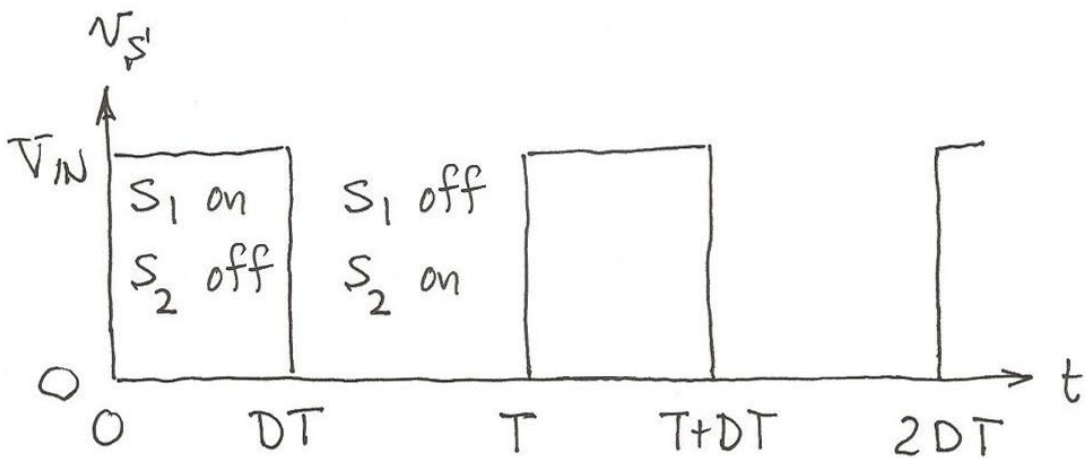
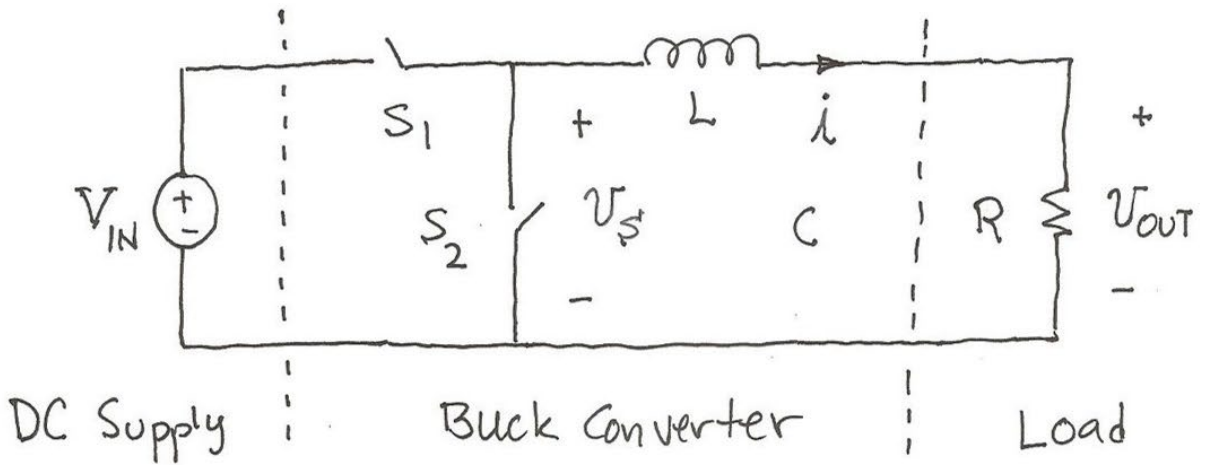


