

6.200 - Lecture 23

Active “LC” Filters

- Successful LC Filters
- Active Filter Motivation
- Sallen-Key Topologies
- Examples

Successful LC Filters

Power System Inductor & Capacitor Banks

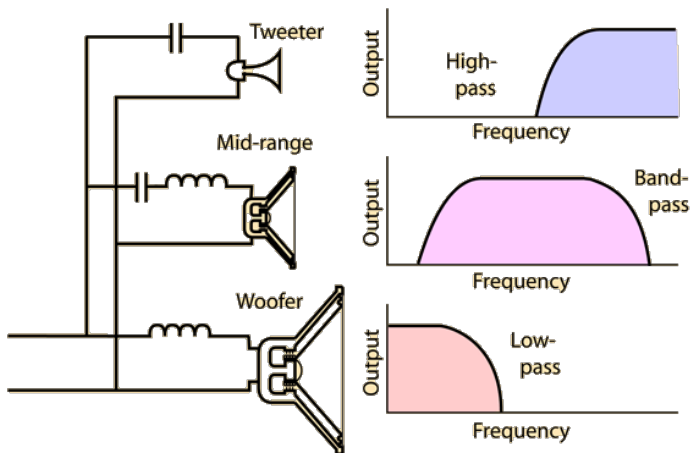


eaton.com

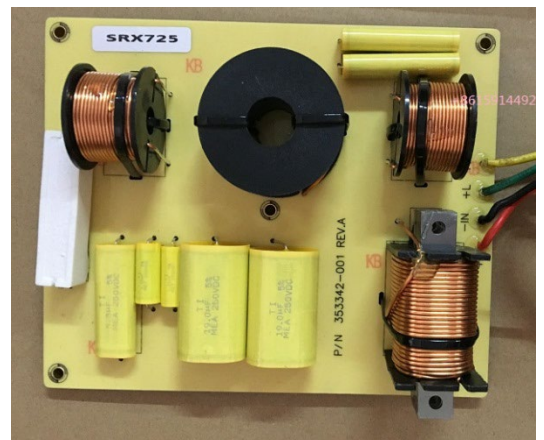


<https://electrical-engineering-portal.com>

Audio Crossover Networks

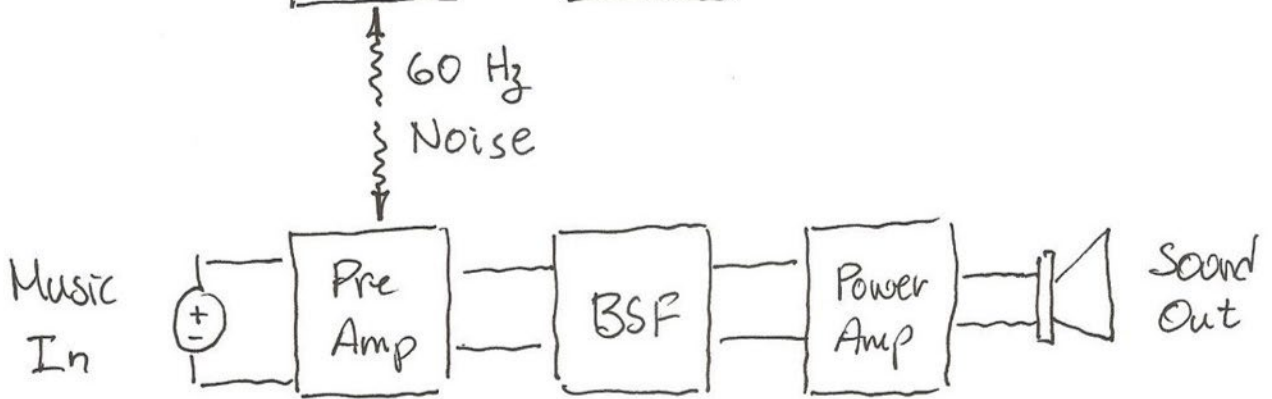
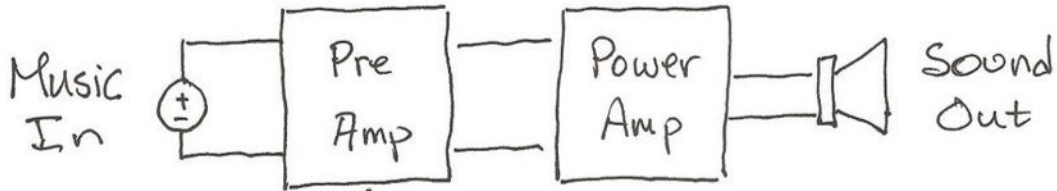


