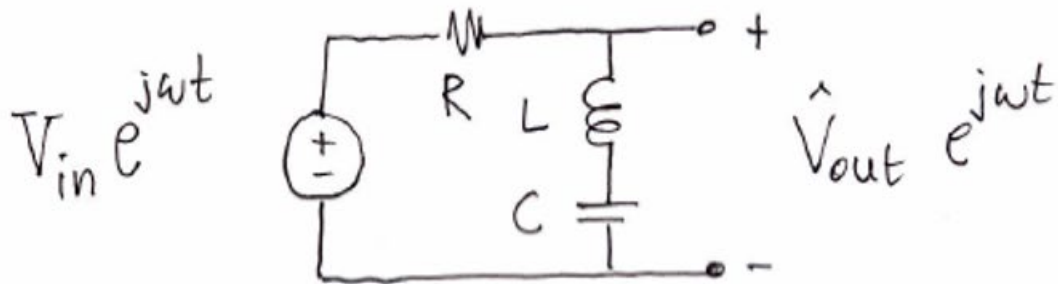


# Demo: LC Comb Filter

$L_1 = 0.1 \text{ H}$  ;  $C_1 = 0.25 \mu\text{F}$  ;  $f_1 = 1 \text{ kHz}$   
 $L_2 = 0.01 \text{ H}$  ;  $C_2 = 0.022 \mu\text{F}$  ;  $f_2 = 10 \text{ kHz}$   
 $R = 10 \text{ k}\Omega$  ;  $Q_1 = Q_2 = 16$

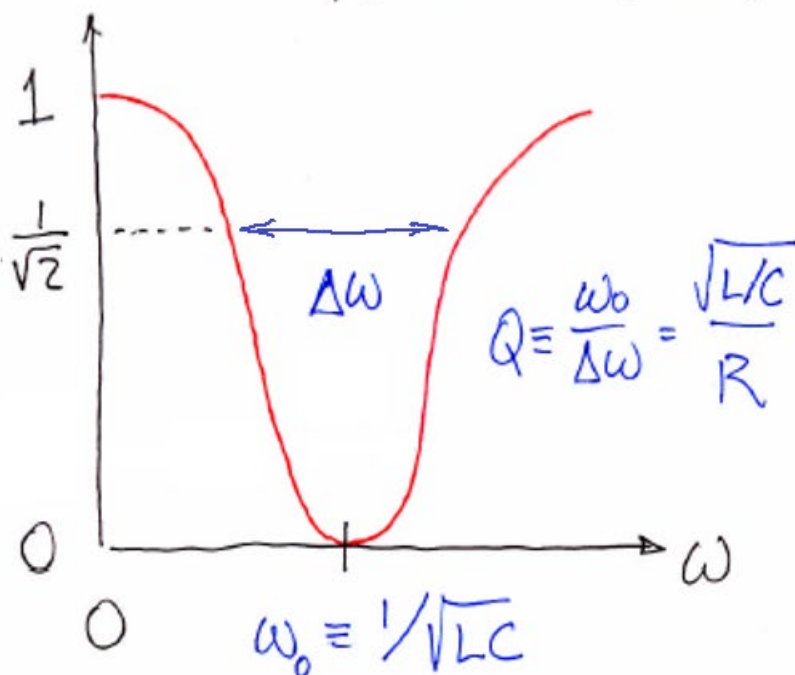


# Theory: Series-LC BSF



$$\frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

$$\left| \frac{\hat{V}_{out}}{\hat{V}_{in}} \right| = \frac{1 - \omega^2 LC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$



Numbers :