6.200 - Lecture 24

Buck Converter

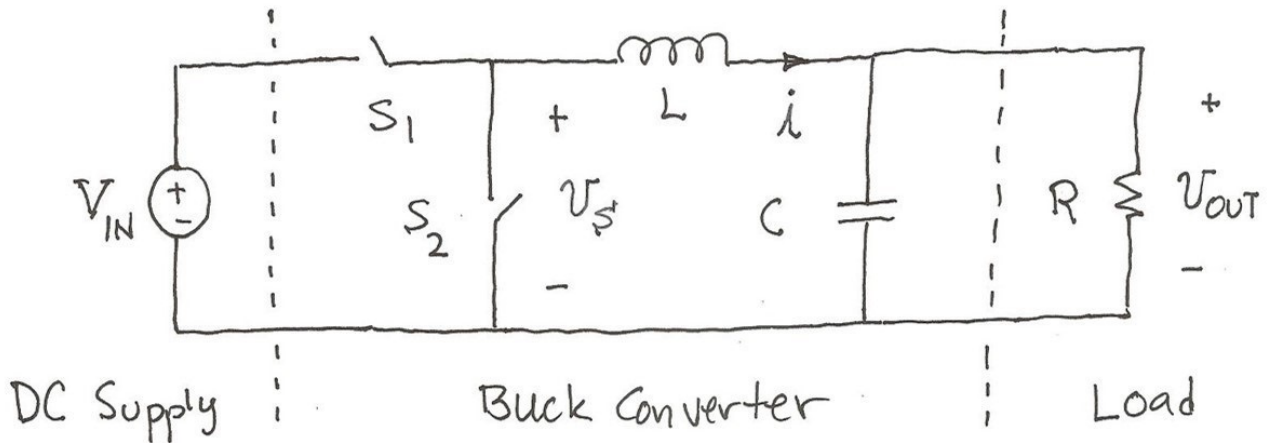
- Principles of operation
- Performance improvement
- Analysis via averaging
- Analysis via filtering
- Natural response

Buck Converter



- * Digital electronics wish to operate at low voltage (V_{out}) to reduce losses and heat.
- * Power is cheaper to deliver at high voltages due to wiring costs at high current.
- * Buck converters bridge the high \rightarrow low voltage gap.

Improved Buck Converter



- * The original buck converter exhibited a significant ripple current through, and hence a significant ripple voltage across, the load.
- * The added capacitor offers a low-impedance path for the ripple current and hence filters out the load ripple current and voltage.
- * But, the converter natural response may now be oscillatory.

Average Analysis

* Assume cyclic continuous ($i > 0$) operation.

* Define an average: $\langle x(t) \rangle \equiv \frac{1}{T} \int_0^T x(s) ds$.

$$\text{Inductor} \Rightarrow \int \dot{V}_S(t) - V_{OUT}(t) = L \frac{di}{dt}$$

$$\langle V_S(t) \rangle - \langle V_{OUT}(t) \rangle = \frac{L}{T} (i(T) - i(0)) \equiv 0$$

$$\langle V_{OUT}(t) \rangle = D \bar{V}_{IN}$$

$$\text{Capacitor} \Rightarrow i(t) - \frac{V_{OUT}(t)}{R} = C \frac{dV_{OUT}}{dt}$$

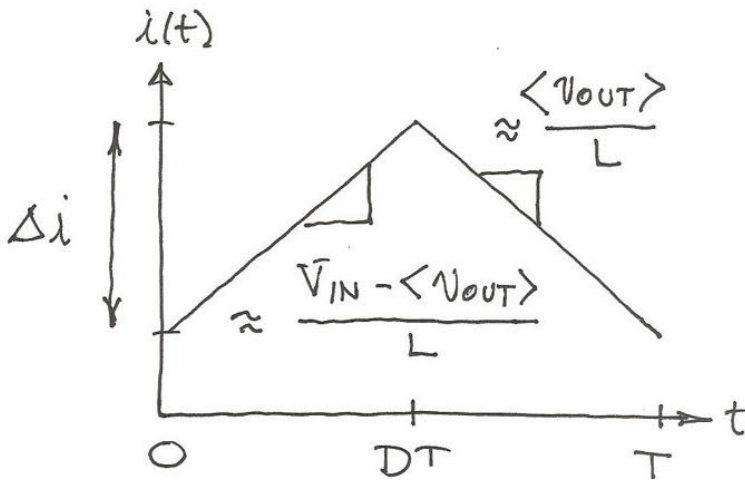
$$\langle i(t) \rangle - \frac{\langle V_{OUT}(t) \rangle}{R} = \frac{C}{T} (V_{OUT}(T) - V_{OUT}(0)) \equiv 0$$

$$\langle i(t) \rangle = \frac{\langle V_{OUT}(t) \rangle}{R} = \frac{D \bar{V}_{IN}}{R}$$

Duty cycle D is used to control V_{OUT} .