<http://hyperphysics.phy-astr.gsu.edu>

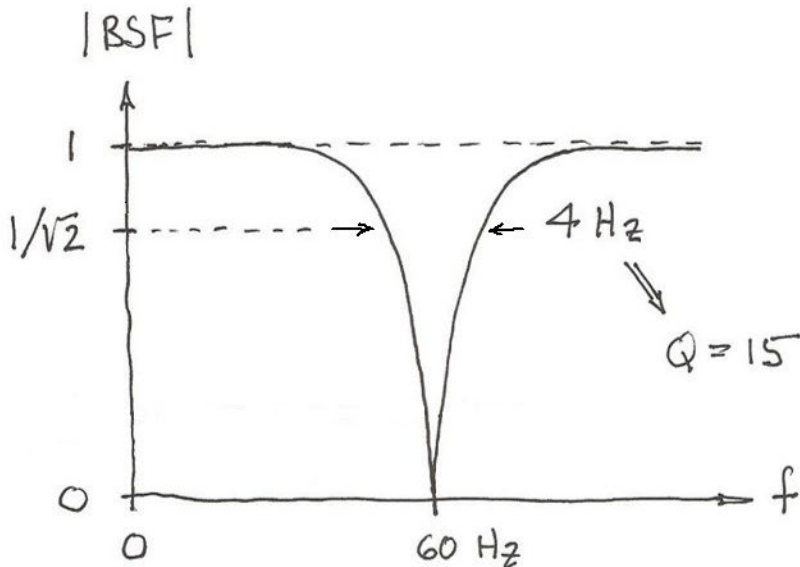


[guangshou shengda audio](http://guangshou-shengda-audio.com)

Motivating Example

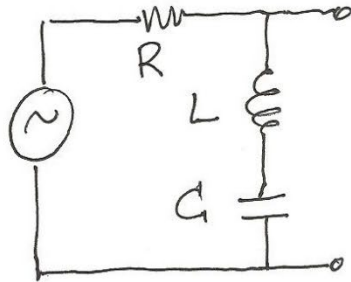


(Best is to eliminate the noise itself.
Next best is to employ a bandstop filter.)