$$L = 0.1 \text{ H}$$

$$C = 0.25 \mu\text{F}$$

$$R = 100 \Omega$$

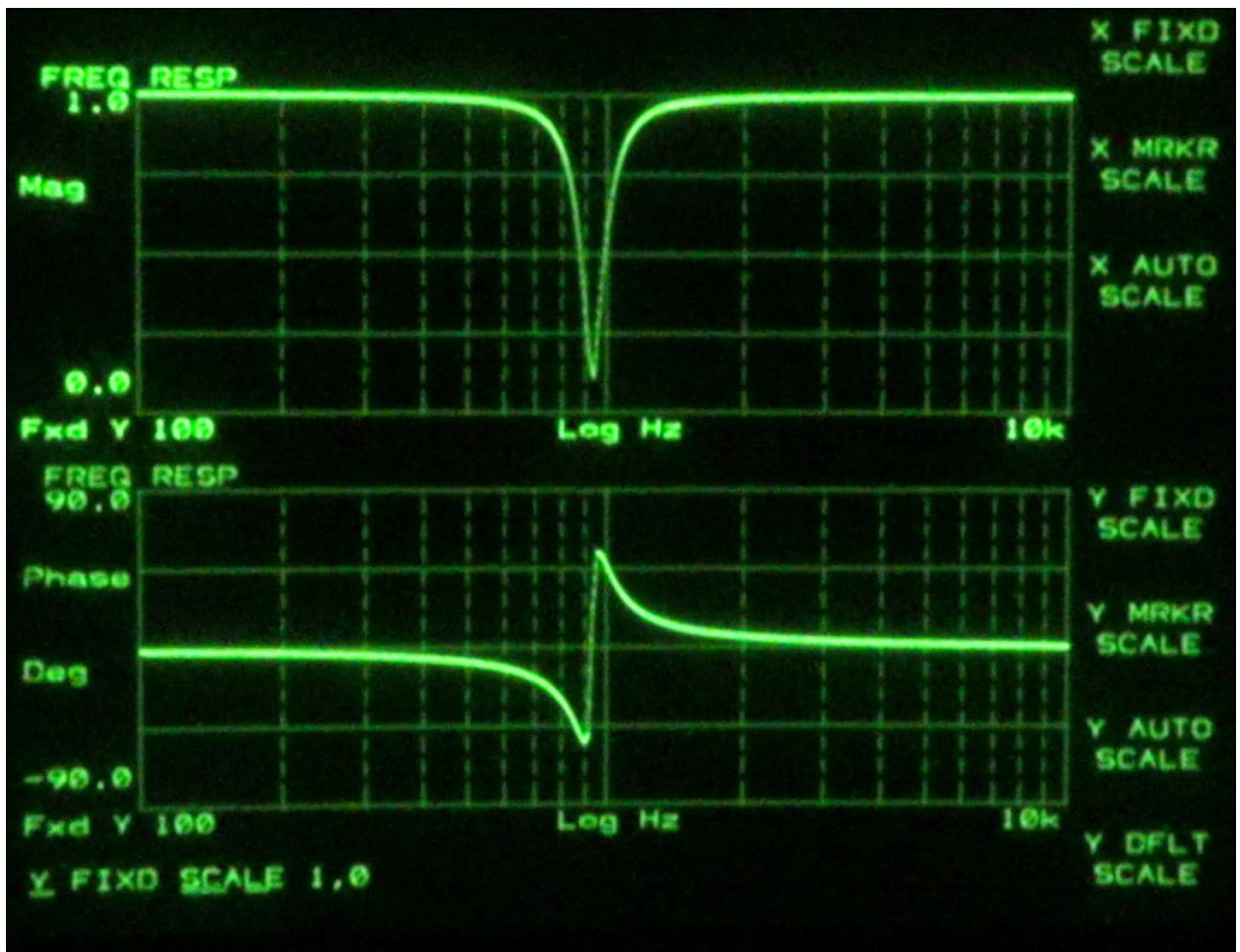
$$\frac{1}{2\pi\sqrt{LC}} = 1 \text{ kHz}$$

