Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.200 – Circuits & Electronics
Spring 2026

Quiz #2

22 April 2026

Name: SOLUTIONS

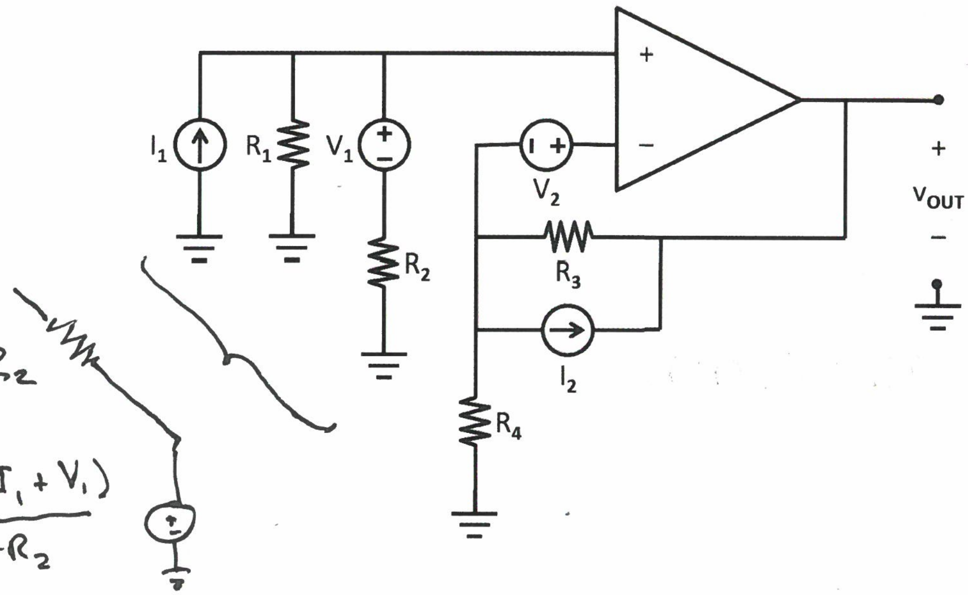
MIT EMail: ______________________ @MIT.EDU

Recitation Time: 12 1 2

- There are 15 pages in this quiz, including this cover page.
- Please put your name and MIT EMail ID in the spaces provided above, and circle the time of your recitation.
- Please do not remove any pages from this quiz.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. *When finished with each part, clearly write your answer for that part into the corresponding answer box or graph.*
- *All numerical answers require proper units.*
- *In order to guarantee receipt of full credit, all answers should be justified by supporting math and/or explanations.*
- This is a closed-book closed-electronics quiz but a single two-sided page of notes is allowed.
- Good luck!

Problem 1: Op-Amp Amplifier - 6%

Shown below is an op-amp amplifier. Assume that the op amp is ideal and determine v_{OUT}.



$R_1 \| R_2$

$$\frac{R_1(R_2 I_1 + V_1)}{R_1+R_2}$$

$$v_{OUT}: \quad \boxed{\frac{R_3+R_4}{R_4} \cdot \left(\frac{R_1(R_2 I_1 + V_1)}{R_1+R_2} - V_2 \right) + R_3 I_2}$$

Use Thevenin / Norton (see above), superposition, and $v_+ \approx v_-$.

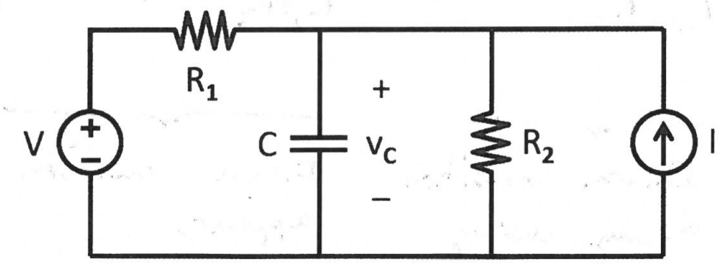
$$\text{Thevenin} \Rightarrow \frac{R_1(R_2 I_1 + V_1)}{R_1+R_2} \cdot \frac{R_3+R_4}{R_4}$$

$$V_2 \Rightarrow -V_2 \cdot \frac{R_3+R_4}{R_4}$$

$$I_2 \Rightarrow R_3 I_2$$

Problem 2: Capacitor Dynamics - 15%

The circuit shown below comprises two sources, two resistors and one capacitor. Assume that prior to time $t = 0$ the conditions $V = 0$ and constant $I \neq 0$ have been established for a very long time such that the circuit operates in steady state by $t = 0$.



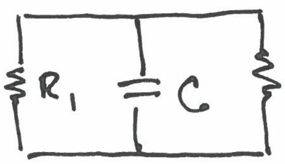
(2A) Determine v_C at $t = 0$.