Passive Band-Stop Filter

Design



$$\omega_0 = 1/\sqrt{LC}$$

$$Z_0 = \sqrt{L/C}$$

$$Q = Z_0/R$$

$$L = 1 \text{ H}$$

$$C = 7 \mu\text{F}$$

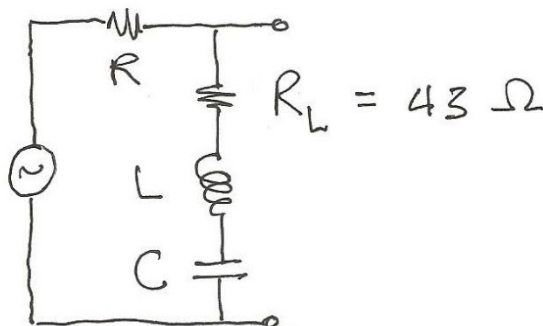
$$R = 25 \Omega$$

$$\omega_0 = 377 \frac{\text{rad}}{\text{s}} = 60 \text{ Hz}$$

$$Z_0 = 377 \Omega$$

$$Q = 15$$

Reality



$$\text{Minimum Gain} = \frac{43}{43+25} = 0.63$$

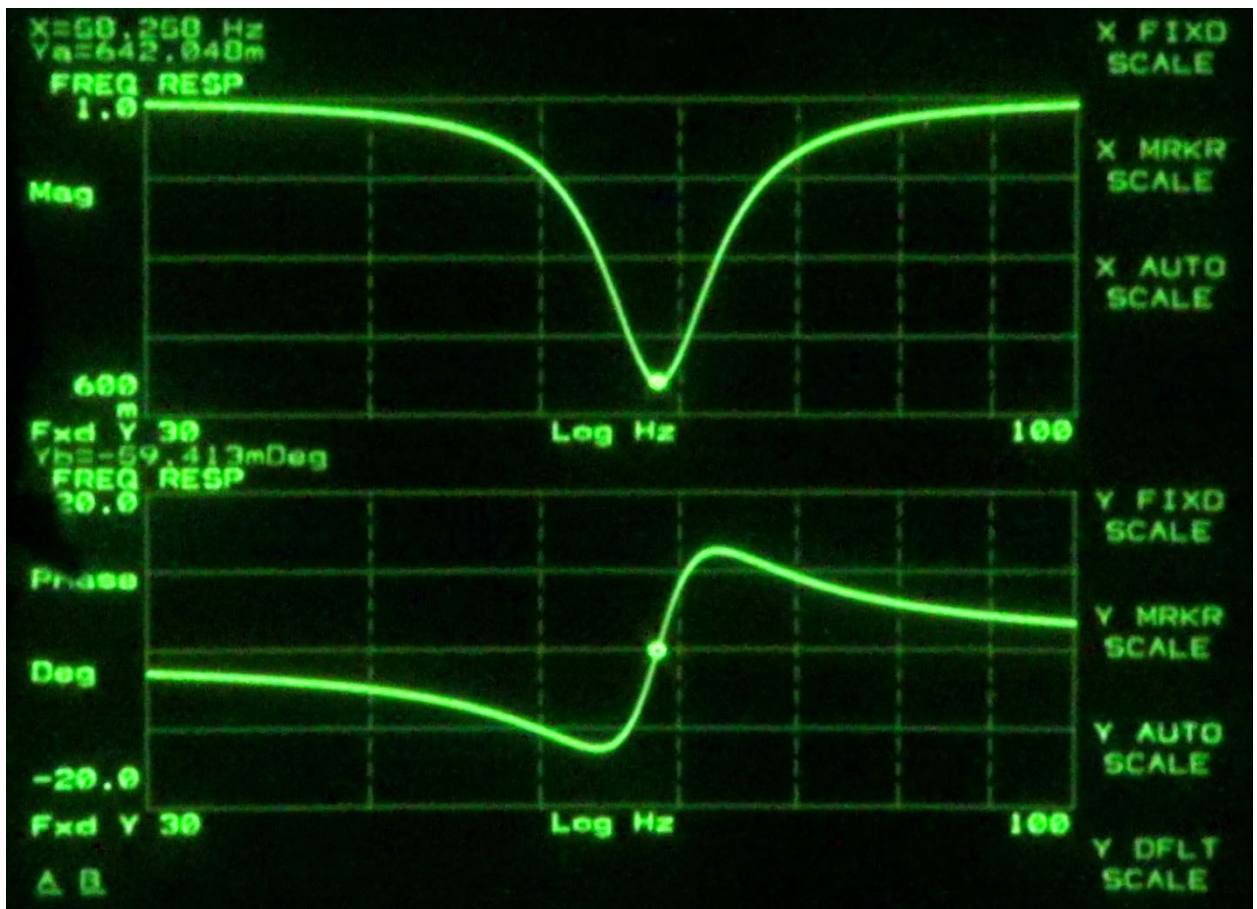
$$Q = Z_0 / (R + R_L) = 5.5$$

Demo

$L = 1 \text{ H}$; $C = 7 \mu\text{F}$; $R = 25 \Omega$ (Hopefully)

$(2\pi\sqrt{LC})^{-1} = 60 \text{ Hz}$; $\sqrt{L/C} = 377 \Omega$

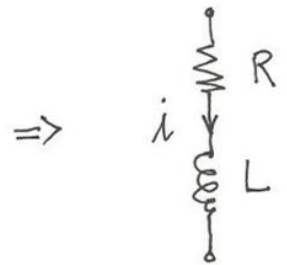
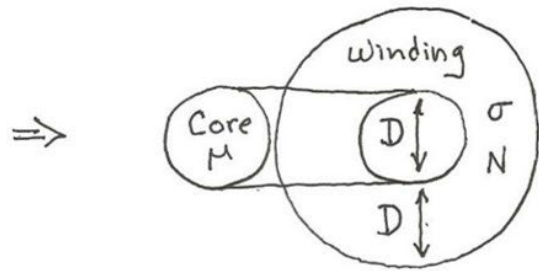
$Q = 15$ (Hopefully)



The DC resistance of the inductor is 43Ω ,
so the stop band will drop only to 0.63
while Q drops to 5.5!