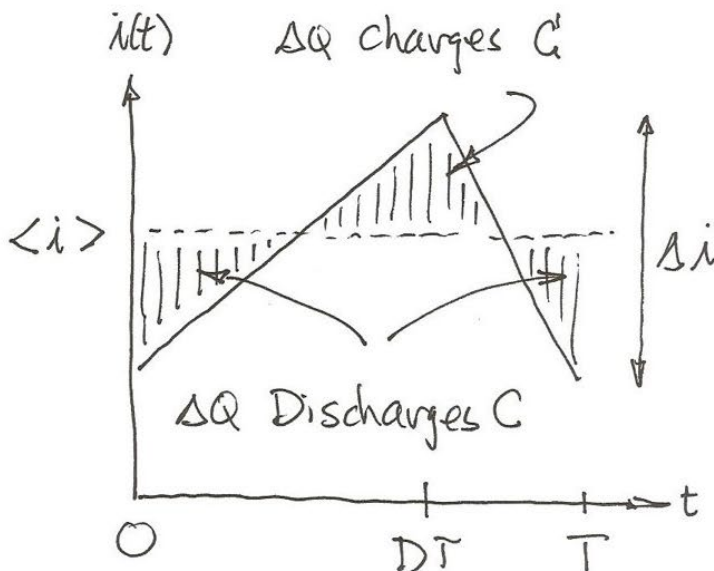
Ripple Analysis

Ideal inductor \Rightarrow inductor current is approximately piecewise linear.



$$\begin{aligned} \Delta i &\approx \frac{\langle V_{OUT} \rangle}{L} (1-D) \\ &= \frac{D(1-D)T \bar{V}_{IN}}{L} \end{aligned}$$

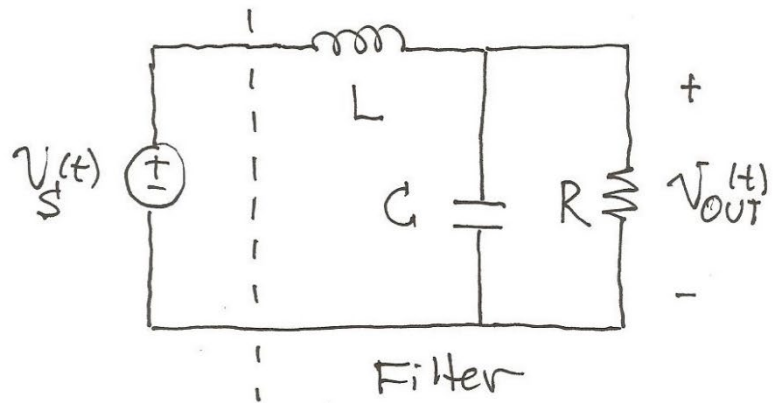
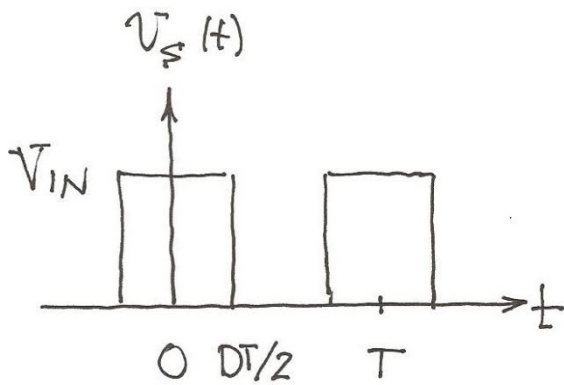
High-frequency inductor current passes largely through capacitor because $\frac{1}{\omega C} \ll R$.



$$\begin{aligned} \Delta Q &\approx \frac{1}{2} \frac{\Delta i}{2} \frac{T}{2} \\ &= \frac{D(1-D)T^2 \bar{V}_{IN}}{8L} \end{aligned}$$

$$\begin{aligned} \Delta V_{OUT} &= \frac{\Delta Q}{C} \\ &= \frac{D(1-D)T^2 \bar{V}_{IN}}{8LC} \end{aligned}$$

Buck Converter → Low-Pass Filter



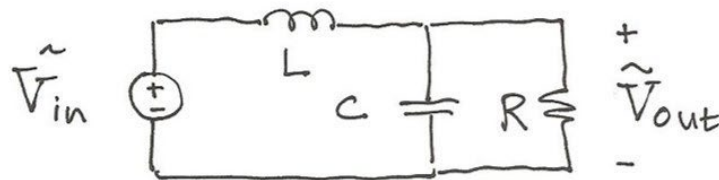
Fourier decomposition of $V_s(t)$:

$$V_s(t) = D V_{IN} + \sum_{n=1}^{\infty} \frac{2 V_{IN} \sin(n\pi D)}{n\pi} \cos(n\omega_s t)$$
$$\omega_s T \equiv 2\pi$$

Filter each Fourier component separately and then recombine results to get complete response.