$v_C(0)$: $\frac{R_2 R_1}{R_1 + R_2} I$

Steady state $\Rightarrow$ $C \frac{dv_c}{dt} = i_c = 0$ $\Rightarrow$ $v_c = \frac{R_2 R_1}{R_1 + R_2} I$

(2B) At $t = 0$, the voltage source steps to a constant such that $V \neq 0$. Determine $v_C(t)$ for $t \geq 0$.

$$v_C(t \geq 0): \quad \boxed{\underbrace{\frac{R_1 R_2 I}{R_1 + R_2} e^{-t/\tau}}_{\text{Initial condition decays away}} + \underbrace{\frac{(V + R_1 I) R_2}{R_1 + R_2}\left(1 - e^{-t/\tau}\right)}_{\text{Final condition appears}} \; ; \; \tau = \frac{R_1 R_2 C}{R_1 + R_2}}$$



$$\Rightarrow \text{RC Time Constant} \quad \tau = \frac{R_1 R_2 C}{R_1 + R_2}$$

$$\text{Final } v_c(t = \infty) = \frac{(V + R_1 I) R_2}{R_1 + R_2}$$

$$\text{Note that } v_c(t \geq 0) = \underbrace{\frac{(V + R_1 I) R_2}{R_1 + R_2}}_{\text{Particular}} - \underbrace{\frac{V R_2}{R_1 + R_2} e^{-t/\tau}}_{\text{Homogeneous}}$$

(2C) At $t = T$ with $T > 0$, the current source steps such that $I = 0$. Determine $v_C(t)$ for $t \geq T$.

$v_C(t \geq T)$: $$\underbrace{\left[\frac{(V+R_1 I)R_2 - V R_2 e^{-T/\tau}}{R_1+R_2}\right] e^{\frac{-(t-T)}{\tau}}}_{\text{Initial condition decays away}} + \underbrace{\frac{V R_2}{R_1+R_2}\left(1 - e^{\frac{-(t-T)}{\tau}}\right)}_{\text{Final condition appears}}$$

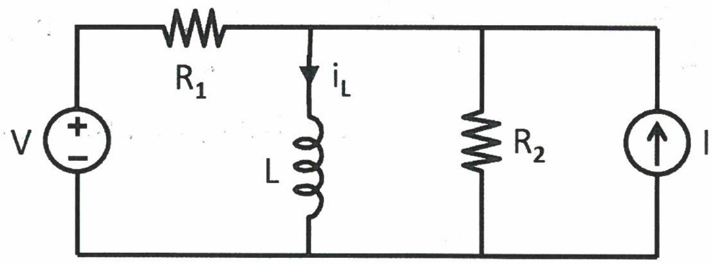
$$\tau = R_1 R_2 C / (R_1 + R_2)$$

Initial condition $v_C(t=T) = \dfrac{(V+R_1 I)R_2}{R_1+R_2} - \dfrac{V R_2}{R_1+R_2} e^{-T/\tau}$

Final $v_C(t=\infty) = \dfrac{V R_2}{R_1+R_2}$

Problem 3: Inductor Dynamics - 15%

The circuit shown below comprises two sources, two resistors and one inductor. Assume that prior to time $t = 0$ the conditions constant $V \neq 0$ and $I = 0$ have been established for a very long time such that the circuit operates in steady state by $t = 0$.



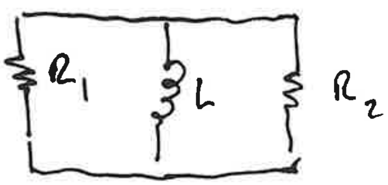
(3A) Determine i_L at $t = 0$.

$i_L(0)$: $\frac{V}{R_1}$

Steady state $\Rightarrow$ $L \frac{di_L}{dt} = v_L = 0$ $\Rightarrow$ $i_L = \frac{V}{R_1}$

(3B) At $t = 0$, the current source steps to a constant such that $I \neq 0$. Determine $i_L(t)$ for $t \geq 0$.

$$\boxed{i_L(t \geq 0):\quad \underbrace{\frac{V}{R_1} e^{-t/\tau}}_{\text{Initial condition decays away}} + \underbrace{\left(I + \frac{V}{R_1}\right)\left(1 - e^{-t/\tau}\right)}_{\text{Final condition appears}} \;;\quad \tau = \frac{L(R_1+R_2)}{R_1 R_2}}$$



$\Rightarrow \frac{L}{R}$ Time constant $\tau = \frac{L(R_1+R_2)}{R_1 R_2}$

Final $i_L(t=\infty) = I + \frac{V}{R_1}$

Note that $i_L(t \geq 0) = \underbrace{I + \frac{V}{R_1}}_{\text{Particular}} - \underbrace{I e^{-t/\tau}}_{\text{Homogeneous}}$

(3C) At $t = T$ with $T > 0$, the voltage source steps such that $V = 0$. Determine $v_C(t)$ for $t \geq T$.