Magnetics Scaling

Inductor



$$L = \frac{N^2 \mu \text{Area}}{\text{Length}} = \frac{N^2 \mu \pi D^2 / 4}{2 \pi D} = \frac{N^2 \mu D}{8}$$

$$R = \frac{N^2 \text{Length}}{\sigma \text{Area}} = \frac{N^2 2 \pi D}{\sigma \pi D^2 / 4} = \frac{8 N^2}{\sigma D}$$

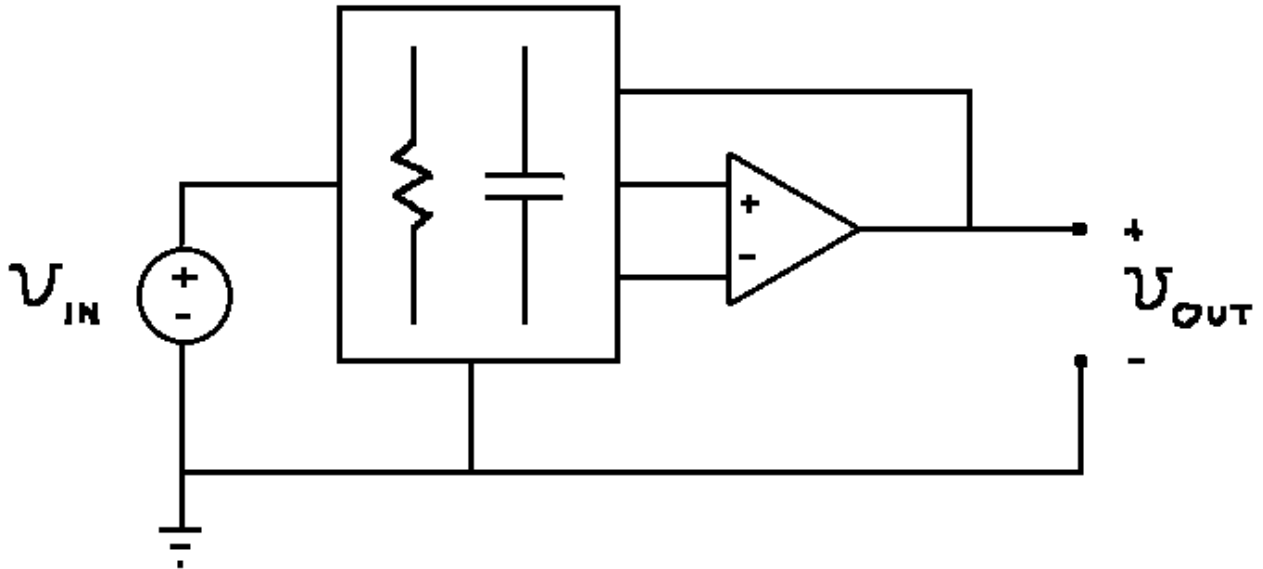
Sinusoidal Steady State $\Rightarrow i = I \cos(\omega t)$

$$Q = 2\pi \frac{\text{Peak Stored Energy}}{\text{Energy Lost Per Cycle}} = 2\pi \frac{L I^2 / 2}{\frac{1}{2} I^2 R \frac{2\pi}{\omega}} = \frac{\omega L}{R} = \frac{\omega \mu \sigma D^2}{64}$$

$Q \sim D^2 \Rightarrow \begin{cases} \text{Bigger is better} \\ \text{Smaller is worse} \end{cases}$

Sallen-Key-Style Filters

R. P. Sallen and E. L. Key, “A Practical Method of Designing RC Active Filters”, *IRE Circuit Theory*, 2:1, 74-85, March 1955

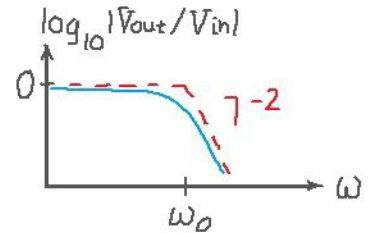


- Simple second-order filters
- Low cost and small size
- “Insensitive” to component variations
- Low power and power handling
- Difficult to tune

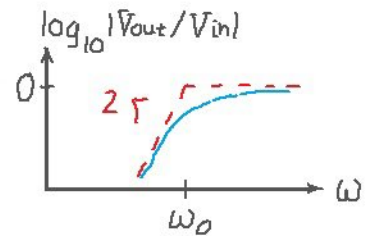
Unity-Gain Second-Order Filters

$$s \equiv j\omega \leftrightarrow d/dt$$

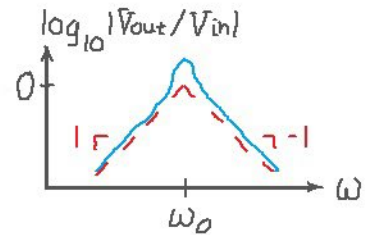
$$\text{LPF: } \frac{V_{out}}{V_{in}} = \frac{\omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$