$$\sqrt{LC} = 632 \Omega$$

$$Q = 6.3$$

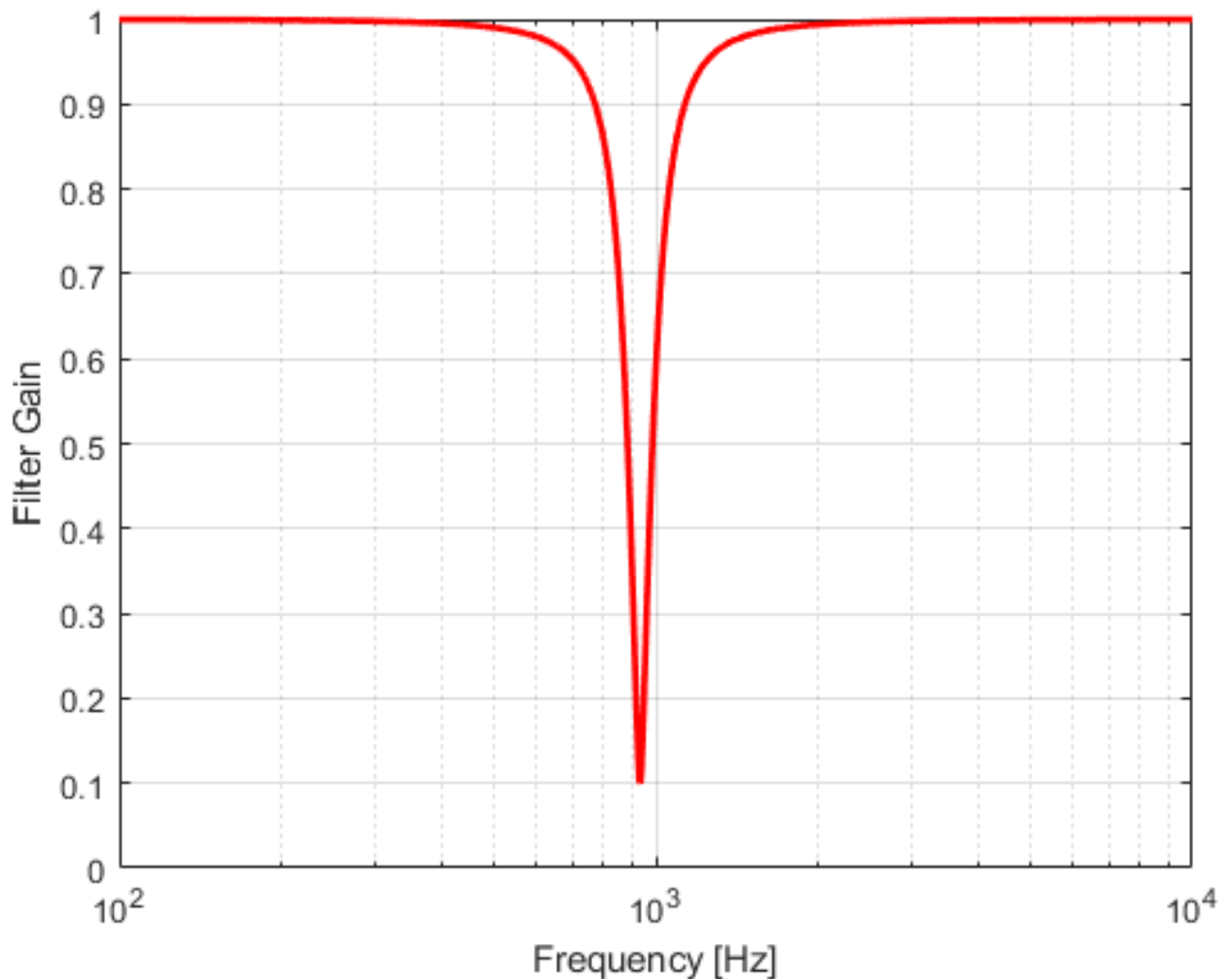
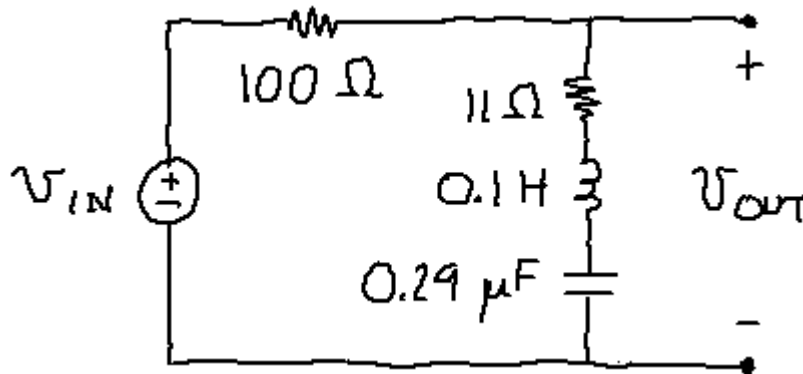
# Demo: Series-LC BSF

$$L = 0.1 \text{ H} ; C = 0.25 \mu\text{F} ; R = 100 \Omega$$
$$(2\pi\sqrt{LC})^{-1} = 1 \text{ kHz} ; \sqrt{L/C} / R = 6.3$$

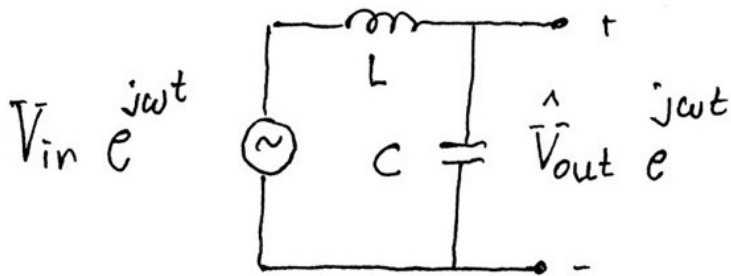


Note that the notch does not go to zero due to the parasitic series resistance of the inductor.