$i_L(t \geq T)$: $\left[I + \frac{V}{R_1} - I e^{-T/\tau}\right] e^{-\frac{(t-T)}{\tau}} + I\left(1 - e^{-\frac{(t-T)}{\tau}}\right)$

$\tau = L(R_1 + R_2)/R_1 R_2$

Initial Condition $i_L(t = T) = I + \frac{V}{R_1} - I e^{-T/\tau}$

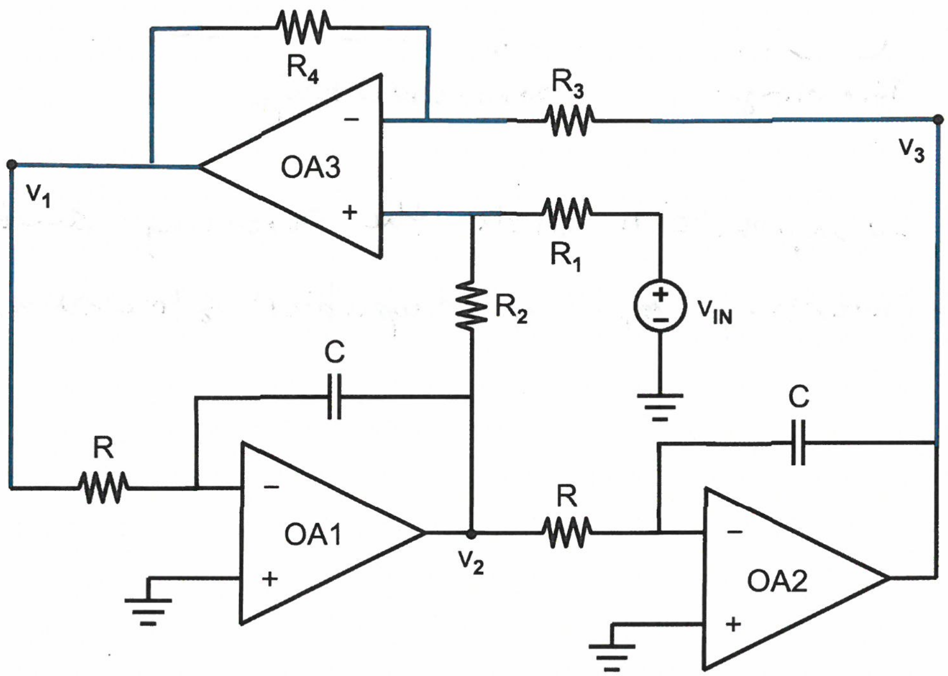
Final $i_L(t = \infty) = I$

Problem 4: Analog Computer – 40%

The circuit shown below can function as an analog computer, solving the second-order differential equation

$$A_2 d^2x/dt^2 + A_1 dx/dt + A_0 x = v_{\text{in}} \qquad (1)$$

Assume its op-amps are ideal.



(4A) Considering the operation of op amps OA1 and OA2, determine both v_2 and v_3 in terms of v_1 and the circuit parameters.

v_2: $-\frac{1}{RC}\int_{-\infty}^{t} v_1(t')\,dt'$	v_3: $\frac{1}{R^2C^2}\int_{-\infty}^{t}\int_{-\infty}^{t'} v_1(t'')\,dt''\,dt'$

Note that reversing these results $\Rightarrow$

$$-RC\frac{dv_2}{dt} = v_1 \; ; \; R^2C^2\frac{d^2v_3}{dt^2} = v_1 \; ; \; -RC\frac{dv_3}{dt} = v_2$$

(4B) Considering the operation of op amp OA3, determine v_1 in terms of v_2, v_3, and v_{IN}.

$$v_1: \quad \underbrace{-\frac{R_4}{R_3} v_3}_{\text{Inverting}} + \underbrace{\frac{R_4+R_3}{R_3}\left(\frac{R_1 v_2 + R_2 v_{IN}}{R_1+R_2}\right)}_{\text{Non-inverting}}$$

Use superposition with the inverting and non-inverting op-amp amplifier structures

(4C) Which of the voltages v_1, v_2, and v_3 should correspond to x in the differential equation? Briefly explain your reasoning.

Selected voltage:	v_3
Explanation:	Assigning x to v_3 then yields $v_3 \sim x$, $v_2 \sim \frac{dx}{dt}$, and $v_1 \sim \frac{d^2x}{dt^2}$

(4D) Following your answer to Part (4C), which matches a voltage to x, match the operation of the circuit to the differential equation above. In doing so determine A_0, A_1, and A_2 in terms of the circuit parameters.