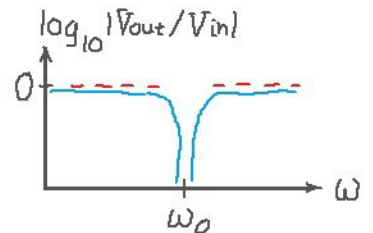
$$\text{HPF: } \frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + s\omega_0/Q + \omega_0^2}$$



$$\text{BPF: } \frac{V_{out}}{V_{in}} = \frac{s\omega_0/Q}{s^2 + s\omega_0/Q + \omega_0^2}$$



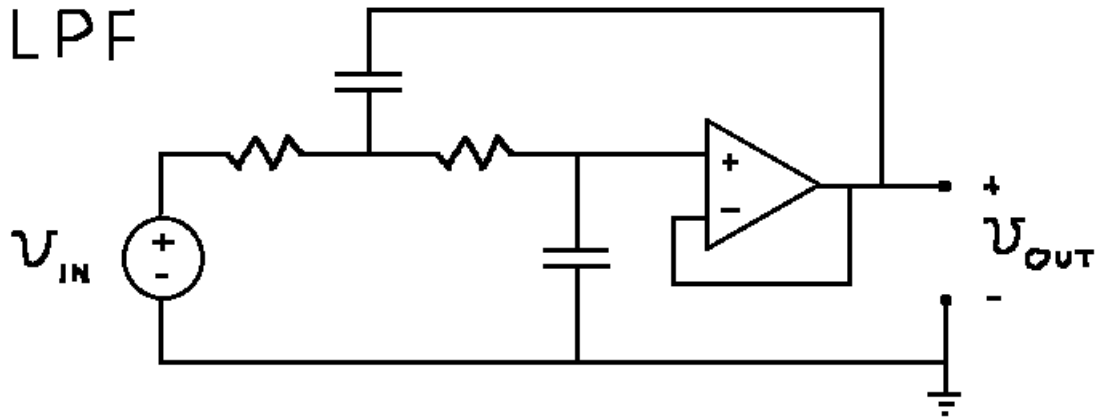
$$\text{BSF: } \frac{V_{out}}{V_{in}} = \frac{s^2 + \omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2}$$



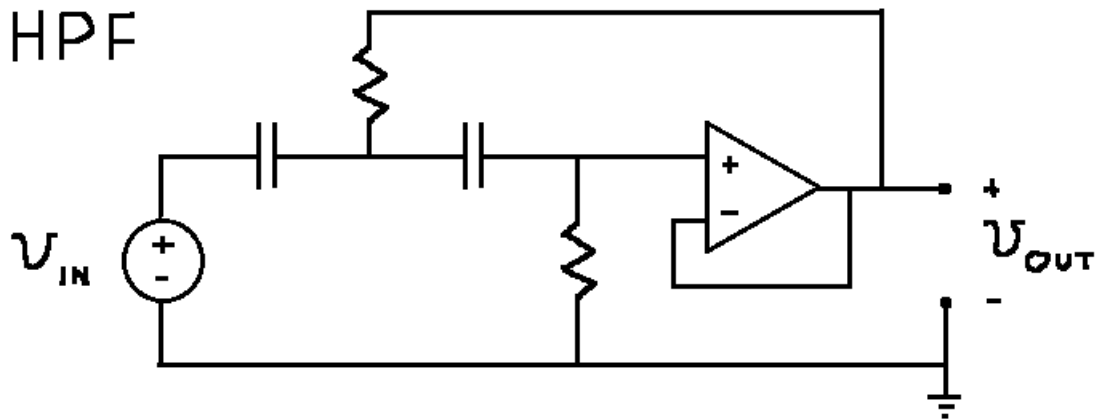
$$\ddot{V}_{out} + \frac{\omega_0}{Q} \dot{V}_{out} + \omega_0^2 V_{out} = \underbrace{\ddot{V}_{in} + \frac{\omega_0}{Q} \dot{V}_{in} + \omega_0^2 V_{in}}_{\text{Selected by filtering type}}$$