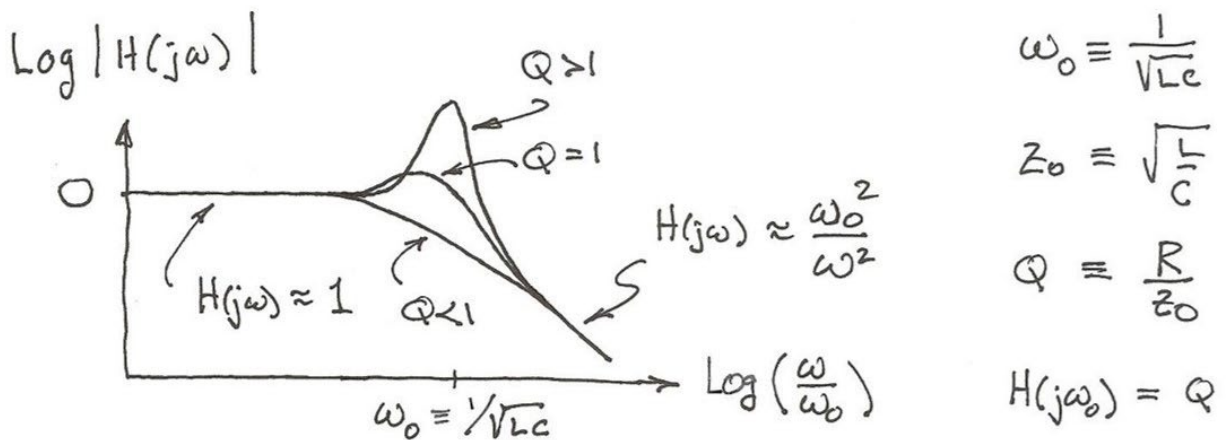
can do this for all signals: $V_{OUT}(t)$, $i(t)$, ...

Filter Analysis (Driven Response)



$$\tilde{V}_{out} = \frac{(R \parallel \frac{1}{j\omega C}) \tilde{V}_{in}}{j\omega L + (R \parallel \frac{1}{j\omega C})} = \frac{\tilde{V}_{in}}{1 - \omega^2 LC + \frac{j\omega L}{R}} \equiv H(j\omega) \tilde{V}_{in}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(1 - \omega^2 LC)^2 + (\frac{\omega L}{R})^2}} \quad \angle H(j\omega) = \tan^{-1}\left(\frac{-\omega L/R}{1 - \omega^2 LC}\right)$$



Demo $\Rightarrow L = 1 \text{ mH} ; C = 141 \text{ } \mu\text{F}$

$\Rightarrow \omega_0 = 2.6 \frac{\text{krad}}{\text{s}} (424 \text{ Hz})$

Filtering Interpretation

- At $\omega=0$, $H(j\omega) = 1 \Rightarrow \tilde{V}_{out} = D\tilde{V}_{IN} = \langle v_{S1}(t) \rangle$.

Same result as before!

- $f_s = 40 \text{ kHz} \Rightarrow \omega_s = 80\pi \frac{\text{krad}}{\text{s}} \gg \omega_o = 2.6 \frac{\text{krad}}{\text{s}}$.

\Rightarrow All switching harmonics are greatly attenuated

\Rightarrow Small ripple!

- Ripple fundamental $\Rightarrow \frac{2\tilde{V}_{IN} \sin(\pi D)}{\pi} \frac{\omega_o^2}{\omega_s^2}$ amplitude.