A_0:	$\frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_2}$
A_1:	$RC \frac{R_1}{R_2}$
A_2:	$R^2C^2 \frac{R_3}{R_3+R_4} \frac{R_1+R_2}{R_2}$

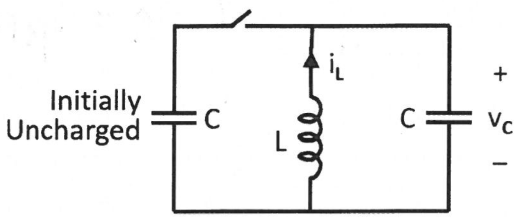
From (4B), $v_1 - \frac{R_4+R_3}{R_3} \cdot \frac{R_1}{R_1+R_2} v_2 + \frac{R_4}{R_3} v_3 = \frac{R_4+R_3}{R_3} \cdot \frac{R_2}{R_1+R_2} v_{IN}$

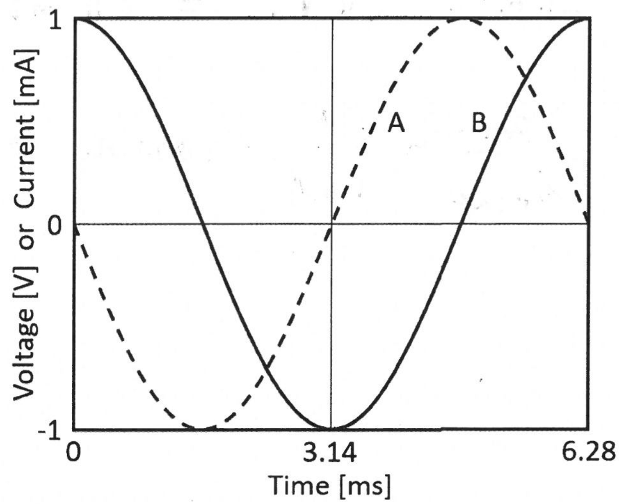
$$v_1 \leftarrow R^2C^2\frac{d^2x}{dt^2}, \quad v_2 \leftarrow -RC\frac{dx}{dt}, \quad v_3 \leftarrow x$$

So, $R^2C^2 \frac{R_3}{R_4+R_3} \frac{R_1+R_2}{R_2} \frac{d^2x}{dt^2} + RC\frac{R_1}{R_2}\frac{dx}{dt} + \frac{R_4}{R_3+R_4}\frac{R_1+R_2}{R_2} x = v_{IN}$

Problem 5: LC Oscillations – 24%

Consider the circuit shown below comprising two capacitors, one inductor, and one switch. The switch is initially open thereby disconnecting one capacitor from the remainder of the circuit. The disconnected capacitor is uncharged. The graph below displays two sinusoidal waveforms, labeled "A" and "B". One waveform shows the capacitor voltage v_C as a function of time while the other waveform shows the inductor current i_L. Note that the vertical axis is accordingly labeled in both voltage and current.





(5A) Which waveform corresponds to the capacitor voltage v_C? (The other waveform therefore corresponds to the inductor current i_L.) Circle the appropriate answer and provide a brief explanation.

v_C waveform:	A	B

Explanation:

$$v_C = -L \frac{di_L}{dt}$$

↑ cos (Dark)

– sin (Dashed)

(5B) Determine values for both C and L.

C: 1 μF	L: 1 H

Period $2\pi\sqrt{LC} = 2\pi$ ms $\Rightarrow \sqrt{LC} = 10^{-3}$ s

$$\frac{v_{Peak}}{i_{Peak}} = \frac{1V}{1\,mA} = 1000\,\Omega \Rightarrow \sqrt{\frac{L}{C}} = 1000\ A$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} L = 1H \\ C = 1\mu F \end{matrix}$$

(5C) Determine the total energy in the system.

Total energy: 0.5 μJ

$$\frac{1}{2} C V_{Pk}^2 = \frac{10^{-6}}{2} J$$

$$\frac{1}{2} L i_{Pk}^2 = \frac{10^{-6}}{2} J$$

(5D) Assume that the switch closes at a point in time at which $v_C = 0$. Determine values for the peak v_C, the peak i_L, and the period of oscillation after that time.

New peak v_C:	$\frac{1}{\sqrt{2}}$ V
New peak i_L:	1 mA
New period:	$\sqrt{2} \times 6.28$ ms

Closing the switch when $v_c = 0$ double the system capacitance while adding no new energy to the system. Thus, the peak i_L remains the same, the new voltage reduces to $\frac{1}{\sqrt{2}}$ V, and the new period increases to $\sqrt{2} \times 6.28$ ms.