Filter Examples

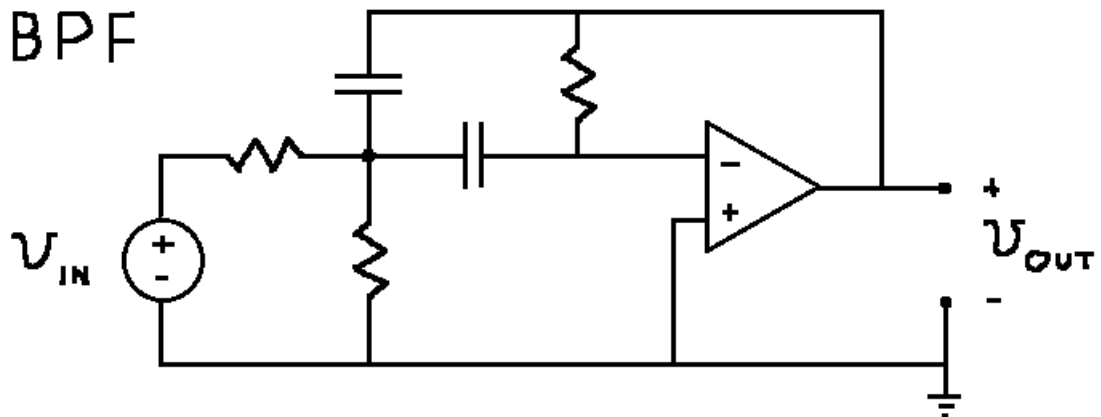
LPF



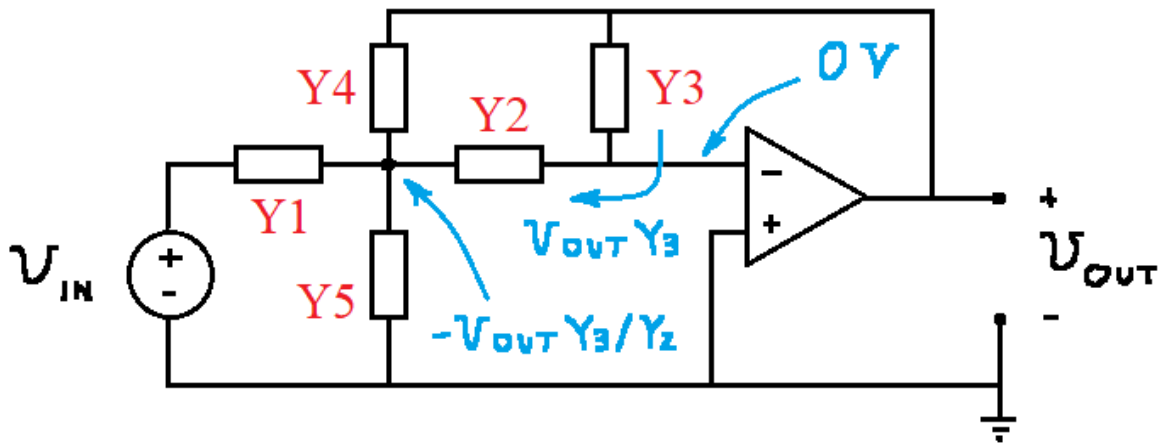
HPF



BPF



Inverting BPF Design

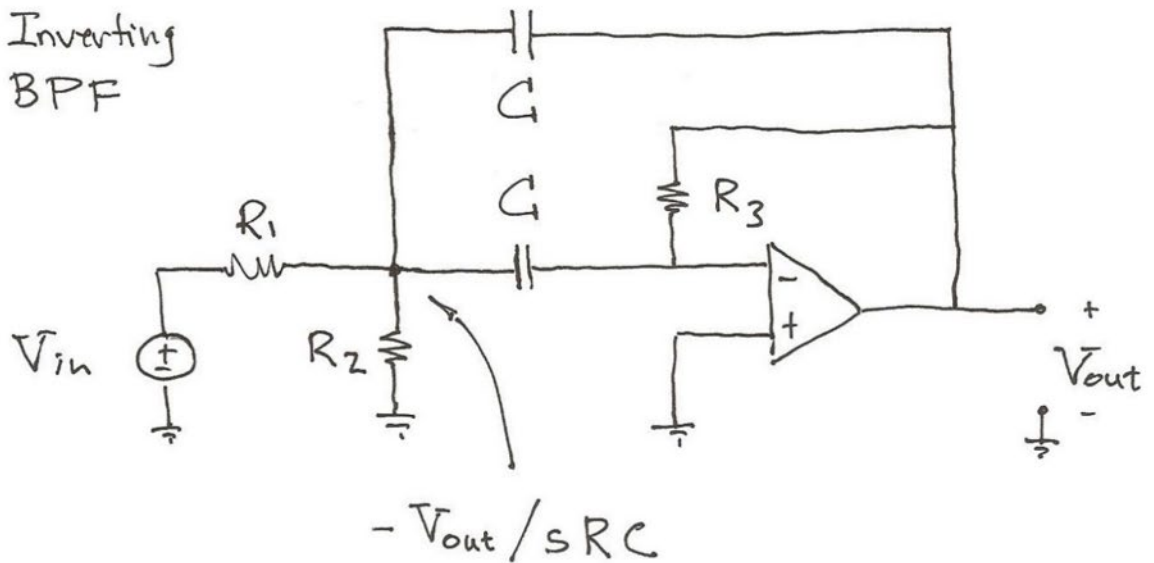
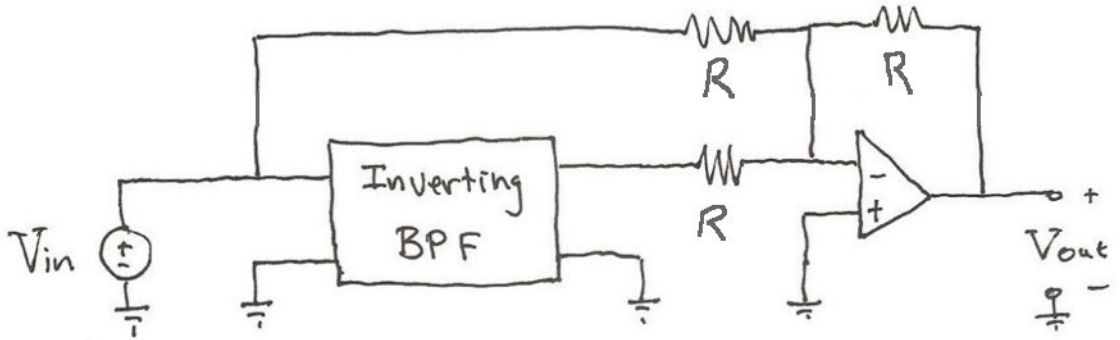
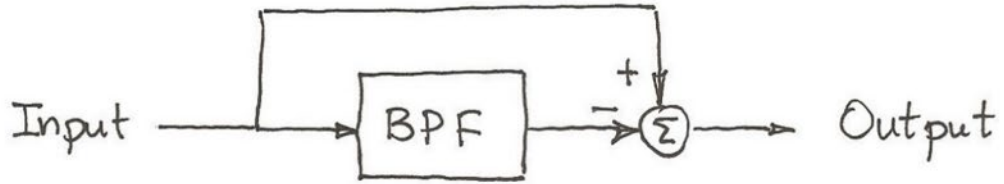


$$\text{KCL} \Rightarrow 0 = (V_{IN} + V_{OUT} Y_3 / Y_2) Y_1 + (V_{OUT} Y_3 / Y_2) Y_5 + (V_{OUT} + V_{OUT} Y_3 / Y_2) Y_4 + (V_{OUT}) Y_3$$

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{-Y_1 Y_2}{Y_1 Y_3 + Y_2 Y_3 + Y_2 Y_4 + Y_3 Y_4 + Y_3 Y_5}$$

- To get s^1 in the numerator choose $Y_1 = 1/R_1$ and $Y_2 = s \cdot C_1$
- To bias the op amp, choose $Y_3 = 1/R_3$
- Now $Y_1 \cdot Y_3$ gives s^0 , and $Y_2 \cdot Y_3$ gives s^1 , in the denominator
- To get s^2 in the denominator choose $Y_4 = s \cdot C_2$
- Arbitrarily choose $Y_5 = 1/R_2$
- Could repeat with $Y_1 = s \cdot C$ and $Y_2 = 1/R$

BSF Design



Inverting BPF Analysis

$$\text{KCL} \Rightarrow \frac{V_{in} + \frac{V_{out}}{sR_3C}}{R_1} + \frac{V_{out}}{R_2} + \frac{V_{out}}{sR_3C} sG + \left[V_{out} + \frac{V_{out}}{sR_3C} \right] sC = 0$$

$$V_{in} \left[\frac{s}{R_1 C} \right] + V_{out} \left[s^2 + \frac{s}{R_3 C} + \frac{s}{R_3 C} + \frac{1}{R_1 R_3 C^2} + \frac{1}{R_2 R_3 C^2} \right] = 0$$

$$V_{out} \left[s^2 + \underbrace{\frac{2}{R_3 C}}_{\omega_0/Q} s + \underbrace{\frac{R_1 + R_2}{R_1 R_2 R_3 C}}_{\omega_0^2} \right] = -V_{in} \left[\underbrace{\frac{1}{R_1 C}}_{G\omega_0/Q} s \right]$$