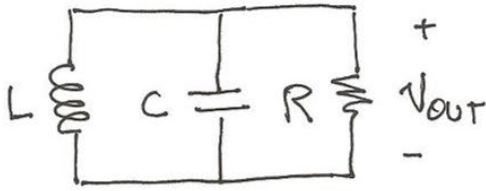
$$\text{For } D = \frac{1}{2}, \text{ amplitude} = \frac{2\tilde{V}_{IN}}{\pi} \frac{T^2}{4\pi^2 LC}$$

$$= \frac{T^2 \tilde{V}_{IN}}{2\pi^3 LC} \approx \frac{T^2 \tilde{V}_{IN}}{62 LC}$$

$$\text{Compare with } \frac{1}{2} \frac{T^2 \tilde{V}_{IN}}{32 LC} = \frac{T^2 \tilde{V}_{IN}}{64 LC} \text{ from}$$

ripple analysis!

Natural Response



$$C \frac{dV_{OUT}}{dt} + \frac{V_{OUT}}{R} + \frac{1}{L} \int_{-\infty}^t V_{OUT} dt = 0$$

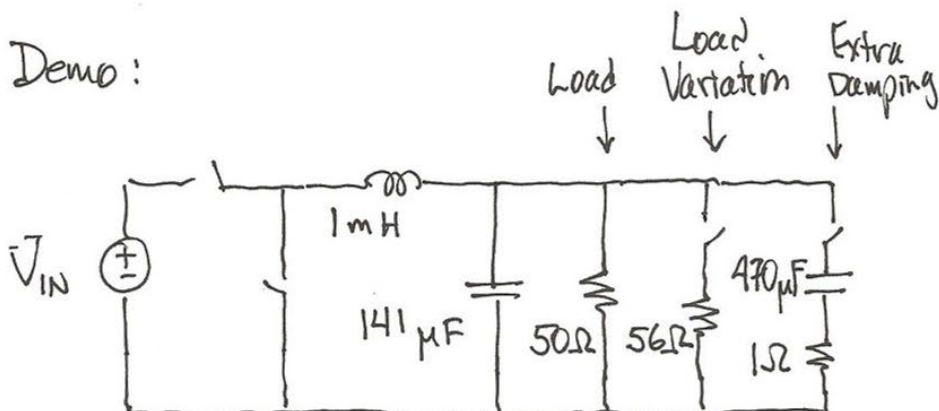
$$\frac{d^2 V_{OUT}}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha} \frac{dV_{OUT}}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} V_{OUT} = 0$$

$$e^{st} \Rightarrow s = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} \approx -\alpha + j\omega_0$$

Lightly Damped

Natural response will appear for :

- * change in input voltage V_{IN} ;
- * change in duty cycle D ;
- * change in load resistance R .