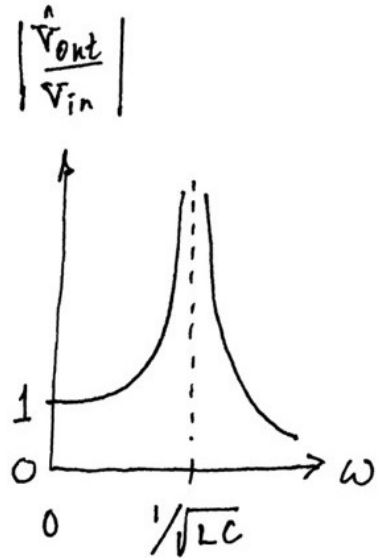
# Numerical: Series-LC BSF



# Resonators



$$\frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{1/j\omega C}{j\omega L + 1/j\omega C} = \frac{1}{1 - \omega^2 LC}$$



Peak voltage limited by parasitic resistances.

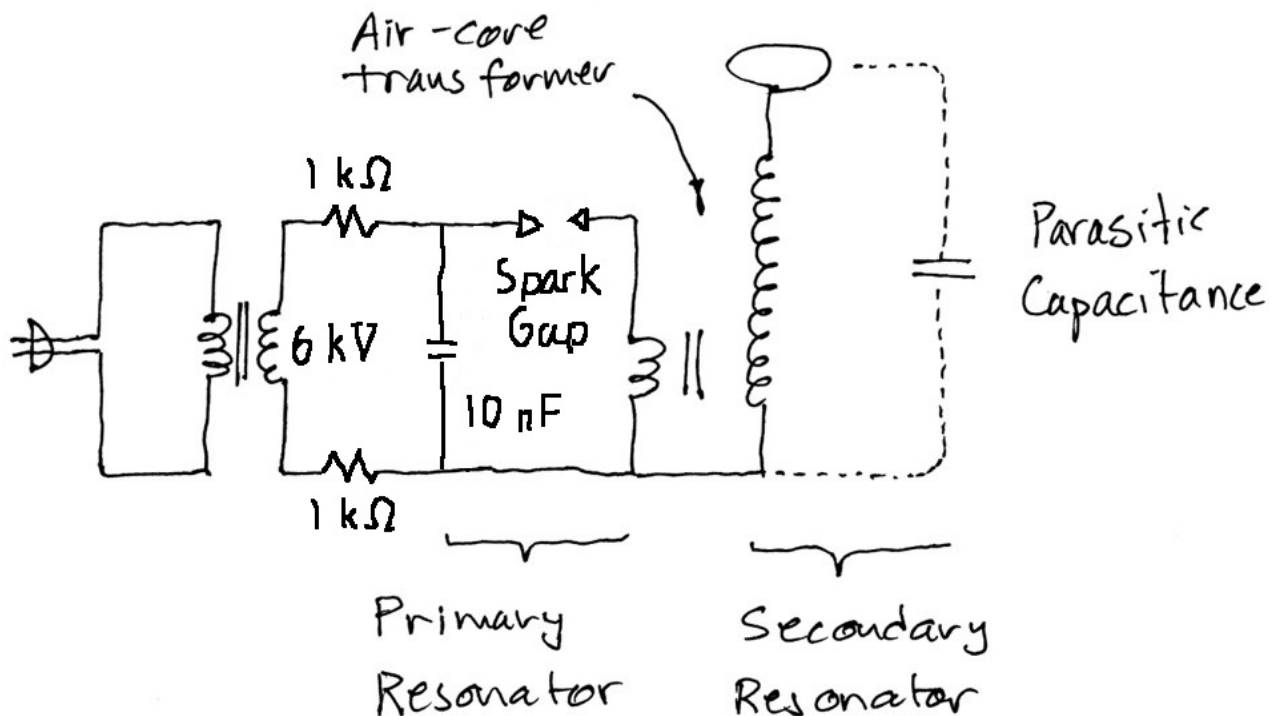
Applications include:

- capacitive shunts along inductive power - grid transmission lines;
- Tesla coil (dual tuned resonator).



# Demo: Tesla Coil

The Tesla Coil employs two coupled resonators, with both resonators tuned to the same frequency.



The resistors protect the transformer, which charges the capacitor (+/-) at 120 Hz. When the capacitor voltage gets high enough, the spark gap sparks, and becomes a short allowing the LC resonator to oscillate, driving the rest of the Tesla coil.

# Demo: Tesla Coil