$$\omega_0^2 = \frac{R_1 + R_2}{R_1 R_2 R_3 C^2}$$

$$R_1 = \frac{Q}{G\omega_0 C}$$

$$Q = \frac{1}{2} \sqrt{\frac{(R_1 + R_2) R_3}{R_1 R_2}}$$

$$R_2 = \frac{Q}{[2Q^2 - Q]\omega_0 C}$$

$$G = \frac{R_3}{2R_1}$$

$$R_3 = \frac{2Q}{\omega_0 C}$$

Demo Design

$$\omega_0 = 2\pi \times 60 \text{ Hz}$$

$$R1 = 20 \text{ k}\Omega$$

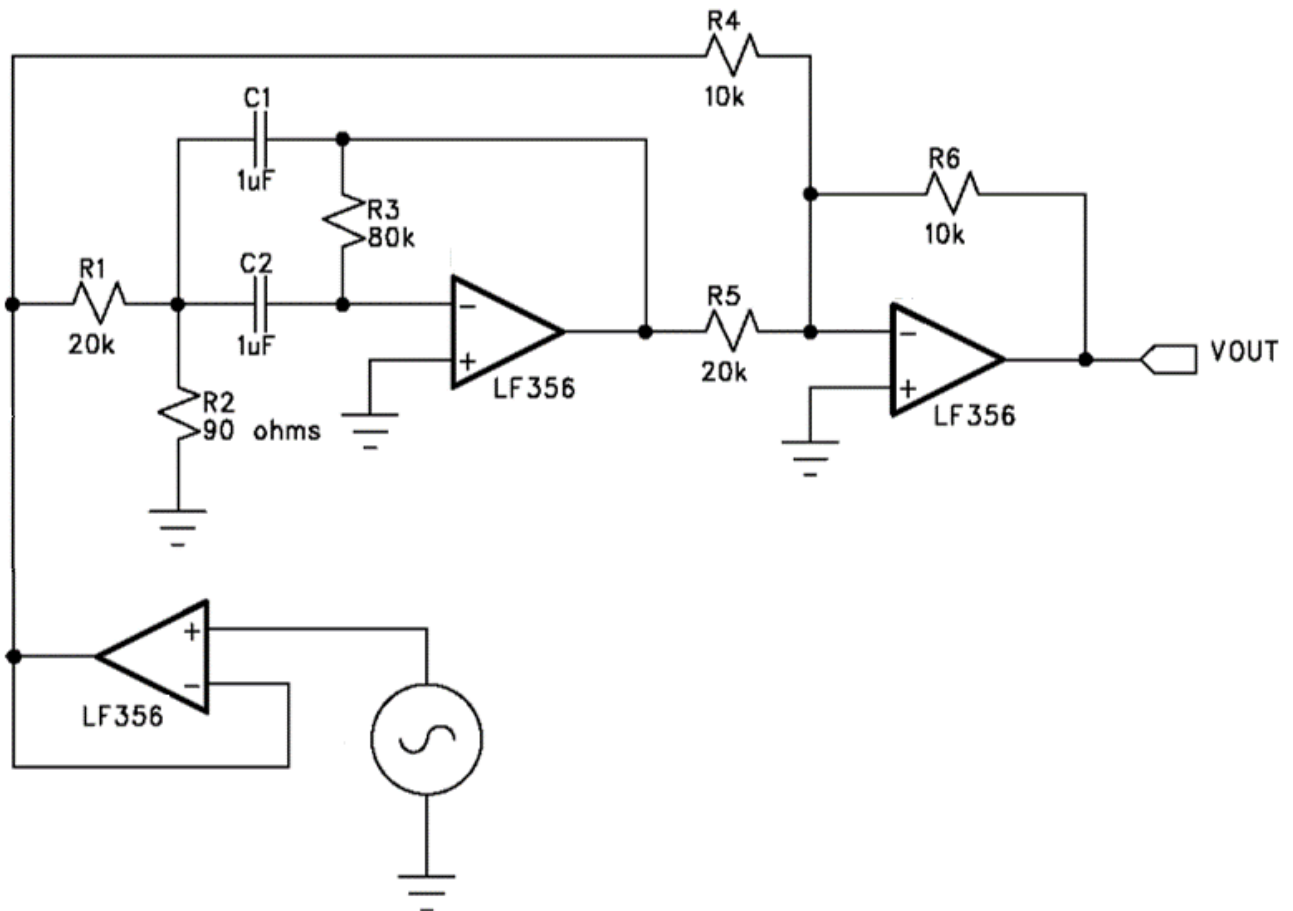
$$Q = 15$$

$$R2 = 90 \Omega$$

$$G = 2$$

$$R3 = 80 \text{ k}\Omega$$

$$C = 1 \mu\text{F}$$



Demo