Demo Ripple Analysis

$$L = 1 \text{ mH}$$

$$D = 0.5$$

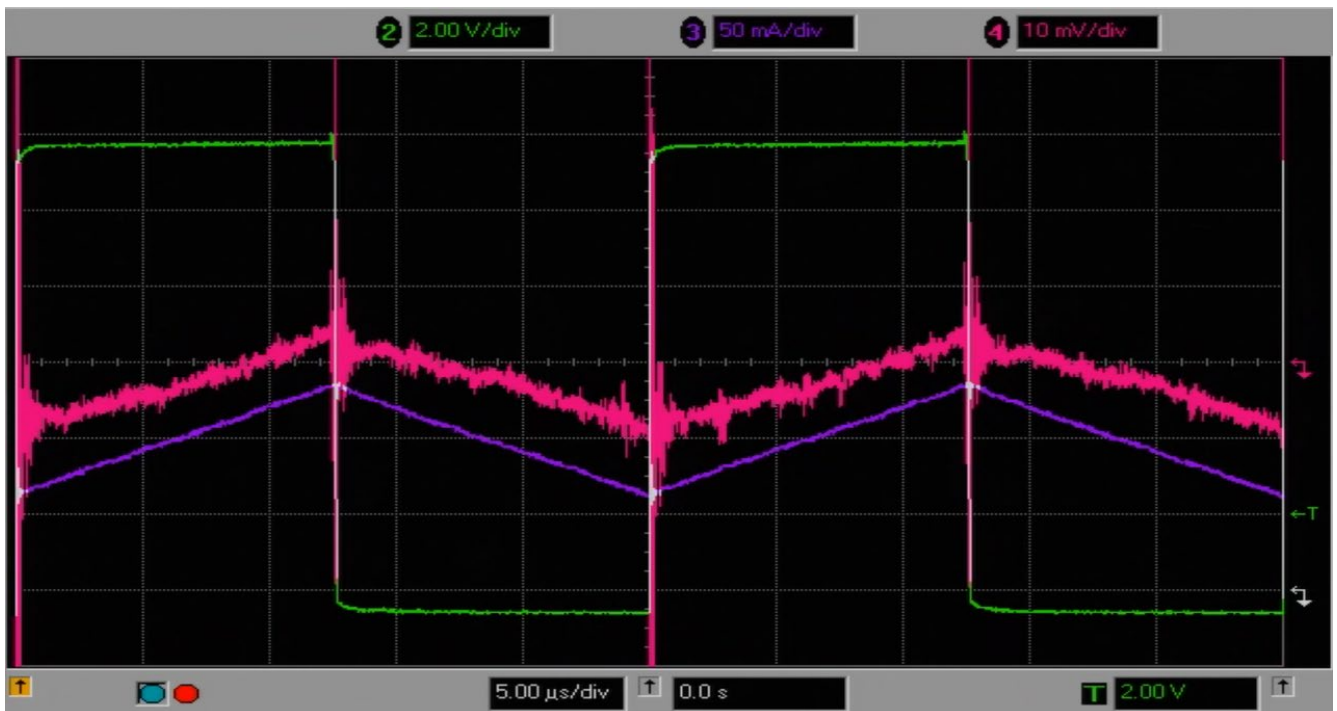
$$V_{IN} = 12 \text{ V}$$

$$C = 141 \mu\text{F}$$

$$T = 25 \mu\text{s}$$

$$\Delta i = \frac{D(1-D)T V_{IN}}{L} = 75 \text{ mA}$$

$$\Delta V_{OUT} = \frac{\Delta i T}{8C} = 13 \text{ mV}$$



Demo Natural Response

$$L = 1 \text{ mH}$$

$$D = 0.5$$

$$V_{IN} = 12 \text{ V}$$

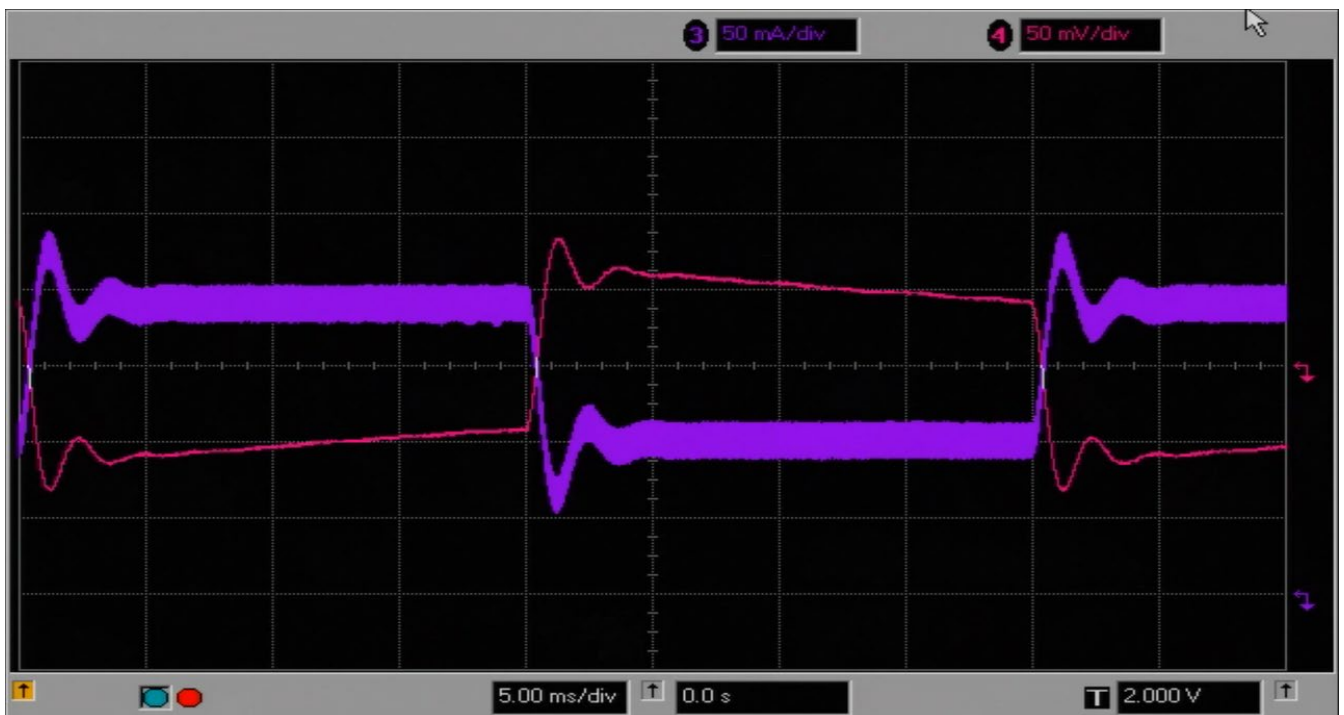
$$C = 141 \mu\text{F}$$

$$T = 25 \mu\text{s}$$

$$R = 50 \Omega$$

$$\text{Period} = 2\pi\sqrt{LC} = 2.4 \text{ ms} \leftrightarrow 424 \text{ Hz}$$

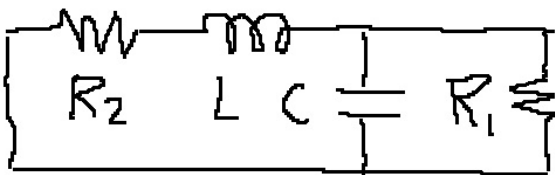
$$Q = \frac{R}{\sqrt{L/C}} = 19 \text{ Without Inductor Loss}$$



Q

$$Q = 2\pi \frac{\text{Stored Energy}}{\text{Energy Lost Per Cycle}}$$

$$\frac{1}{Q} = \frac{1}{2\pi} \frac{1}{\text{Stored Energy}} \sum_i \text{Cycle Loss}_i$$



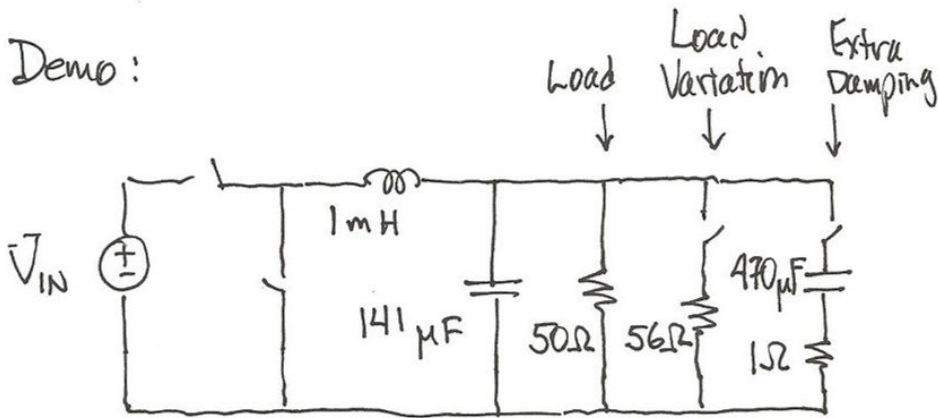
A circuit diagram showing a series RLC circuit. It consists of a resistor labeled R_2 , an inductor labeled L , a capacitor labeled C , and another resistor labeled R_1 connected in series.

$$Q_1 = \frac{R_1}{\sqrt{L/C}} \quad Q_2 = \frac{\sqrt{L/C}}{R_2}$$

$$L = 1 \text{ mH} \quad C = 141 \mu\text{F} \quad R_1 = 50 \Omega \quad R_2 = 0.5 \Omega$$

$$Q_1 = 19 \quad Q_2 = 5.3 \quad Q = 4.2$$

Damping



Objective is to reduce the oscillatory load - step response.

- * Does not affect average (DC) behavior because of series capacitor.
- * Does not carry much ripple current (get hot) because $1\Omega \gg 1/(\omega_s \cdot 141\mu\text{F})$.
- * Provides significant damping because 1Ω and $1/(\omega_N \cdot 470\mu\text{F}) \ll 50\Omega$.

Demo Damping

Damping network $\Rightarrow R = 1 \Omega$; $C = 470 \mu\text{F}$

* $1/\omega_N C = 0.8 \Omega$ at $\omega_N = 424 \text{ Hz}$ \Rightarrow small
but not strictly negligible. Ignore anyway.

* $Q = \frac{R}{Z_0} = 0.4 \Rightarrow$ Overdamped

